Key 100

Physics 234 Exam # 2, Wed. 4/9/03

Name:______________________________

Section A. Circle one of the answers (4 points each).

1. The speed of ultraviolet radiation in free space compared to the speed of visible light in free space is

(a) much greater (b) a little greater (c) the same (d) a little less

(e) much less

V is independent of λ
$V = c/n$

2. A ray strikes a slab of glass (n=1.55) at an angle of incidence of 35°. The angle of refraction is

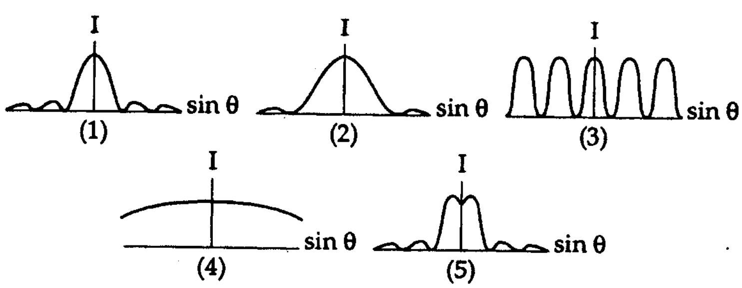
(a) 22° (b) 63° (c) 27° (d) 90° (e) none of these

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1}{1.55}\sin(35°)\right) = 21.7°$

3. Light of wavelength 650 nm is incident on a slit of width 25.0 μm. At what angle is the second diffraction minimum observed

(a) 0.052° (b) 1.5° (c) 2.2° (d) 3.0° (e) 3.7°

$a\sin\theta = m\lambda \quad \theta = \sin^{-1}\left(\frac{2\lambda}{a}\right) = \sin^{-1}\left(\frac{2(0.650)}{25}\right) = 2.98°$



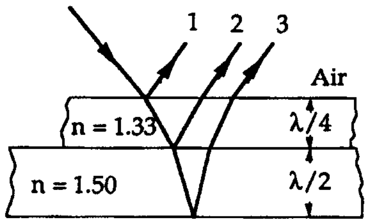
4. The graph which represents the widest single slit is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

$a\sin\theta = m\lambda$ first min
$a\sin\theta = \lambda$ fixed λ → small a ⇔ big θ

16

Section B. Circle one of the answers (6 points each).



$n_2 > n_1$ π phase change
$n_3 > n_2$ π phase change
$n_4 < n_3$ 0 phase change

5. light of wavelength λ is incident on two thin films that are in contact and surrounded by air. The top layer is λ/4 thick and has n = 1.33 and the bottom layer is λ/2 has n = 1.50. At normal incidence, the reflected rays which are in phase with each other are

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3 (e) none of these

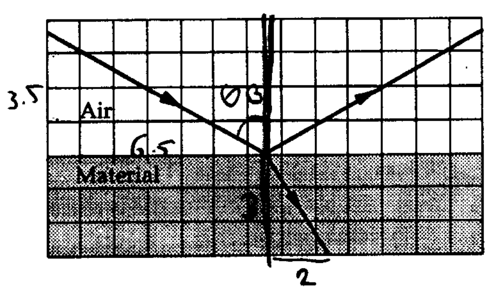
$\pi \to \frac{\lambda}{2}$

$\pi + \frac{2L}{\lambda/n_2}2\pi = \pi + \frac{2\lambda/4}{\lambda/n_2}2\pi = \pi + \pi n_2 = \pi(2.33)$

$0 + \left(\frac{2L}{\lambda n_2} + \frac{2L_2}{\lambda n_1}\right)2\pi$

$2\pi\left(\frac{2\lambda/4\, n_2}{\lambda} + \frac{2\lambda/2\, n_3}{\lambda}\right)$

$= \pi(n_2 + 2n_3)$ 2.66 + 3.00 5.66π



6. A ray of light is shown being reflected at the surface of a material. If the reflected ray is completely plane polarized the index of refraction of the material is approximately

(a) 1.3 (b) 1.9 (c) 0.63 (d) 0.8 (e) 1.5

$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$ $\tan\theta_B = \frac{6.5}{3.5} = \frac{n_2}{n_1} \Rightarrow n_2 = n_1 \tan\theta_B$

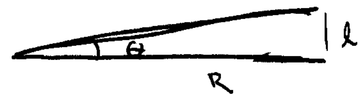
$n_2 = n_1\left(\frac{6.5}{3.5}\right) \approx 1.86$

12

7. The headlights of an oncoming car are 1.2 m apart. What is the maximum distance from the car at which you can resolve the lights as two independent sources if diameter of your pupil is 5 mm and wavelength of the light is 555 nm?

(a) 8.9 km (b) 22 km (c) 4.4 km (d) 5.4 km (e) 13 km

$\theta_R = 1.22 \frac{\lambda}{d}$

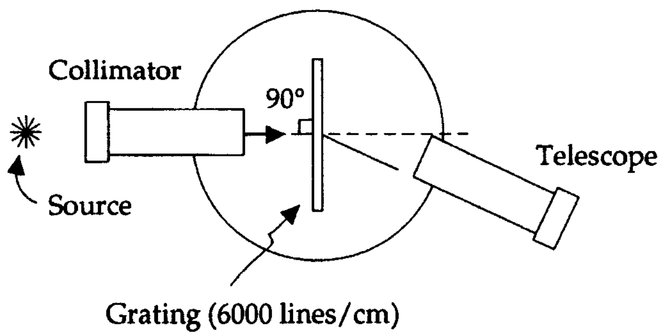


$l = R\theta = R\, 1.22 \frac{\lambda}{d}$

$R = \frac{l d}{1.22\lambda} = \frac{(1.2m)\, 5\times10^{-3}m}{1.22 \quad 550\times10^{-9}m}$

$= 8.9km * \frac{3}{5} = 5.3\ m$

1 cm has



8. The angle at which the telescope must be located to observe the second order image of light of wavelength 589.3 nm is approximately

(a) 18° (b) 45° (c) 75° (d) 85° (e) 37°

$d \sin\theta = m\lambda$

1 cm has 6000 lines

⇓

$d = \frac{1cm}{6000} = 1.667\times10^{-6}\ m$

$\theta = \sin^{-1}\left(\frac{2(0.5893)}{1.667}\right)$

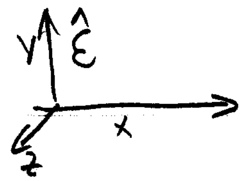
$= 44.9$

12

9. (15 pts.) A plane electromagnetic wave, with wavelength 3.0 m, travels in vacuum in the positive x direction with its electric field $\vec{E}$, of amplitude 300 V/m, directed along the y axis. (a) What is the frequency f of the wave? (b) What are the direction and amplitude of the magnetic field associated with the wave? (c) What are the values of k and ω if $E = E_m \sin(kx - \omega t)$? (d) What is the time-averaged rate of energy flow in watts per square meter associated with this wave? (e) If the wave falls on a perfectly absorbing sheet of area 2.0 m^2, at what rate is momentum delivered to the sheet and what is the radiation pressure exerted on the sheet?

10. (15 pts.) A beam of polarized light is sent through a system of two polarizing sheets. Relative to the polarization direction of that incident light, the polarizing directions of the sheets are at angles θ for the first sheet and 90° for the second sheet. If 0.10 of the incident intensity is transmitted by the two sheets, what is θ?

$\vec{S} \propto \vec{E}\times\vec{B}$



9

34-25 (a) $f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\ m/s}{3.0\ m} = 1.0\times10^{8}\ hz$ 3

(b) $B \parallel \hat{z}$ $\vec{S} \propto \vec{E}\times\vec{B}$ 3

15 $B = \frac{E_m}{c} = \frac{300\ V/m}{3.0\times10^{8}} = 1.0\times10^{-6}\ T$

(c) $-k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0m} = 2.1$ rads 3

$-\omega = 2\pi f = 2\pi\times10^{8}$ rad/sec $= 6.3\times10^{8}$ rads/s

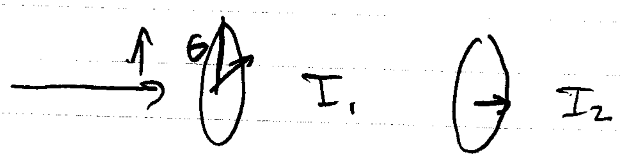
(d) $I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\ V/m)^2}{2(4\pi\times10^{-7}\ H/m)(3.0\times10^{8}\ m/s)} = 119\ W/m^2$ 3

=

(e) $\frac{dp}{dt} = \frac{IA}{c} = \frac{119\ W/m^2\ (2m^2)}{3.0\times10^{8}} = 8.0\times10^{-7}\ N$ 3

Pressure $= \frac{8.0\times10^{-7}N}{2.0m^2} = 4.0\times10^{-7}\ Pa$

10



$I_1 = I_0 \cos^2\theta$

$I_2 = (I_0 \cos^2\theta) \cos^2(90-\theta)$

$\cos 90 \cos\theta - \sin 90 \sin\theta$

$I_0 \cos^2\theta \cos^2 90$

$\cos\theta \sin\theta = \frac{1}{2}\sin(2\theta)$

$I_2 = I_0 \cos^2\theta \sin^2\theta$

10

$I_2 = \frac{I_0}{4} \sin^2(2\theta)$

$\frac{I_2}{I_0} = \frac{1}{4}\sin^2(2\theta) = 0.10$

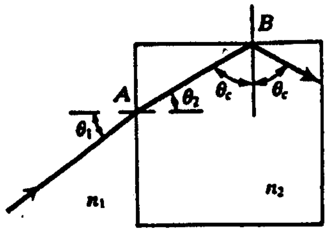
$2\theta = \sin^{-1}(\sqrt{0.40})$

$2\theta = \sin^{-1}(0.6324)$

$2\theta = 39.2^\circ$

$\theta = 19.6^\circ$

11. (15 pts.) Total internal reflection is used to keep light rays within a glass fiber. Suppose a light ray in air strikes the end of a fiber (n=1.31). What is the maximum value of θ_1 that will ensure that the ray undergoes total internal reflection at the point B in the fiber?



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_c = 90° - \theta_2$

$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ $n_1 \Rightarrow$ glass, $n_2 \rightarrow$ air

$\sin\theta_c = \frac{n_{air}}{n_{glass}} = \frac{n_1}{n_2}$ $\sin\theta_c = \sin(90° - \theta_2)$

$= \sin(90°)\cos\theta_2 - \cos(90)\sin(\theta_2)$

$= \cos\theta_2$

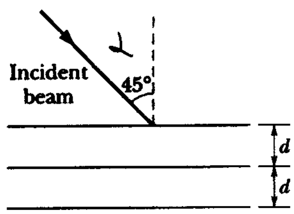
$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)$ $\sin\theta_2 = \sqrt{1+\cos^2\theta_2}$

$= \sqrt{1-\sin^2\theta_c}$

$= \sin^{-1}\left(\frac{n_2}{n_1}\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}\right)$

$= \sin^{-1}\left(\sqrt{\left(\frac{n_2}{n_1}\right)^2 - 1}\right)$

$= \sin^{-1}\left(\sqrt{1.31^2 - 1}\right) = 57.8°$



12. (15pts) Let a beam of x-rays of wavelength 0.125 nm be incident on an NaCl crystal as shown (d = 0.252 nm). Through what angles must the crystal be rotated about an axis normal to the plane of the page for these reflecting planes to give maxima

$2d \sin\theta = m\lambda$

$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{0.125\times10^{-9}\ m}{2(0.252\times10^{-9})}\right)$

$= \sin^{-1}(0.248\ m)$

$m_{min} = 1$

$m_{max} = \frac{1}{0.248} = 4.0$

$m = 1, 2, 3, 4$

← angle between plane and incident beam

$m=1 \rightarrow \theta = 14.36°$ $\alpha = 45 - \theta = 45 - 14.36 = 30.6°$ clockwise

$m=2 \rightarrow \theta = 29.7°$ $\alpha = 15°$ clockwise

$m=3 \rightarrow \theta = 48°$ $\alpha' = -3 \rightarrow +3$ counter clockwise

$m=4 \rightarrow \theta = 82.8°$ $\alpha'' = 37.8°$ counter clockwise

Method 1

$\theta_c = \sin^{-1}\left(\frac{n_2'}{n_1}\right) = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

$= \sin^{-1}\left(\frac{1}{1.31}\right) = 49.76°$

$\theta_2 = 90° - \theta_c = 40.238$

$\cos\theta_2 = \cos(40.238)$
$= 0.7634$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)$

$= \sin^{-1}\left(\frac{1.31}{n}\sin(40.238)\right)$

$= 57.8°$

Method 2 pure algebra

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_2 = 90° - \theta_c$ $\quad \theta_c = 49.76$

$\sin\theta_c = \frac{n_2'}{n_1'} = \frac{n_1}{n_2} = \frac{1}{1.31}$

$\cos\theta_2 = \cos 90° \cos\theta_c + \sin 90° \sin\theta_c = \sin\theta_c = \frac{n_1}{n_2} = \frac{1}{1.31} = 0.7633$

$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 = \frac{n_2}{n_1}\sqrt{1-\cos^2\theta_2}$ (0.6459)

$= \frac{n_2}{n_1}\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}$

$= \left(\frac{n_2}{n_1}\right)\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}$

$= \sqrt{\left(\frac{n_2}{n_1}\right)^2 - 1}$

$= \sqrt{1.31^2 - 1} = 0.846$

$\theta_1 =$

$\frac{n_2}{n_1}\sin(40.238)$

$1.31\sqrt{1-\left(\frac{1}{1.31}\right)^2}$

$\sqrt{1.31^2 - 1}$

0.846