# Chapter 13

Vibrations
and
Waves

# Hooke's Law

- $F_s = - k x$
  - $F_s$ is the spring force
  - k is the spring constant
    - It is a measure of the stiffness of the spring
      - A large k indicates a stiff spring and a small k indicates a soft spring
  - x is the displacement of the object from its equilibrium position
    - x = 0 at the equilibrium position
  - The negative sign indicates that the force is always directed opposite to the displacement

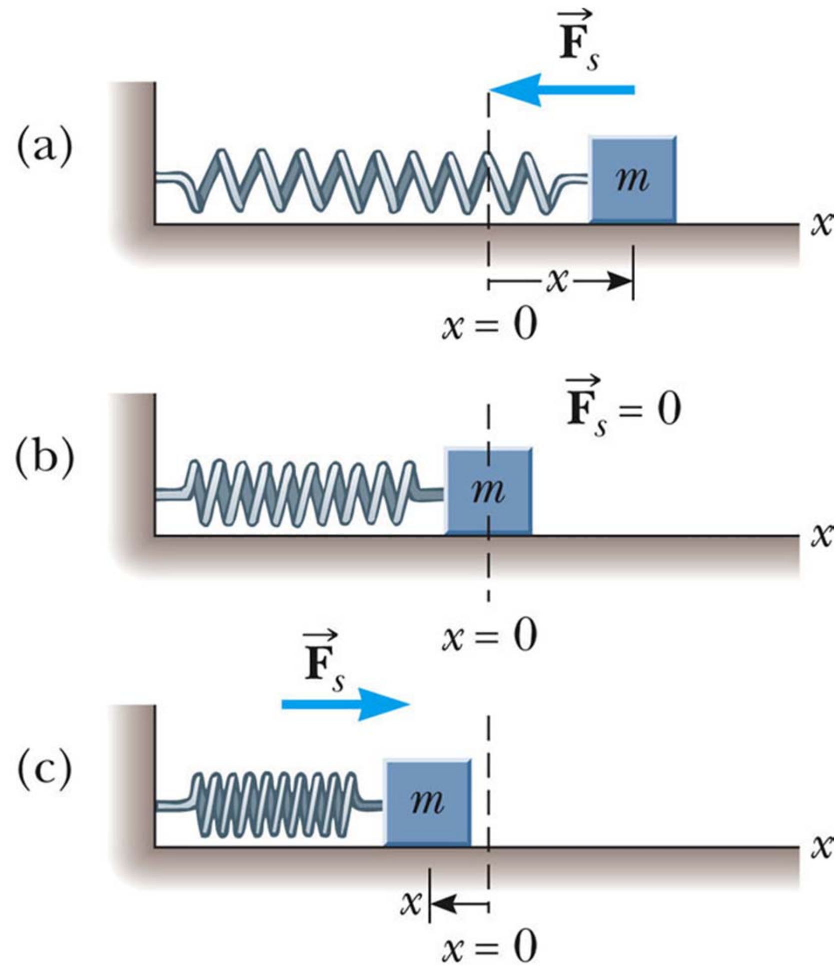
# Hooke's Law Force

- The force always acts toward the equilibrium position
  - It is called the *restoring force*
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

# Hooke's Law Applied to a Spring – Mass System

- When x is positive (to the right), F is negative (to the left)
- When x = 0 (at equilibrium), F is 0
- When x is negative (to the left), F is positive (to the right)

Please replace with active fig. 13.1



# Motion of the Spring-Mass System

- Assume the object is initially pulled to a distance A and released from rest
- As the object moves toward the equilibrium position, F and a decrease, but v increases
- At $x = 0$, F and a are zero, but v is a maximum
- The object's momentum causes it to overshoot the equilibrium position

# Motion of the Spring-Mass System, cont

- The force and acceleration start to increase in the opposite direction and velocity decreases
- The motion momentarily comes to a stop at $x = -A$
- It then accelerates back toward the equilibrium position
- The motion continues indefinitely

# Simple Harmonic Motion

- Motion that occurs when the net force along the direction of motion obeys Hooke's Law
  - The force is proportional to the displacement and always directed toward the equilibrium position
- The motion of a spring mass system is an example of Simple Harmonic Motion

# Simple Harmonic Motion, cont.

- Not all periodic motion over the same path can be considered Simple Harmonic motion
- To be Simple Harmonic motion, the force needs to obey Hooke’s Law

# Amplitude

- Amplitude, A
  - The amplitude is the maximum position of the object relative to the equilibrium position
  - In the absence of friction, an object in simple harmonic motion will oscillate between the positions $x = \pm A$

# Period and Frequency

- The period, T, is the time that it takes for the object to complete one complete cycle of motion
  - From x = A to x = - A and back to x = A
- The frequency, $f$, is the number of complete cycles or vibrations per unit time
  - $f = 1 / T$
  - Frequency is the reciprocal of the period

# Acceleration of an Object in Simple Harmonic Motion

- Newton's second law will relate force and acceleration
- The force is given by Hooke's Law
- $F = -k x = m a$
  - $a = -kx / m$
- The acceleration is a function of position
  - Acceleration is *not* constant and therefore the uniformly accelerated motion equation cannot be applied

# Example 1

A vertical spring stretches 9.6 cm when a 1.2 kg block is hung from its end. Calculate the spring constant of the spring.

# Position as a Function of Time

- If $F = - k x$ the equation of motion is given by

$$x = A \cos(\omega t)$$

  - $x$ is the position at time $t$
  - $x$ varies between $+A$ and $-A$
  - $\omega$– angular frequency of oscillations, units: radians per second [rad/s]
  - $\omega = 2\pi f$; $f$ – frequency of oscillations, units: $s^{-1}$ or Hertz [Hz] $f = 1/T$ , $T$ period of oscillations

# Motion Equations

- $x = A \cos \omega t$
- $v = = -A \omega \sin \omega t$
- $a = -A\omega^2 \cos \omega t = -\omega^2 x$
- $-kx = ma$, $-kx = -m\omega^2 x \quad k = m\omega^2$
- $\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$

# Example 2

A 0.35 kg mass vibrates according to the equation $x = 0.25 \text{ m} \cos (0.393 \text{rad/s})t$. Determine (a) the period, (b) the spring constant, (c) the maximum speed of the mass (d) the position, the velocity of the mass at $t=1$ s

# Example 3

A 1.6 kg piston in an automobile engine travels back and forth over a distance of 18 cm. Find the maximum velocity of the piston when the auto engine is running at the rate of 3600 rev/min.

# Elastic Potential Energy

- A compressed spring has potential energy
  - The compressed spring, when allowed to expand, can apply a force to an object
  - The potential energy of the spring can be transformed into kinetic energy of the object

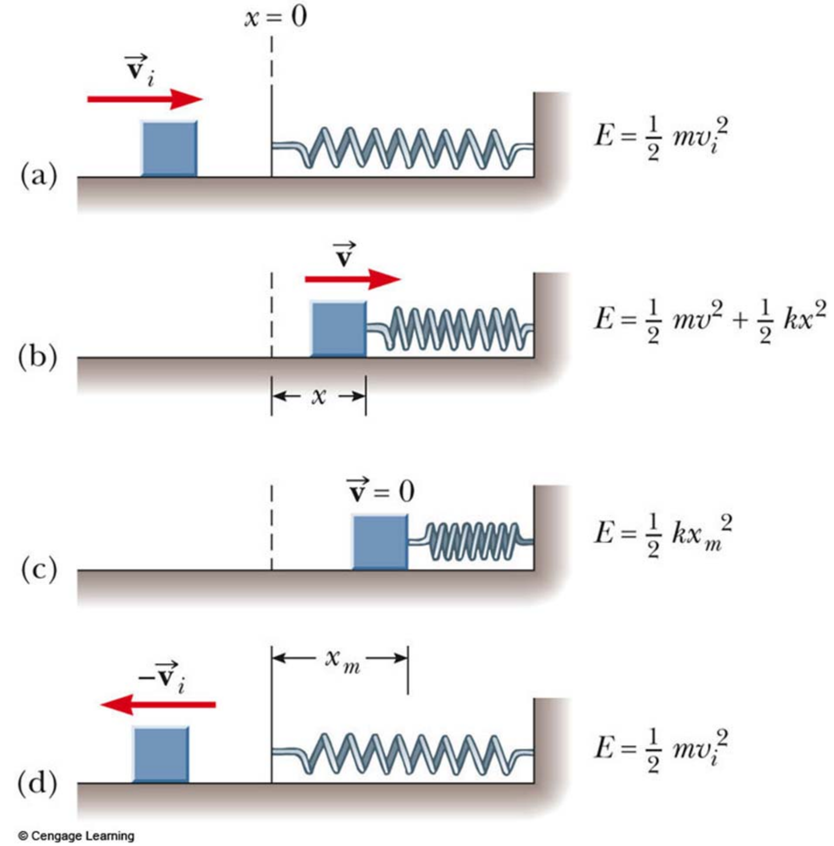
# Elastic Potential Energy, cont

- The energy stored in a stretched or compressed spring or other elastic material is called *elastic potential energy*
  - $PE_s = \frac{1}{2}kx^2$
- The energy is stored only when the spring is stretched or compressed
- Elastic potential energy can be added to the statements of Conservation of Energy and Work-Energy

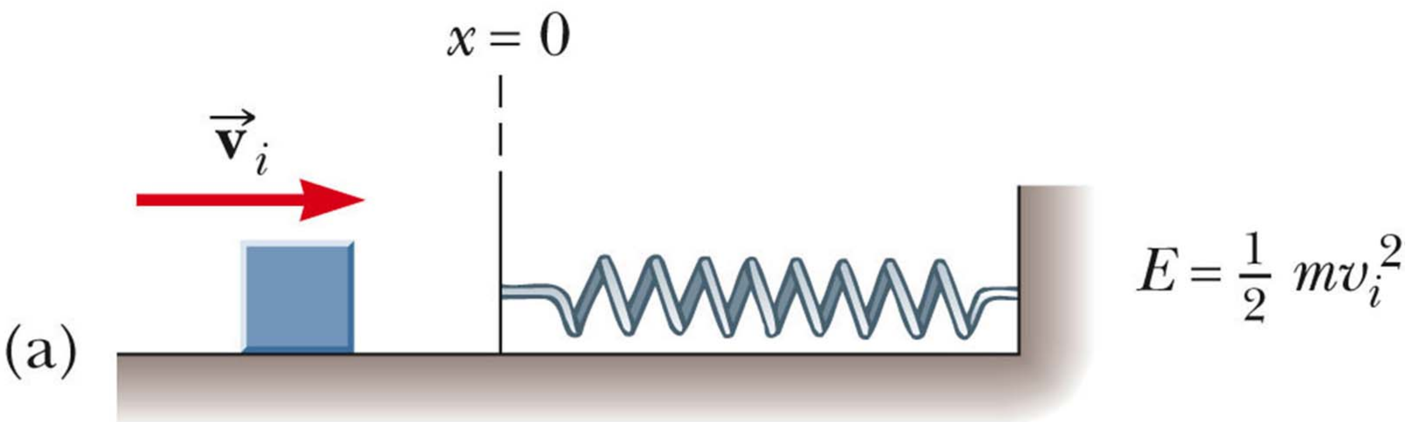
# Energy in a Spring Mass System

Please replace with fig. 13.4

- A block sliding on a frictionless system collides with a light spring
- The block attaches to the spring
- The system oscillates in Simple Harmonic Motion



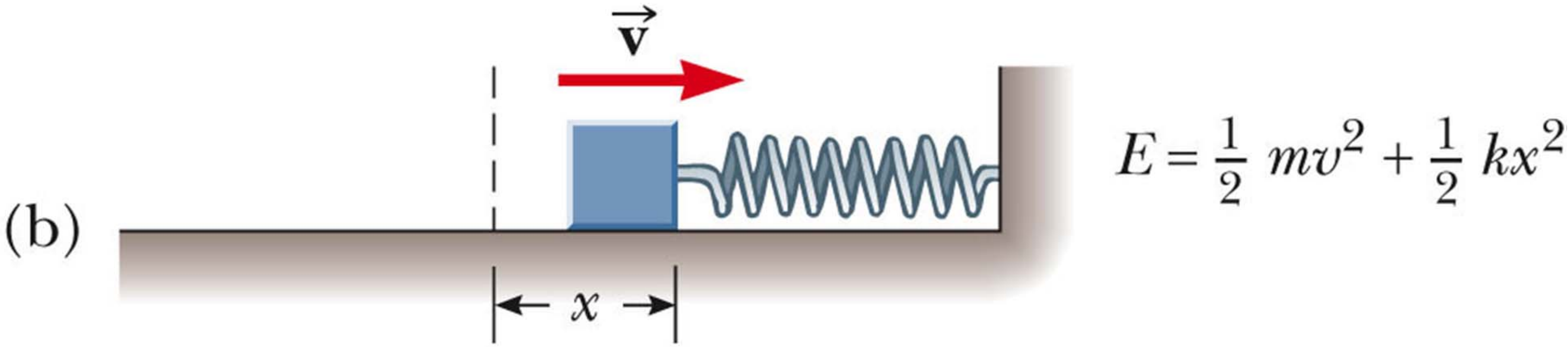
# Energy Transformations






- The block is moving on a frictionless surface
- The total mechanical energy of the system is the kinetic energy of the block

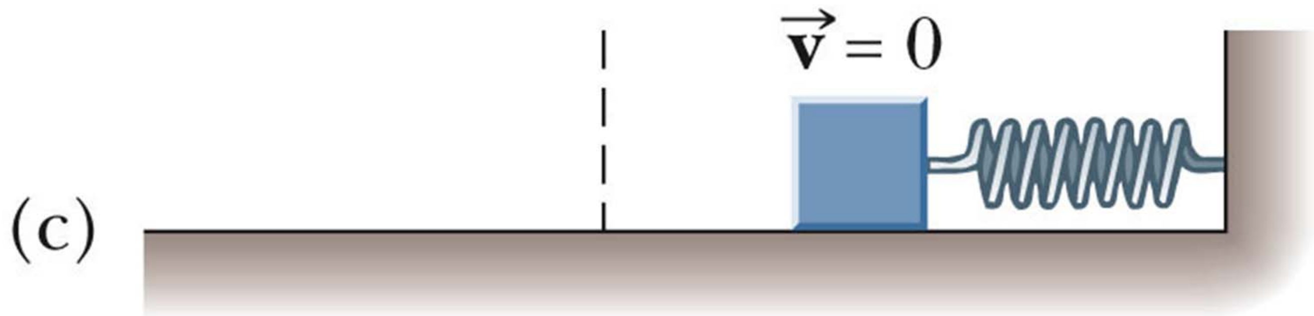
# Energy Transformations, 2






- The spring is partially compressed
- The energy is shared between kinetic energy and elastic potential energy
- The total mechanical energy is the sum of the kinetic energy and the elastic potential energy

# Energy Transformations, 3

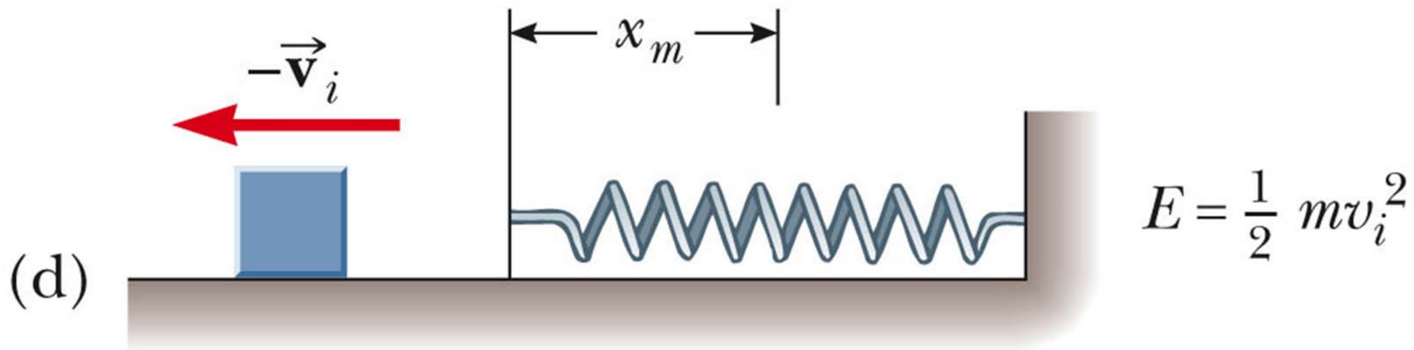



$$E = \frac{1}{2} k x_m^{\ 2}$$



- The spring is now fully compressed
- The block momentarily stops
- The total mechanical energy is stored as elastic potential energy of the spring

# Energy Transformations, 4






- When the block leaves the spring, the total mechanical energy is in the kinetic energy of the block
- The spring force is conservative and the total energy of the system remains constant

# Velocity as a Function of Position

- Conservation of Energy allows a calculation of the velocity of the object at any position in its motion

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

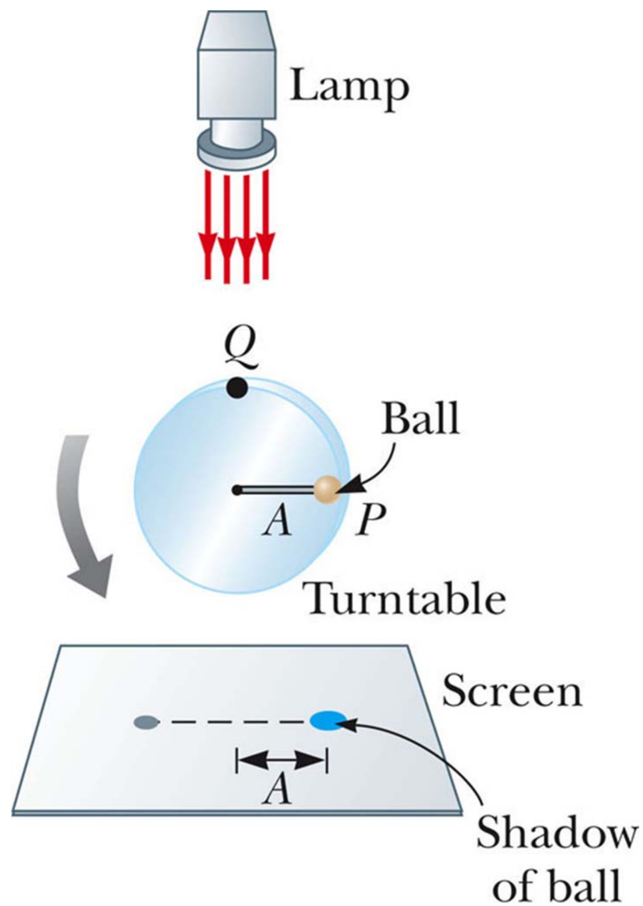
  - Speed is a maximum at x = 0
  - Speed is zero at x = ±A
  - The ± indicates the object can be traveling in either direction

# Example 4

A 0.5 kg object connected to a light spring with a spring constant of 20 N/m oscillates on a frictionless horizontal surface. (a) Find the period of oscillations. (b) Calculate the total energy of the system and maximum speed of the object if the amplitude of the motion is 3 cm. © What is the velocity of the object when the displacement is 2 cm.

# Simple Harmonic Motion and Uniform Circular Motion

- A ball is attached to the rim of a turntable of radius A
- The focus is on the shadow that the ball casts on the screen
- When the turntable rotates with a constant angular speed, the shadow moves in simple harmonic motion



# Period and Frequency from Circular Motion

- Period $T = 2\pi\sqrt{\dfrac{m}{k}}$
  - This gives the time required for an object of mass m attached to a spring of constant k to complete one cycle of its motion
- Frequency $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$
  - Units are cycles/second or Hertz, Hz

# Angular Frequency

- The angular frequency is related to the frequency

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

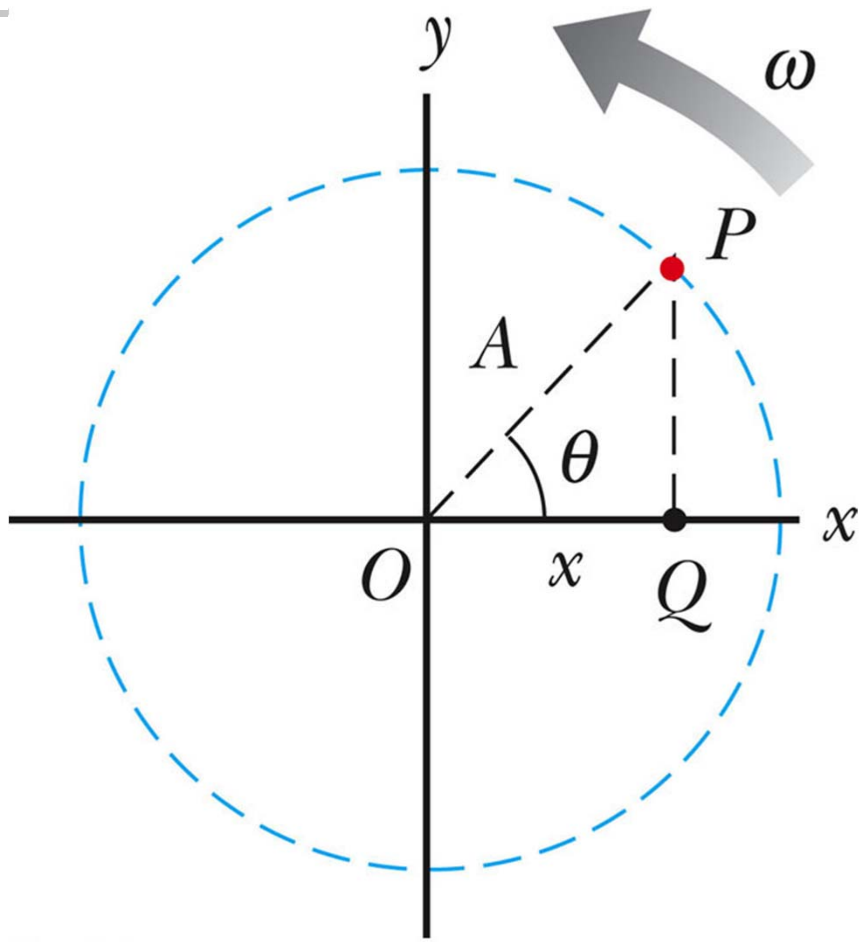
- The *frequency* gives the number of cycles per second
- The *angular frequency* gives the number of radians per second

# Effective Spring Mass

- A graph of $T^2$ versus m does not pass through the origin
- The spring has mass and oscillates
- For a cylindrical spring, the *effective* additional mass of a light spring is 1/3 the mass of the spring

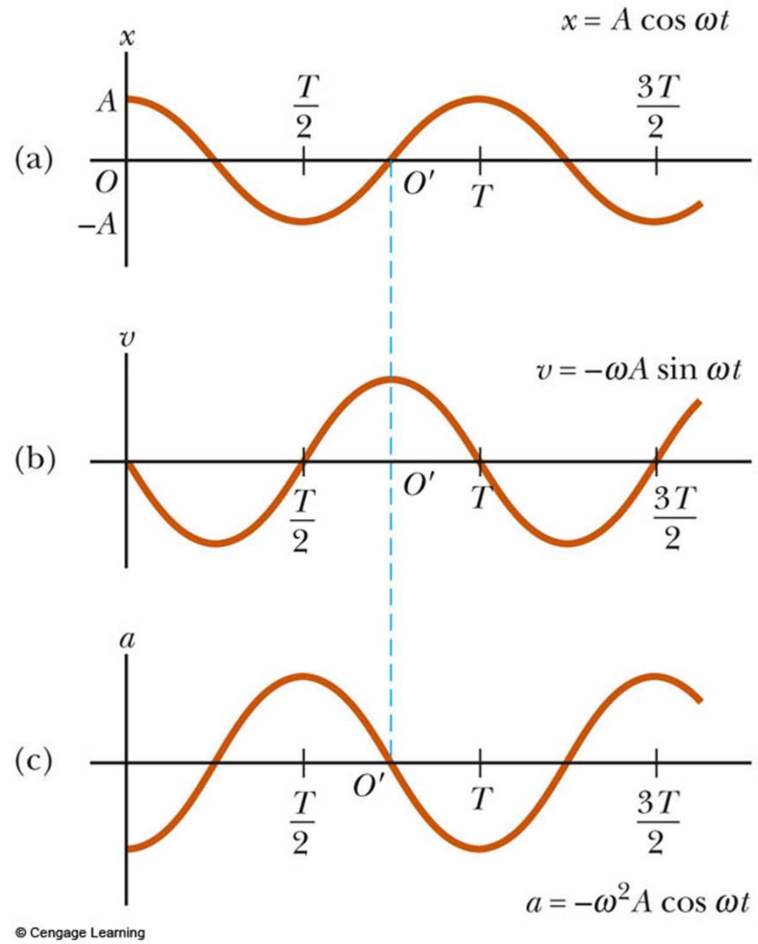
# Motion as a Function of Time

- Use of a *reference circle* allows a description of the motion
- $x = A \cos(2\pi f t)$
  - x is the position at time t
  - x varies between +A and -A



# Graphical Representation of Motion

- When x is a maximum or minimum, velocity is zero
- When x is zero, the velocity is a maximum
- When x is a maximum in the positive direction, a is a maximum in the negative direction



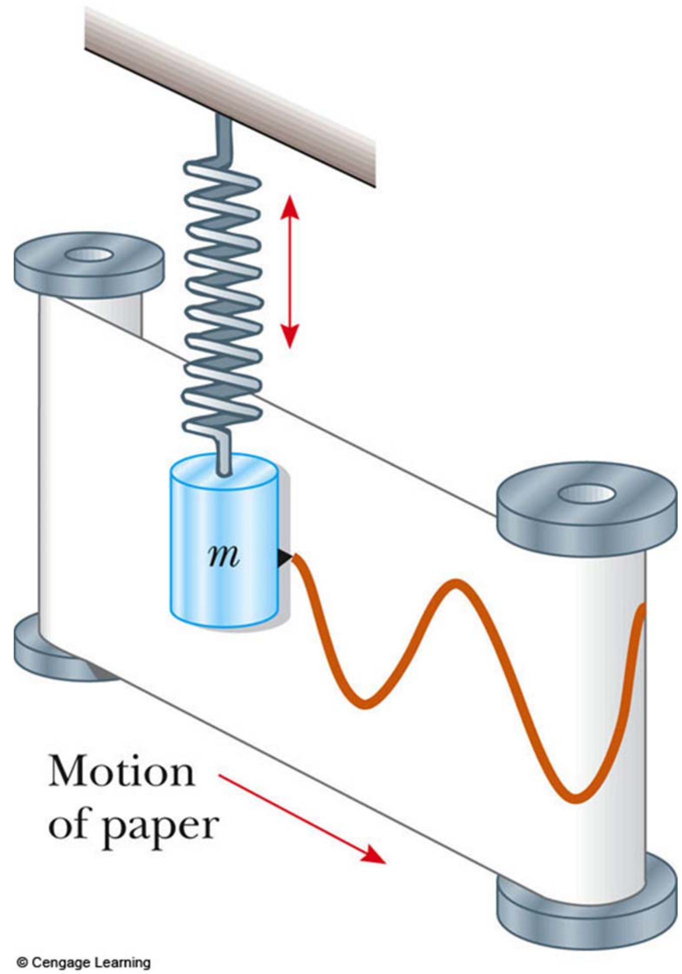
# Motion Equations

- Remember, the uniformly accelerated motion equations cannot be used
- $x = A \cos(2\pi f t) = A \cos \omega t$
- $v = -2\pi f A \sin(2\pi f t) = -A\,\omega \sin \omega t$
- $a = -4\pi^2 f^2 A \cos(2\pi f t) =$

  $-A\omega^2 \cos \omega t$

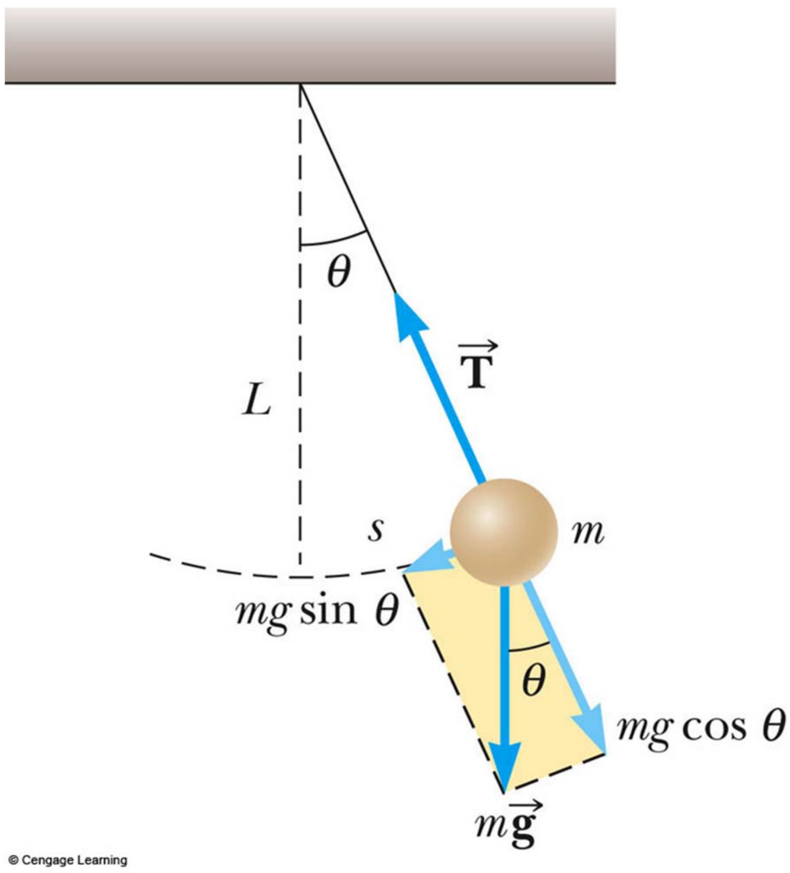
# Verification of Sinusoidal Nature

- This experiment shows the sinusoidal nature of simple harmonic motion
- The spring mass system oscillates in simple harmonic motion
- The attached pen traces out the sinusoidal motion



# Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion
  - $F_t = - mg \sin \theta$



# Simple Pendulum, cont

- In general, the motion of a pendulum is not simple harmonic
- However, for small angles, it becomes simple harmonic
  - In general, angles < 15° are small enough
  - $\sin \theta = \theta$
  - $F_t = - mg\,\theta$
    - This force obeys Hooke's Law

# Period of Simple Pendulum

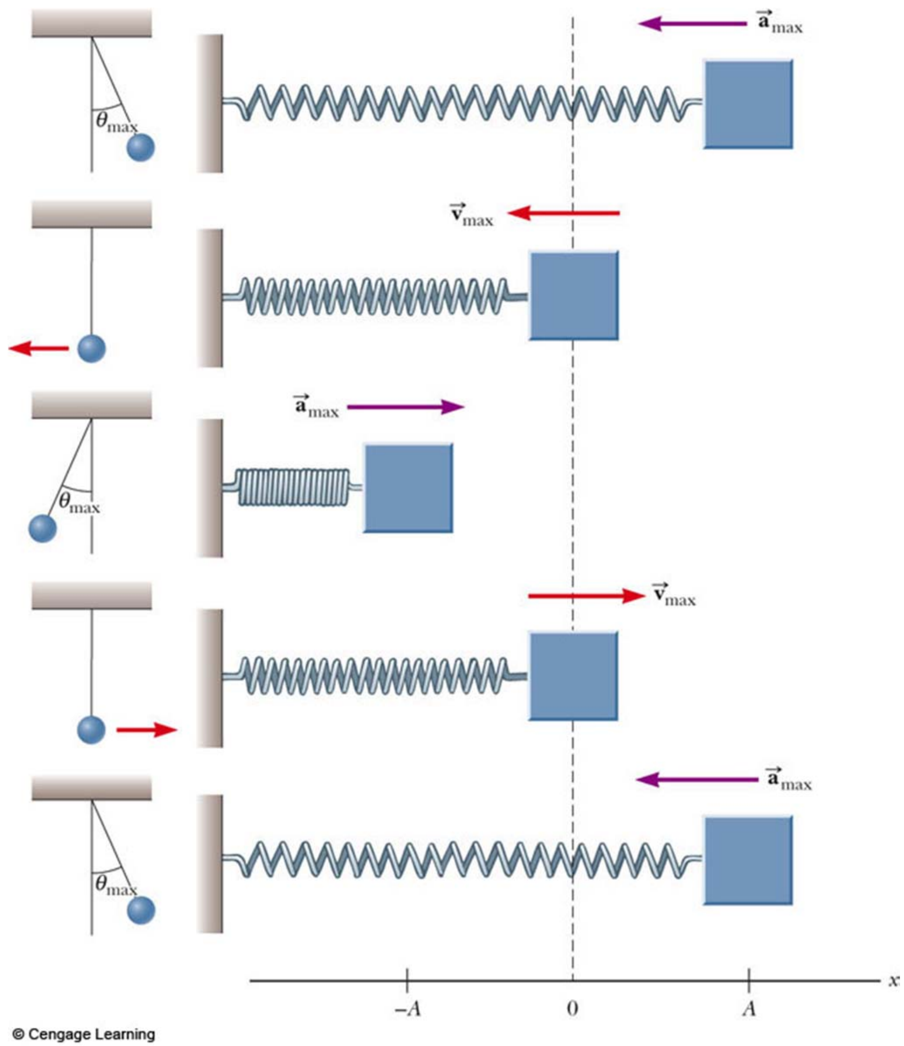
$$T = 2\pi\sqrt{\frac{L}{g}}$$

- This shows that the period is independent of the amplitude
- The period depends on the length of the pendulum and the acceleration of gravity at the location of the pendulum

# Example 5

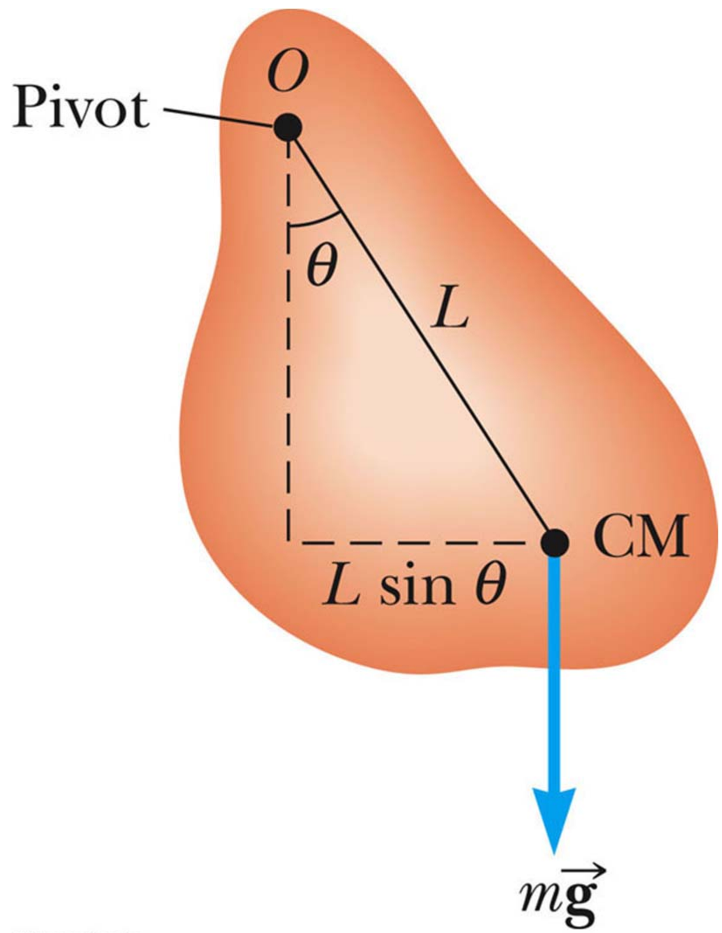
Using a small pendulum of length 0.171 m, a geophysicist counts 72 complete swings in 60 s. What is the value g in this location?

# Simple Pendulum Compared to a Spring-Mass System



# Physical Pendulum

- A physical pendulum can be made from an object of any shape
- The center of mass oscillates along a circular arc



# Period of a Physical Pendulum

- The period of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

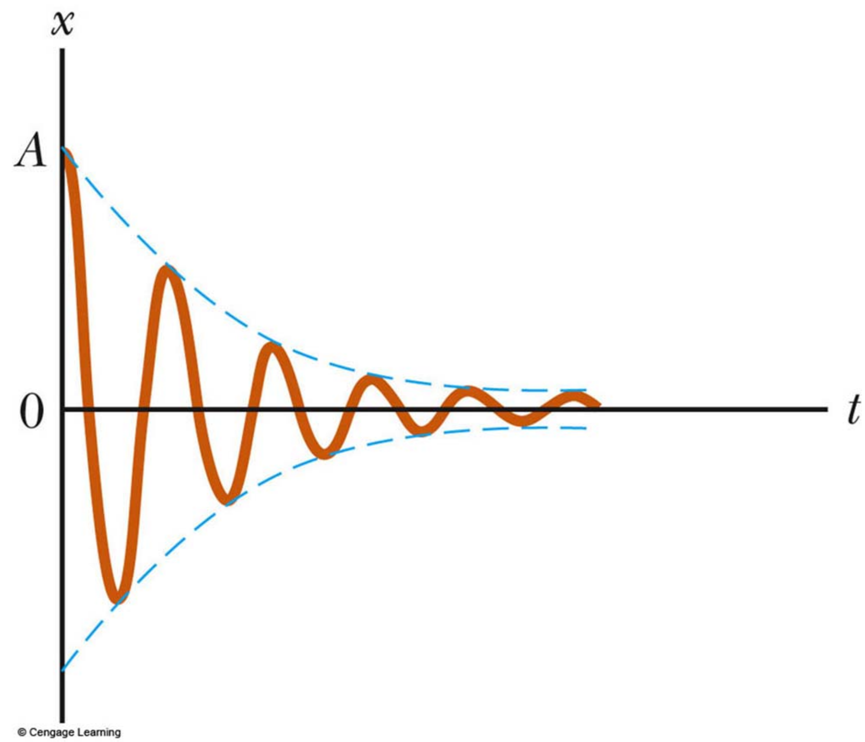
  - I is the object’s moment of inertia
  - m is the object’s mass
- For a simple pendulum, $I = mL^2$ and the equation becomes that of the simple pendulum as seen before

# Damped Oscillations

- Only ideal systems oscillate indefinitely
- In real systems, friction retards the motion
- Friction reduces the total energy of the system and the oscillation is said to be *damped*

# Damped Oscillations, cont.

- Damped motion varies depending on the fluid used
  - With a low viscosity fluid, the vibrating motion is preserved, but the amplitude of vibration decreases in time and the motion ultimately ceases
    - This is known as *underdamped* oscillation

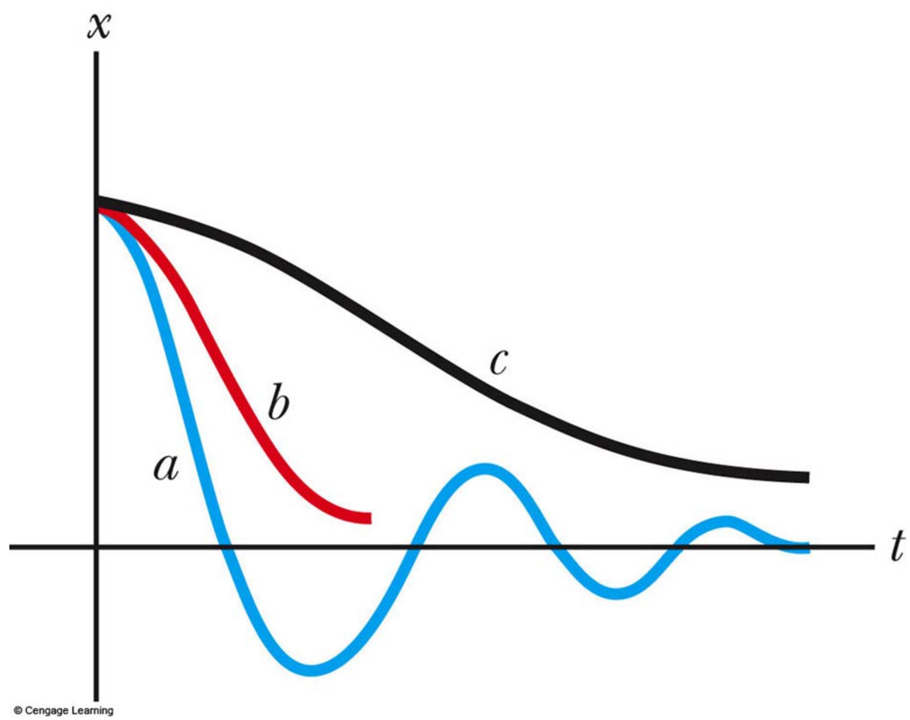


# More Types of Damping

- With a higher viscosity, the object returns rapidly to equilibrium after it is released and does not oscillate
  - The system is said to be *critically damped*
- With an even higher viscosity, the piston returns to equilibrium without passing through the equilibrium position, but the time required is longer
  - This is said to be *overdamped*

# Graphs of Damped Oscillators

- Plot a shows an underdamped oscillator
- Plot b shows a critically damped oscillator
- Plot c shows an overdamped oscillator

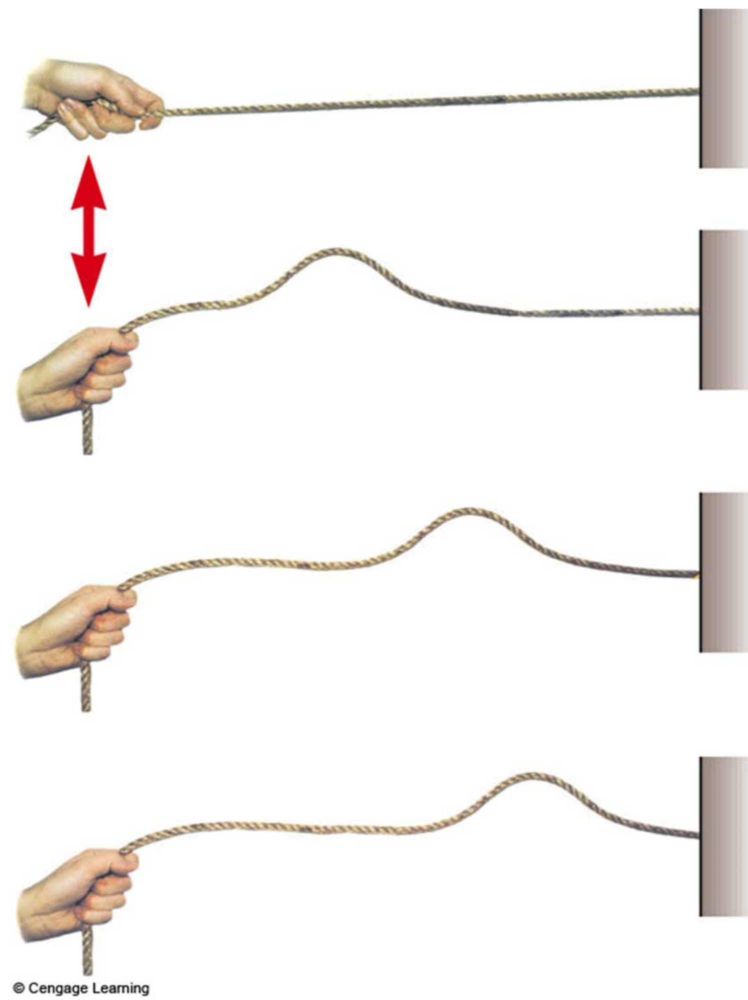


# Wave Motion

- A wave is the motion of a disturbance
- Mechanical waves require
  - Some source of disturbance
  - A medium that can be disturbed
  - Some physical connection between or mechanism though which adjacent portions of the medium influence each other
- All waves carry energy and momentum

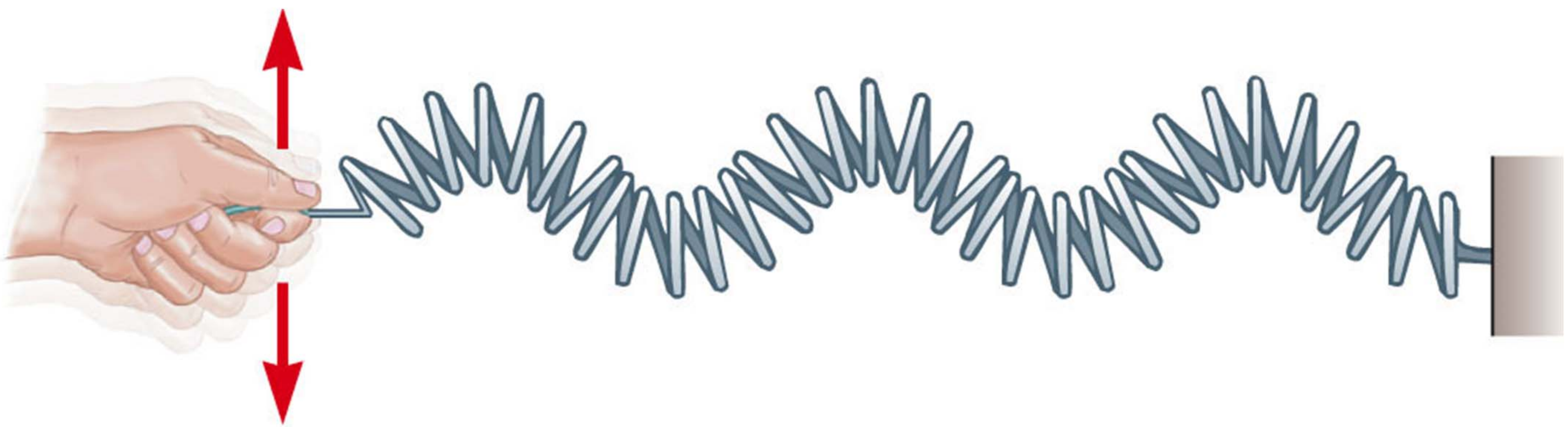
# Types of Waves – Traveling Waves

- Flip one end of a long rope that is under tension and fixed at one end
- The pulse travels to the right with a definite speed
- A disturbance of this type is called a *traveling wave*



# Types of Waves – Transverse

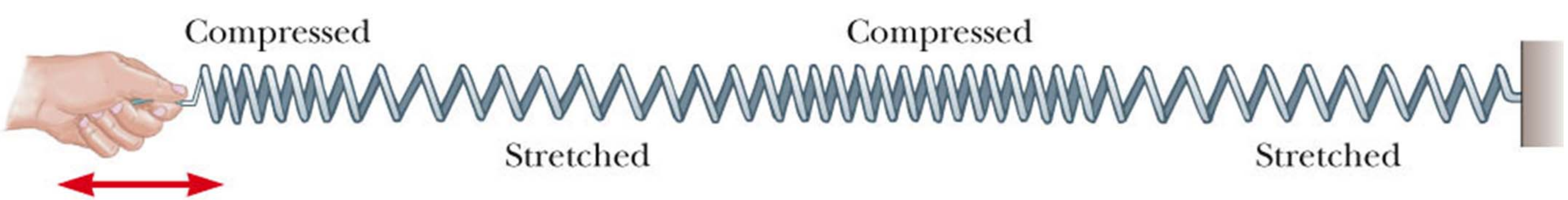
- In a transverse wave, each element that is disturbed moves in a direction perpendicular to the wave motion



(a) Transverse wave

# Types of Waves – Longitudinal

- In a longitudinal wave, the elements of the medium undergo displacements parallel to the motion of the wave
- A longitudinal wave is also called a compression wave



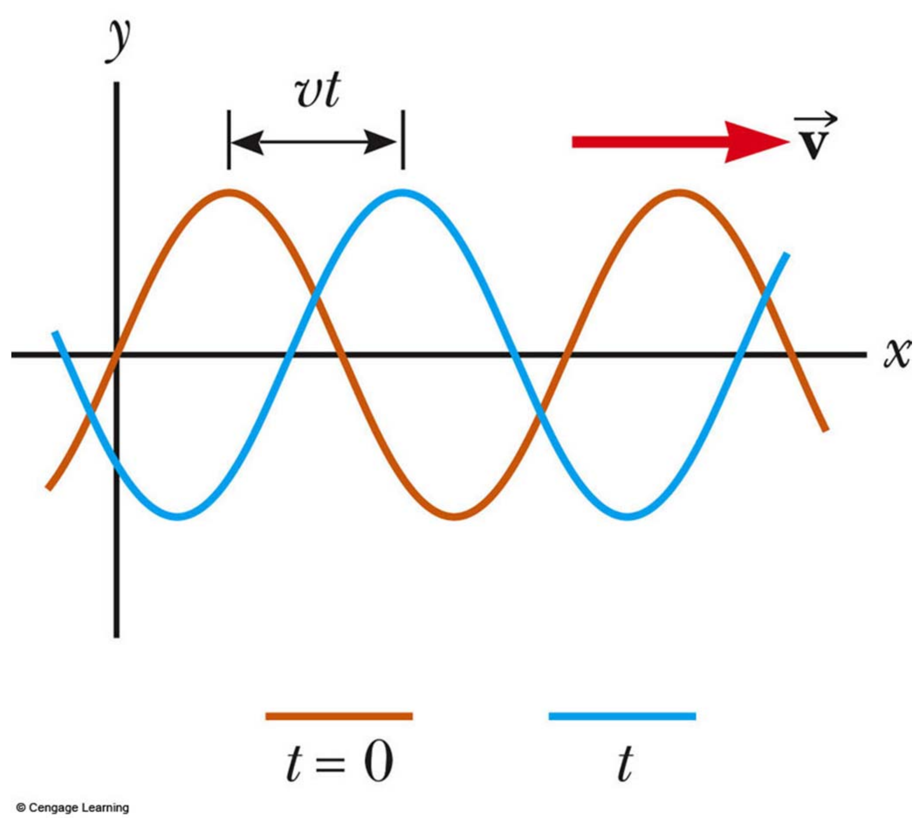

(b) Longitudinal wave

# Other Types of Waves

- Waves may be a combination of transverse and longitudinal
- A **soliton** consists of a solitary wave front that propagates in isolation
  - First studied by John Scott Russell in 1849
  - Now used widely to model physical phenomena

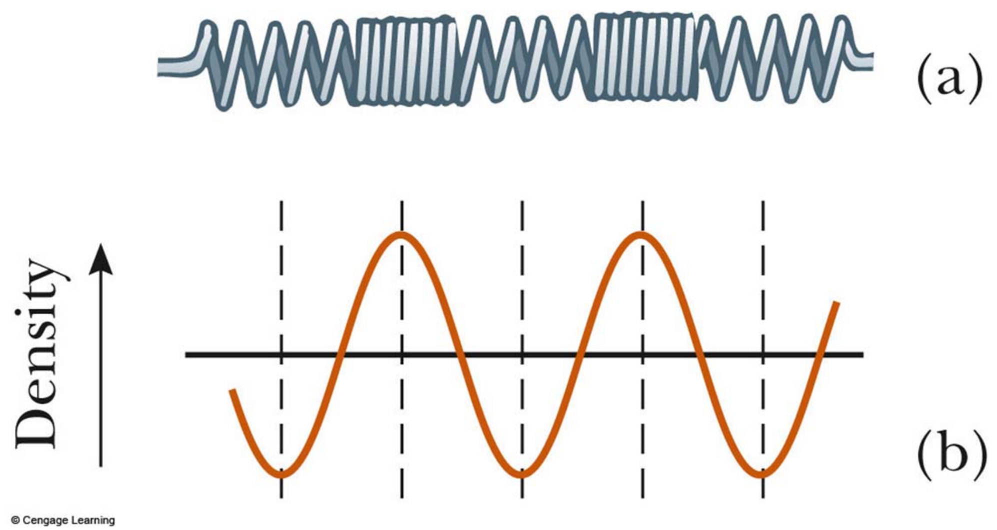
# Waveform – A Picture of a Wave

- The brown curve is a "snapshot" of the wave at some instant in time
- The blue curve is later in time
- The high points are *crests* of the wave
- The low points are *troughs* of the wave



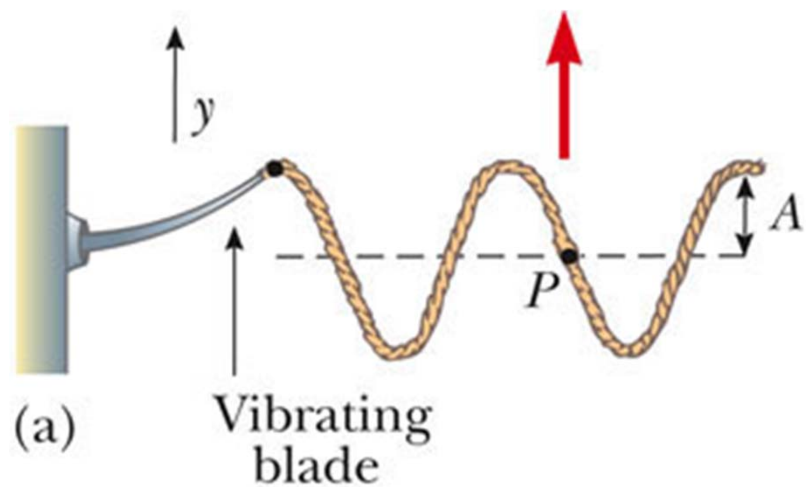
# Longitudinal Wave Represented as a Sine Curve

- A longitudinal wave can also be represented as a sine curve
- Compressions correspond to crests and stretches correspond to troughs
- Also called density waves or pressure waves



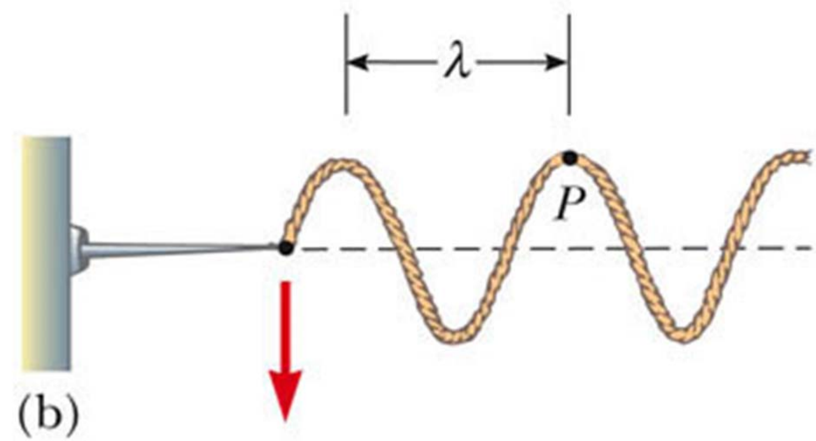
# Description of a Wave

- A steady stream of pulses on a very long string produces a continuous wave
- The blade oscillates in simple harmonic motion
- Each small segment of the string, such as P, oscillates with simple harmonic motion



# Amplitude and Wavelength

- Amplitude is the maximum displacement of string above the equilibrium position
- Wavelength, λ, is the distance between two successive points that behave identically



# Speed of a Wave

- $v = f\lambda$
  - Is derived from the basic speed equation of distance/time
- This is a general equation that can be applied to many types of waves

# Speed of a Wave on a String

- The speed on a wave stretched under some tension, F

$$v = \sqrt{\frac{F}{\mu}} \quad where \quad \mu = \frac{m}{L}$$

  - $\mu$ is called the linear density
- The speed depends only upon the properties of the medium through which the disturbance travels

# Example

What is the speed of a transverse wave in a rope of length 2 m and mass 60 g under a tension of 500 N?

# Example

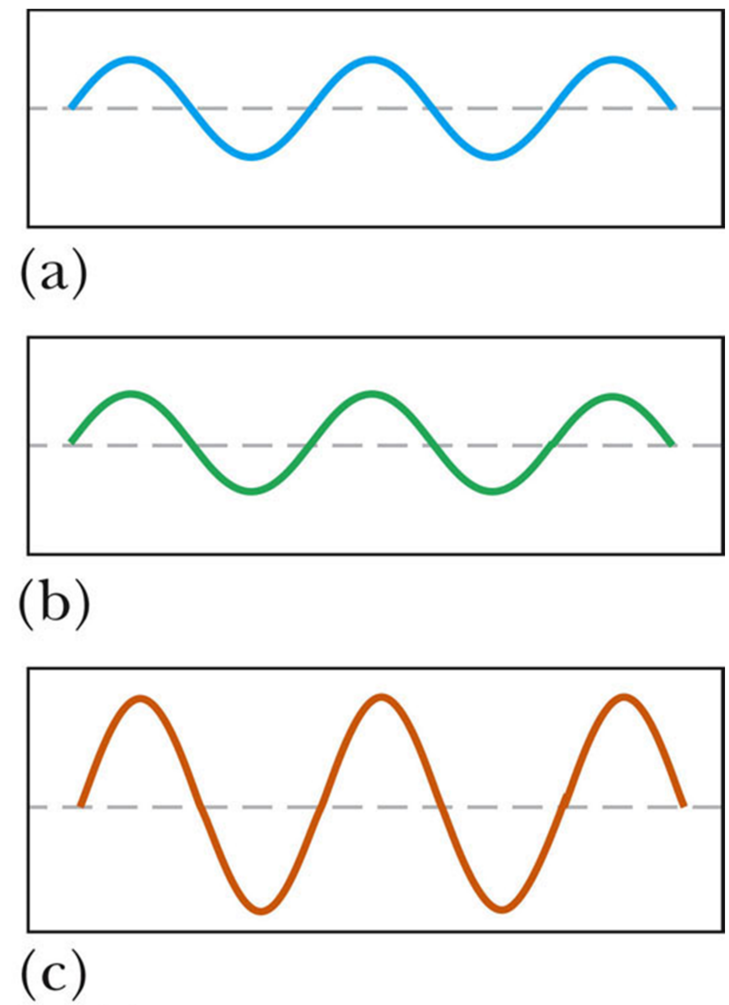
Stretched string has a mass per unit length of 5 g/cm. What tension has to be applied to the string to produce a sinusoidal wave of frequency 100 Hz and wavelength of 62 cm?

# Interference of Waves

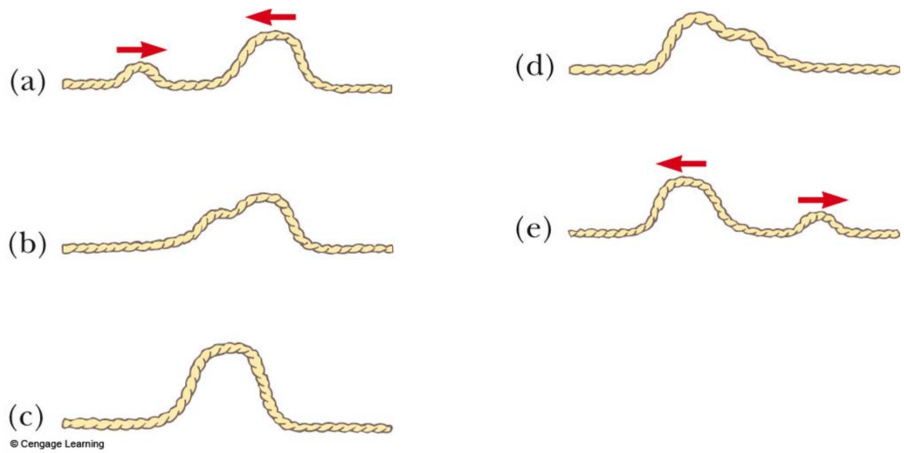
- Two traveling waves can meet and pass through each other without being destroyed or even altered
- Waves obey the *Superposition Principle*
  - When two or more traveling waves encounter each other while moving through a medium, the resulting wave is found by adding together the displacements of the individual waves point by point
  - Actually only true for waves with small amplitudes

# Constructive Interference

- Two waves, a and b, have the same frequency and amplitude
  - Are *in phase*
- The combined wave, c, has the same frequency and a greater amplitude



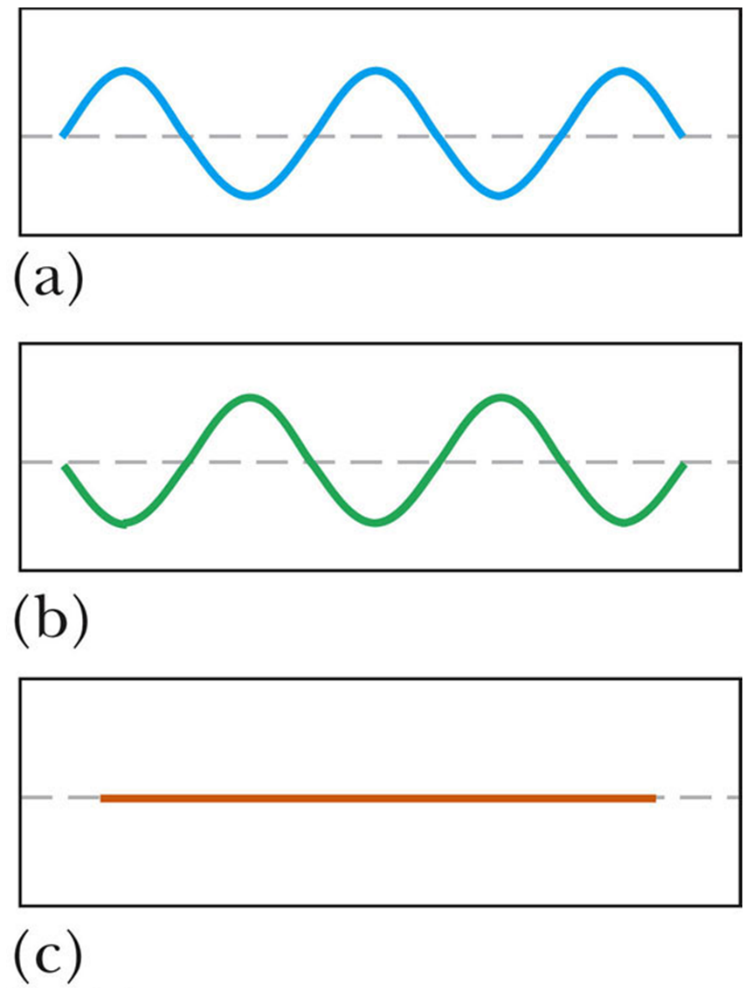
# Constructive Interference in a String



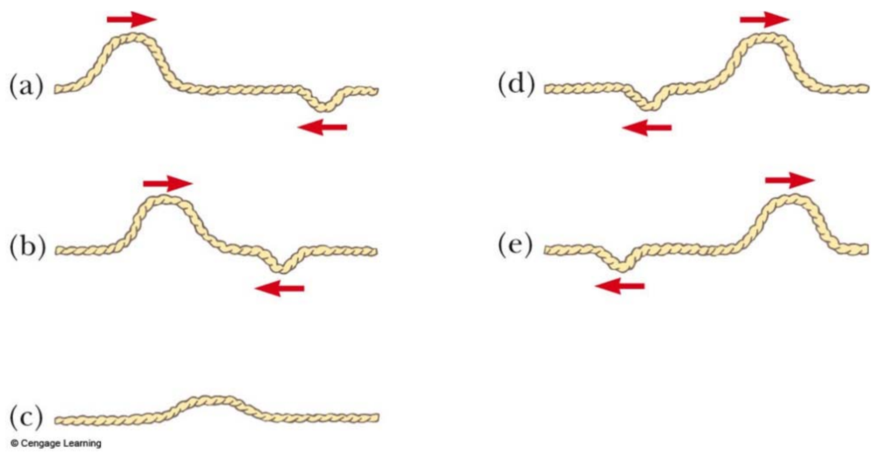

- Two pulses are traveling in opposite directions
- The net displacement when they overlap is the sum of the displacements of the pulses
- Note that the pulses are unchanged after the interference

# Destructive Interference

- Two waves, a and b, have the same amplitude and frequency
- They are 180° out of phase
- When they combine, the waveforms cancel



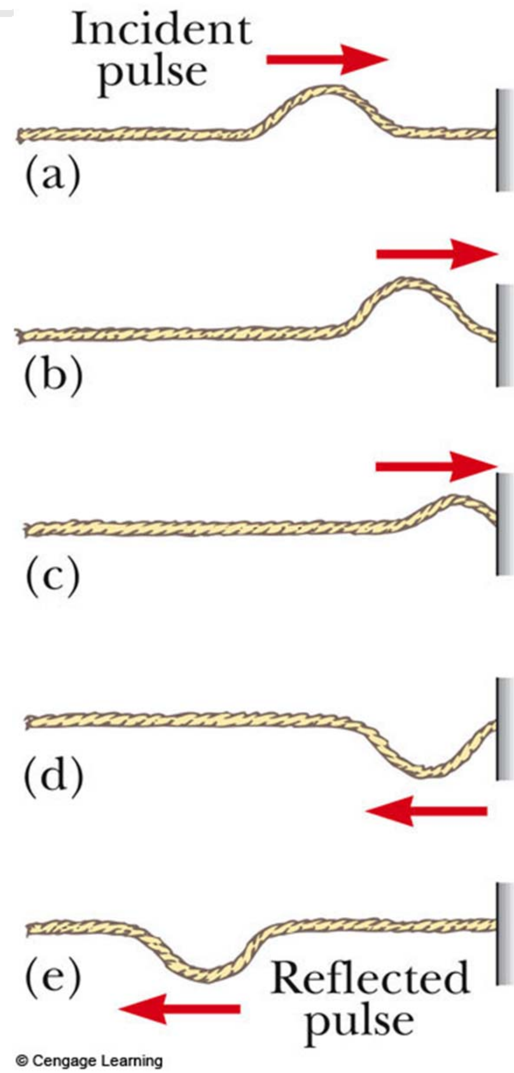
# Destructive Interference in a String




- Two pulses are traveling in opposite directions
- The net displacement when they overlap is decreased since the displacements of the pulses subtract
- Note that the pulses are unchanged after the interference

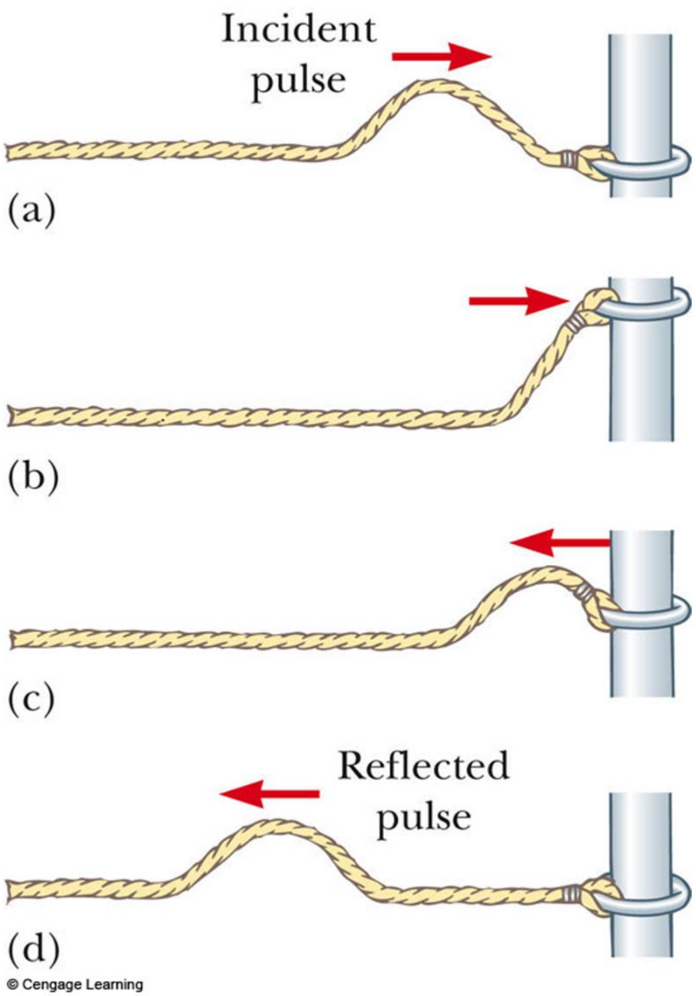
# Reflection of Waves – Fixed End

- Whenever a traveling wave reaches a boundary, some or all of the wave is reflected
- When it is reflected from a fixed end, the wave is inverted
- The shape remains the same

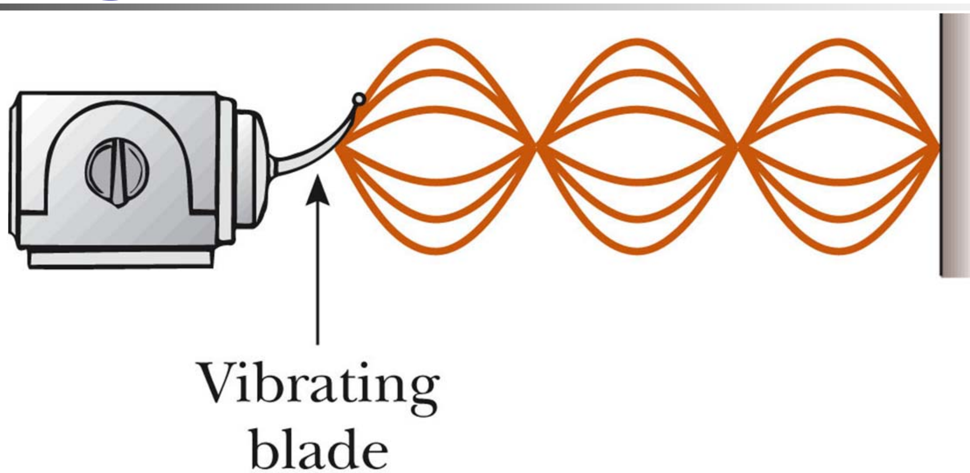


# Reflected Wave – Free End

- When a traveling wave reaches a boundary, all or part of it is reflected
- When reflected from a free end, the pulse is not inverted



# Standing Waves on a String

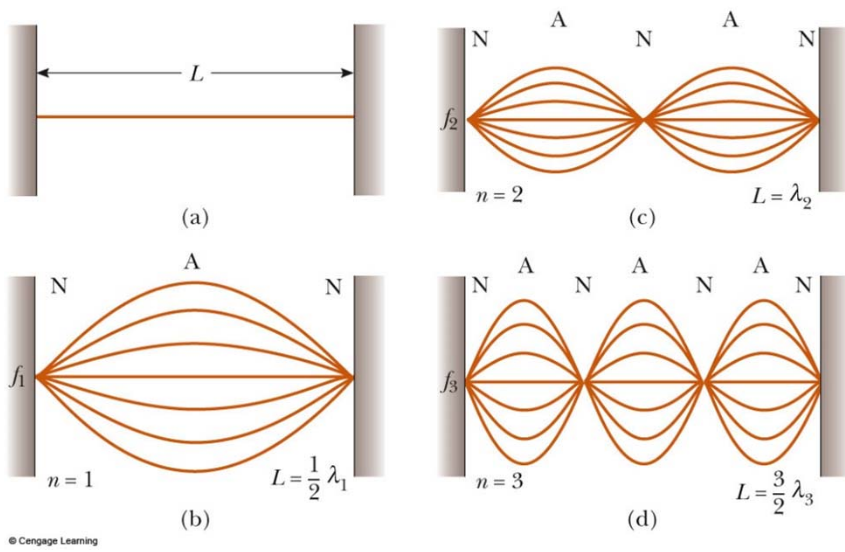





- Nodes must occur at the ends of the string because these points are fixed

# Standing Waves on a String, final

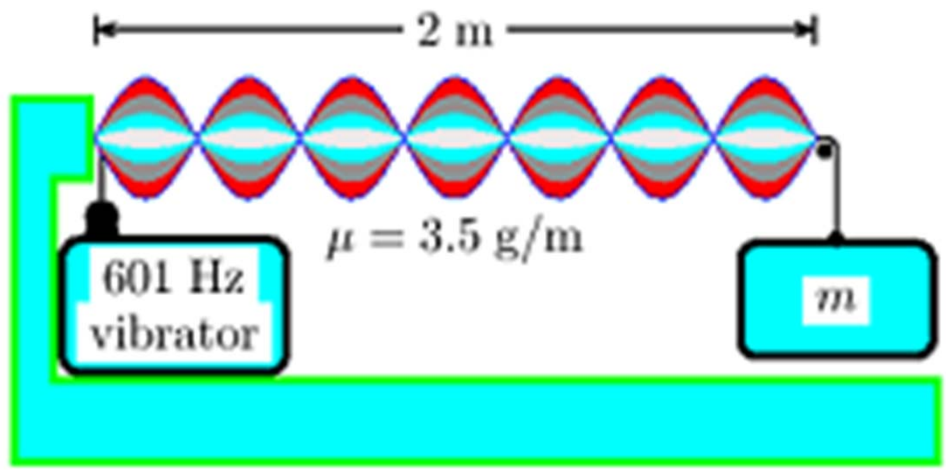
- One loop: $n=1$ $L=\lambda/2, \lambda=2L$
- Two loops: $n=2$; $L=\frac{3}{2}\lambda$ $\lambda=\frac{2L}{3}$
- n loops: $\lambda=\frac{2L}{n}$ $\lambda=\frac{v}{f}=\frac{2L}{n}$
- Solve for $f=\frac{v}{2L}n$ $f_1=\frac{v}{2L}$
- The lowest frequency of vibration $f_1$ is called the *fundamental frequency*




$$f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

# Example

The figure represents the vibrations in the string. A vibrator of constant frequency 601 Hz is attached near one end of the string. The length of the string is 2 m. a) What is the length of the standing wave? b) What is the velocity of this wave? c) What is the tension that will produce the standing wave shown in the figure? d. Find mass m.



# Standing Waves on a String – Frequencies

- $f_1$, $f_2$, $f_3$ form a harmonic series
  - $f_1$ is the fundamental and also the first harmonic
  - $f_2$ is the second harmonic
- Waves in the string that are not in the harmonic series are quickly damped out
  - In effect, when the string is disturbed, it "selects" the standing wave frequencies

# Example


A high E string on a certain guitar measures 64 cm in length and has a fundamental frequency of 329 Hz. When a guitarist presses down so that the string is in contact with the first fret, the string is shortened so it plays an F note that has a frequency of 349 Hz. How far is the fret from the nut?