Chapter 11

Equilibrium and Elasticity

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Goals for Chapter 11

- To study the conditions for equilibrium of a body
- To understand center of gravity and how it relates to a body’s stability
- To solve problems for rigid bodies in equilibrium

Introduction

- Many bodies, such as bridges, aqueducts, and ladders, are designed so they do not accelerate.

- Real materials are not truly rigid. They are elastic and do deform to some extent.

Conditions for Equilibrium

The net force equals zero

- $$\sum \vec{\mathbf{F}} = 0$$
- If the object is modeled as a particle, then this is the only condition that must be satisfied

The net torque equals zero

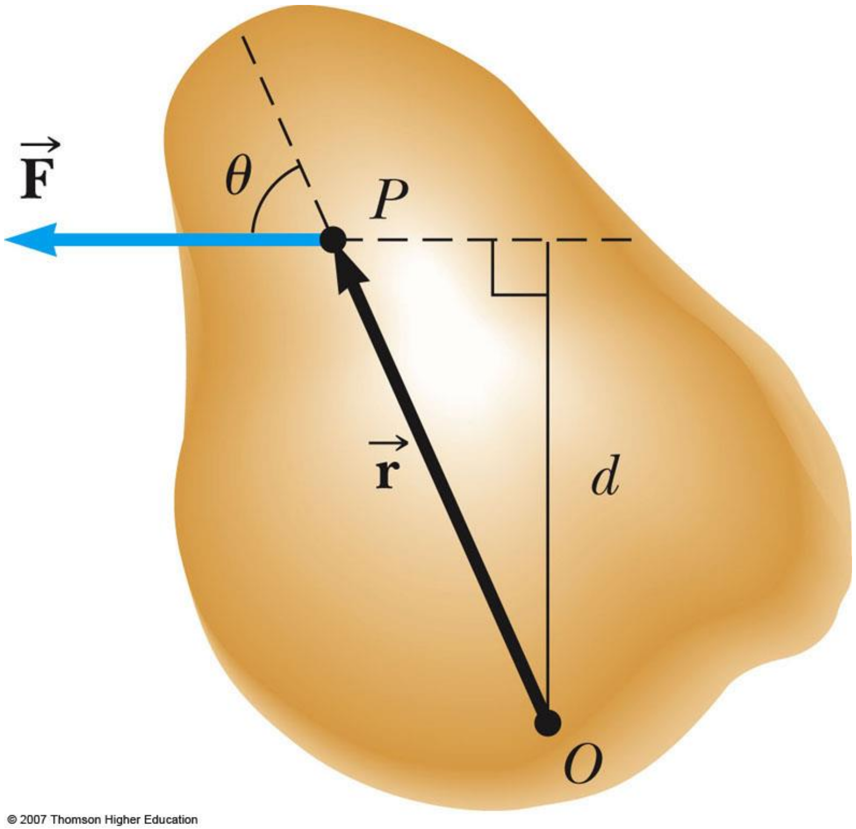
- $$\sum \vec{\tau} = 0$$
- This is needed if the object cannot be modeled as a particle

These conditions describe the rigid objects in equilibrium analysis model

Torque

$$\vec{\tau} = \vec{\mathbf{F}} \times \vec{\mathbf{r}}$$

- Use the right hand rule to determine the direction of the torque
- The tendency of the force to cause a rotation about O depends on F and the moment arm *d*



Translational Equilibrium

The first condition of equilibrium is a statement of translational equilibrium

It states that the translational acceleration of the object's center of mass must be zero

- This applies when viewed from an inertial reference frame

Rotational Equilibrium

The second condition of equilibrium is a statement of rotational equilibrium

It states the angular acceleration of the object to be zero

This must be true for any axis of rotation

Static vs. Dynamic Equilibrium

In this chapter, we will concentrate on static equilibrium

- The object will not be moving
- $v_{CM} = 0$ and $\omega = 0$

Dynamic equilibrium is also possible

- The object would be rotating with a constant angular velocity
- The object would be moving with a constant v_{CM}

Equilibrium Equations

We will restrict the applications to situations in which all the forces lie in the xy plane

- These are called coplanar forces since they lie in the same plane

There are three resulting equations

- $\Sigma F_x = 0$
- $\Sigma F_y = 0$
- $\Sigma \tau = 0$

Conditions for equilibrium

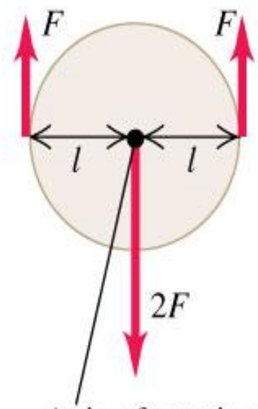
- *First condition*: The sum of all the forces is equal to zero:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

- *Second condition*: The sum of all torques about any given point is equal to zero.

(a) This body is in static equilibrium.

Equilibrium conditions:

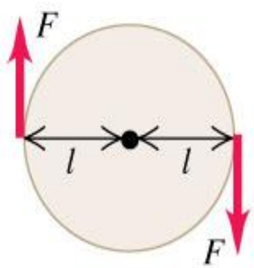


First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

Axis of rotation (perpendicular to figure)

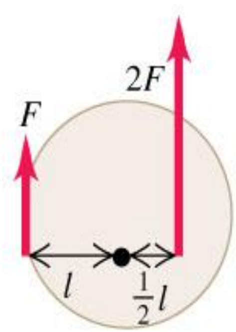
(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.



First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.

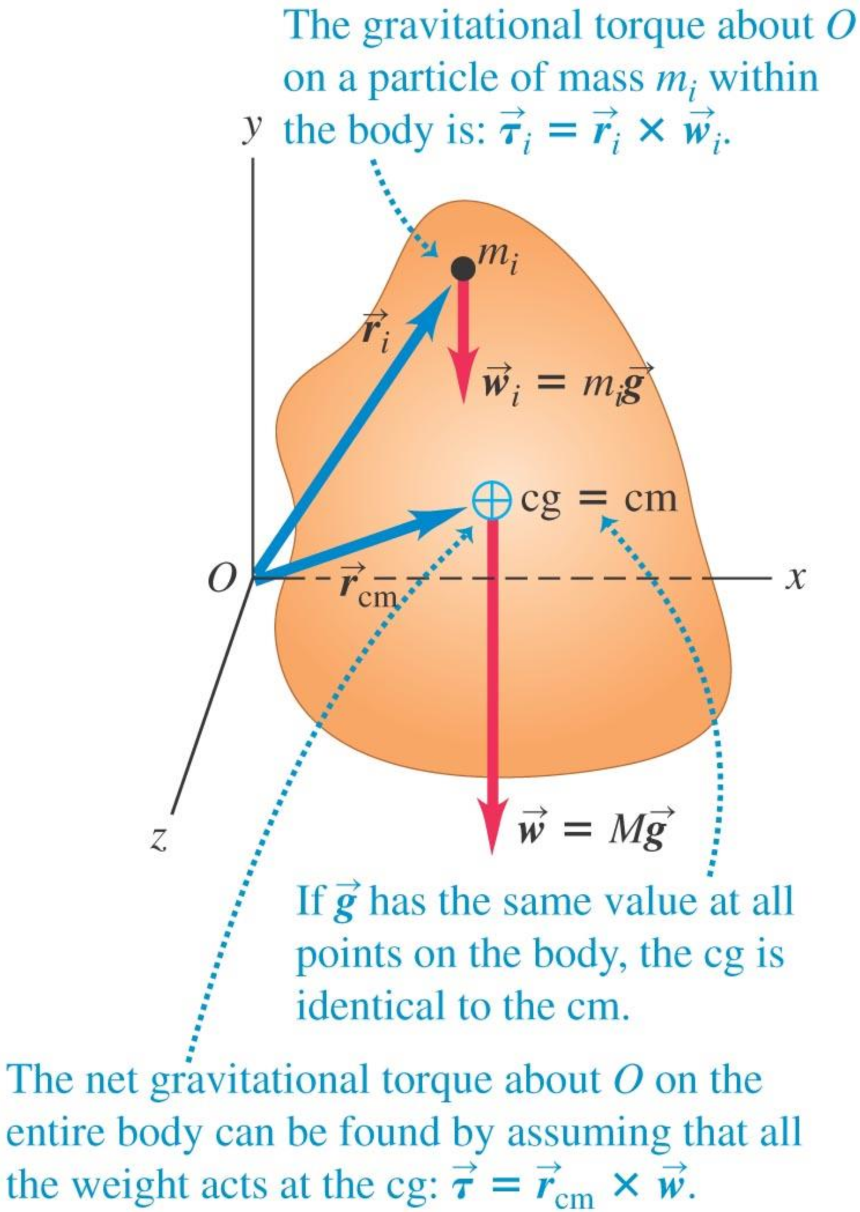


First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

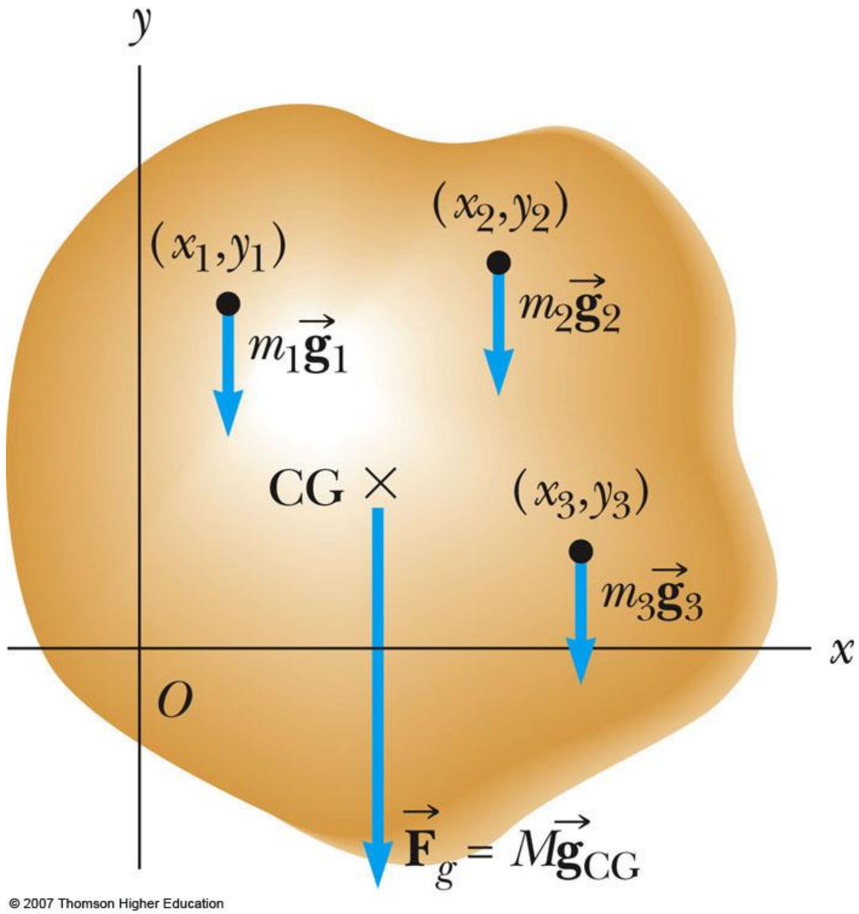
Center of gravity

- We can treat a body's weight as though it all acts at a single point—the *center of gravity*.
- If we can ignore the variation of gravity with altitude, the center of gravity is the same as the center of mass.



Center of Gravity

All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG)



Center of Gravity, cont

The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object

If g is uniform over the object, then the center of gravity of the object coincides with its center of mass

If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Problem-Solving Strategy – Equilibrium Problems

- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Apply the second condition for equilibrium: $\Sigma\tau = 0$
- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma\tau = 0$

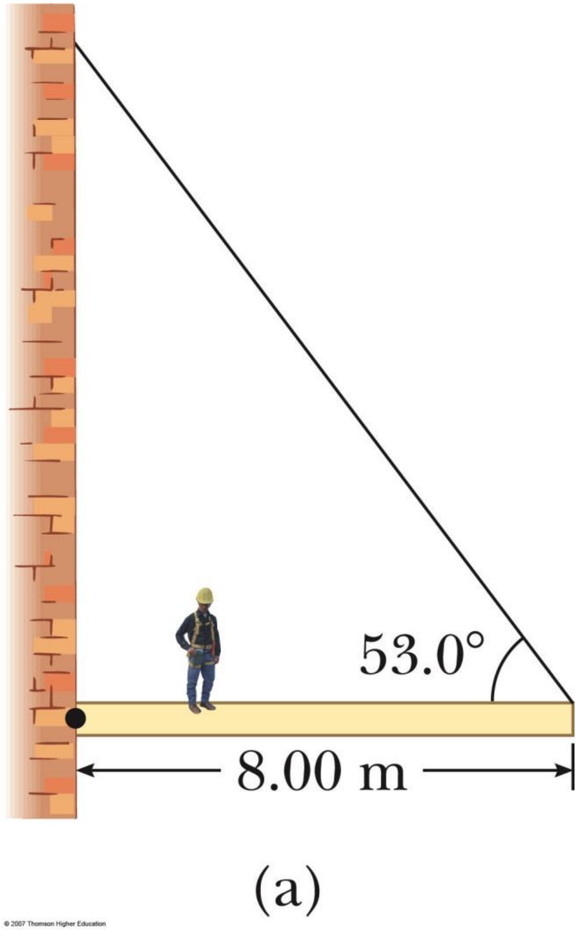
Horizontal Beam Example

The beam is uniform and weighs 200 N

- So the center of gravity is at the geometric center of the beam

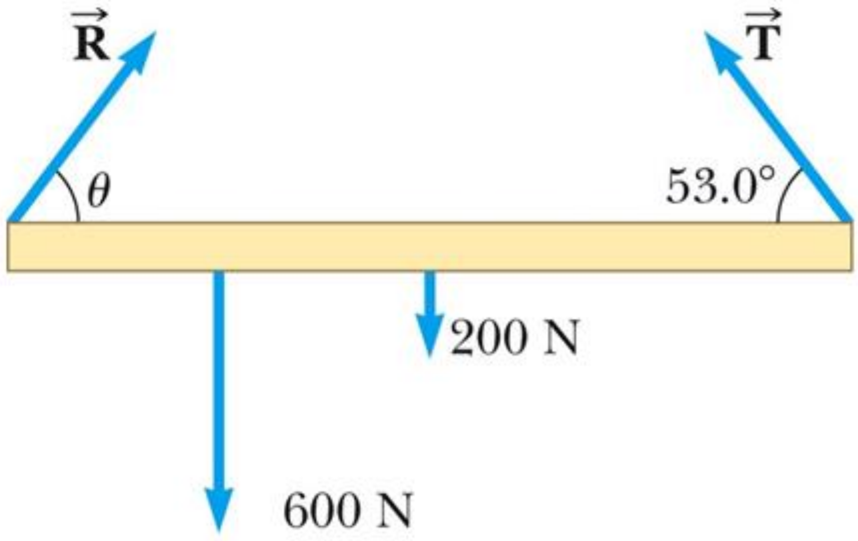
The 600 N person is standing on the beam at a distance 2 m from the wall.

Find the force exerted by the wall on the beam?



Horizontal Beam

- Draw a free body diagram
- Use the pivot in the problem (at the wall) as the pivot
- Note there are three unknowns (T, R, θ)



$R_x = T\cos 53^0$ $\quad$ $R_y - 600N – 200N + T\sin 53^0 = 0$ $\quad$ $R_y = 550\ N$

$T \cdot 8m \cdot \sin 53^0 – 200N \cdot 4m - 600N \cdot 2m = 0$ $\quad$ $T = 313\ N$

$R_x = 313N \cos 53^0 = 188.4\ N$ $\quad$ $R_y = 600N + 200N - 313\ N\sin 53^0 = 550\ N$

$R = 581\ N$ $\quad$ $\theta = \tan^{-1}(R_y/R_x) = 71^0$

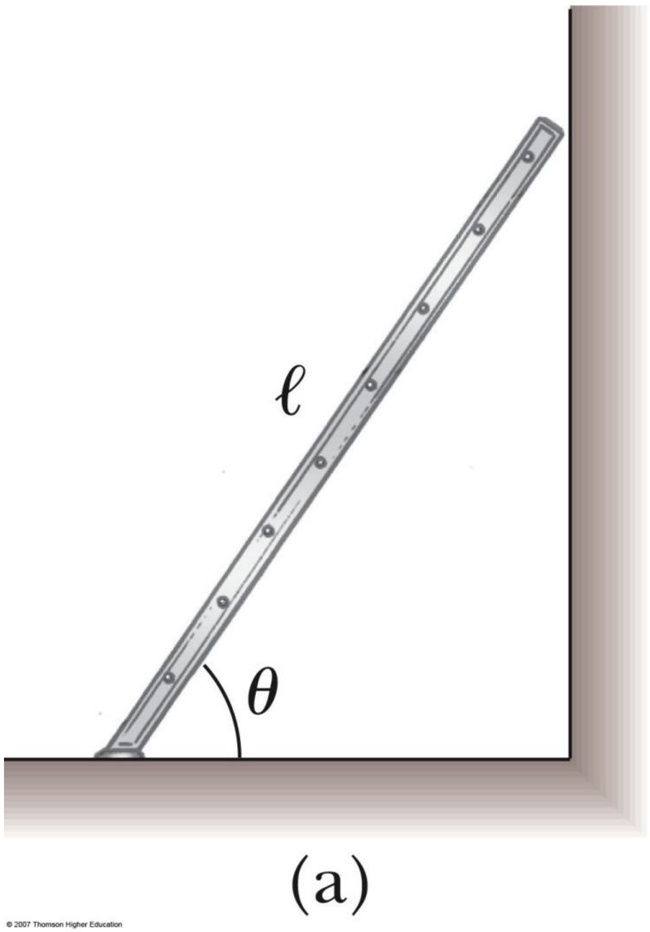
Ladder Example

The 350 N ladder is uniform

- So the weight of the ladder acts through its geometric center (its center of gravity)

a. Find the magnitude of the force that the wall exerts on the ladder if $\theta = 38^0$.

b. If the ladder is on the verge of slipping, what is the coefficient of static friction ?



- Analyze: Draw a free body diagram for the ladder
- The frictional force is $f_s = \mu_s n$

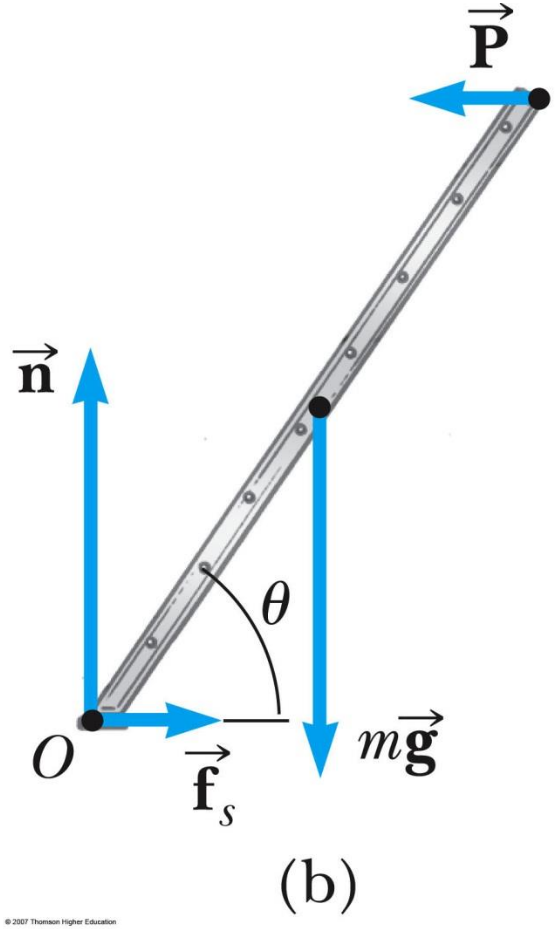
Ladder - Example

(a) $P \cdot L \cdot \sin 38^0 - 350N \cdot \frac{1}{2} L \cdot \sin 52^0 = 0$

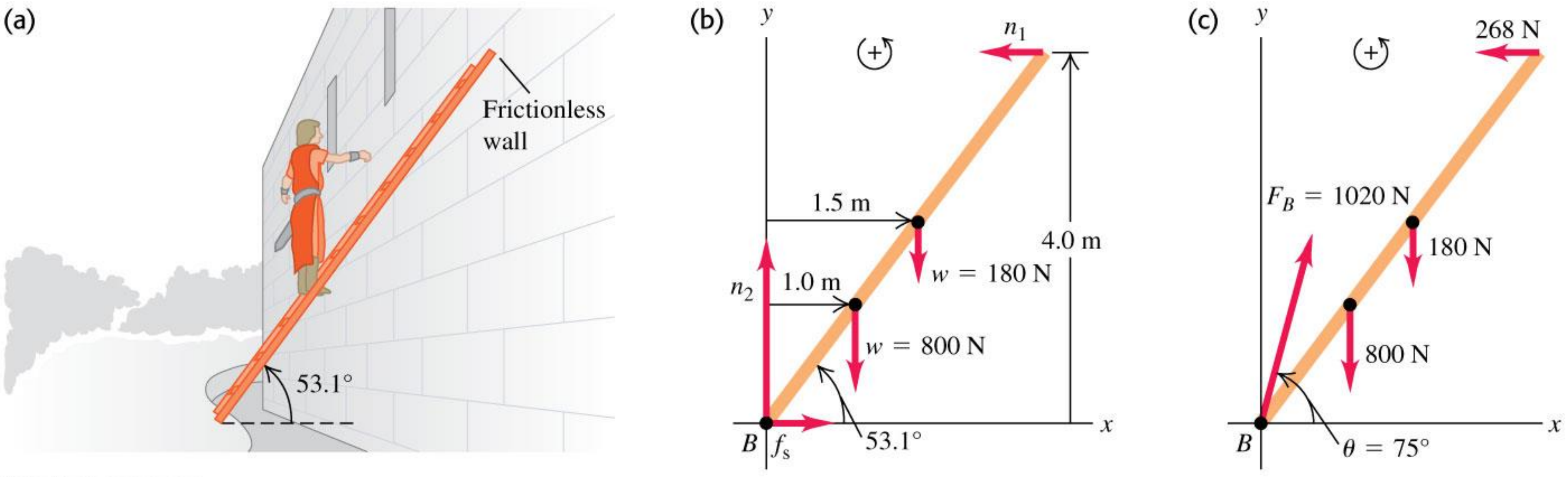
$P = 224N$

(b) $224\ N = \mu_s \cdot n \quad n = 350\ N$

$\mu_s = 0.64$

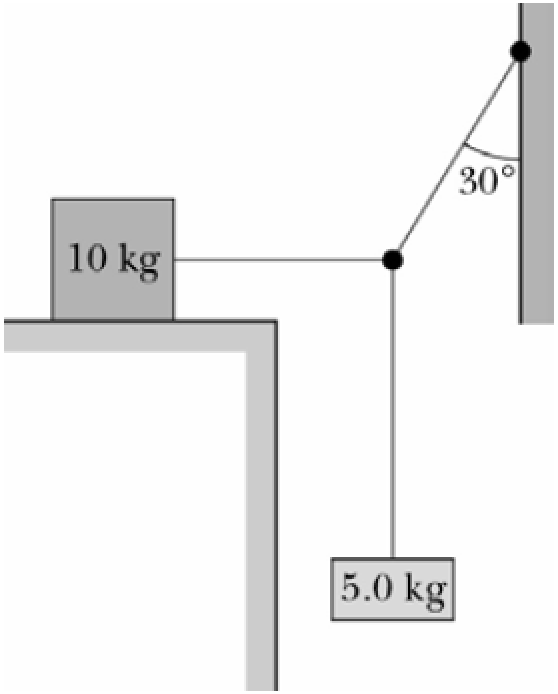


A 800N-man is climbing a uniform ladder that is 5m long and weighs 180 N. The ladder makes an angle of 53.1°. Find the normal and friction forces on the base of the ladder.



Example 3

The system in the figure is in equilibrium, but it begins to slip if any additional mass is added to 5-kg object. What is the coefficient of static friction between the 10-kg object and the plane on which it rests?



$T_3 = 5kg \cdot 9.8m/s^2 = 49\ N$

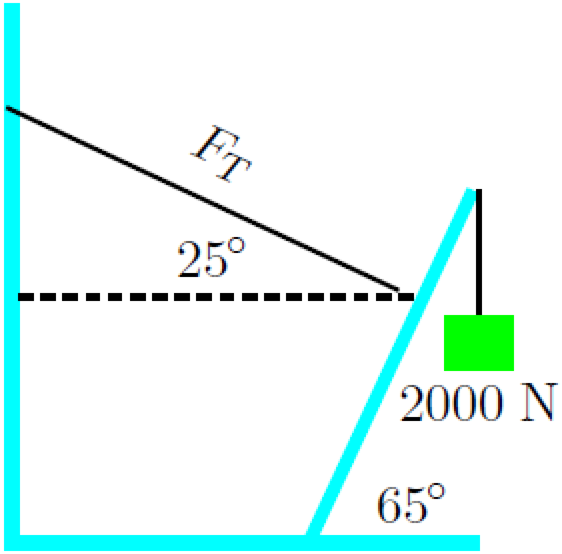
$T_1 \sin 60^0 = T_3 = 49\ N \quad T_1 = 57\ N$

$T_1 \cos 60^0 = T_2 \quad T_2 = 28.5N$

$T_2 = \mu_s \cdot n \quad n = 10kg \cdot 9.8m/s^2 = 98\ N$

$\mu_s = 0.29$

Example 4



A 1200 N boom of length 1.6 m is supported by a cable as shown. The boom is pivoted at the bottom, The cable is attached a distance 1.2 m from the pivot and a 2000 N weight hangs from the boom's top.

a. Find the tension in the cable.

b. Find the magnitude of the force that the pivot exerts on the boom.

$F_T \cdot 1.2m \cdot \sin(25^0+65^0) – 2000N \cdot 1.6m \cdot \sin 25^0 - 1200m \cdot 0.8m \cdot \sin 25^0 = 0$ $F_T = 1465\ N$

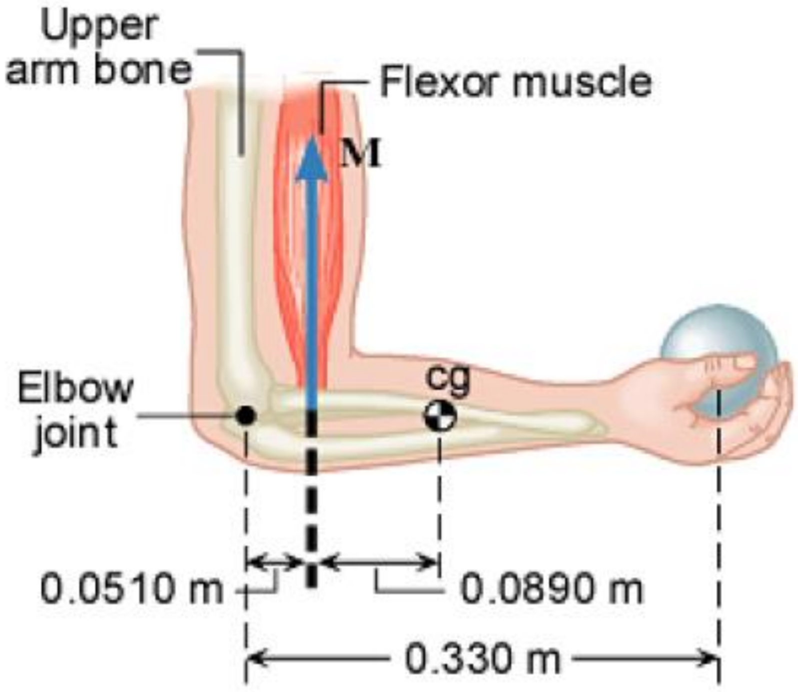
$F_x = F_T \cos 25^0 = 1328N$

F_y $-2000N-1200N+ F_T \sin 25^0 =0$

$F_y = 2581\ N$

$F = 2902\ N$

Example 5



A man holds a 178 N ball in his hand with his forearm horizontal. He can support the ball because of the flexor muscle force **M** which is Perpendicular to the forearm. The forearm weighs 22 N.

Find (a) the magnitude of **M** and
(b) the magnitude and direction of the force applied by the arm upper bone to the forearm at the elbow joint.

M·0.051m-22N·0.14m-178N·0.33m = 0

M = 1212 N

F + M+-22N-178 N =0

F = 1012 N down

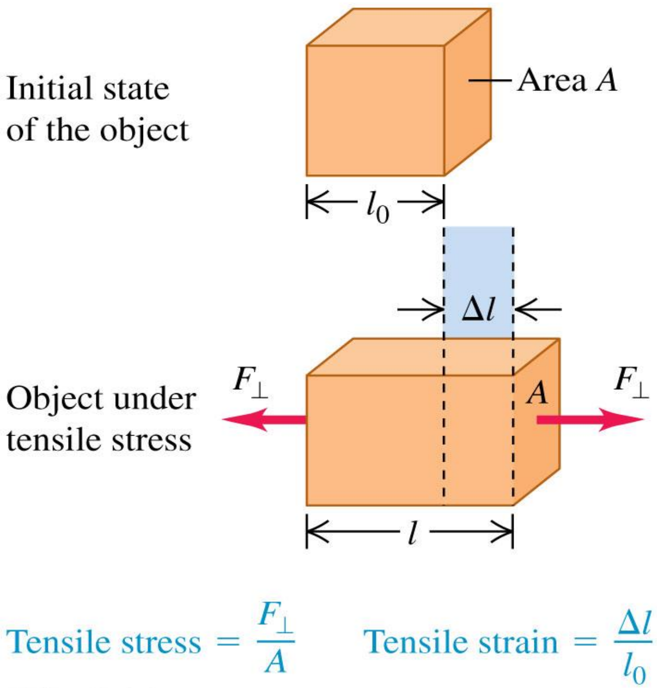
Strain, stress, and elastic moduli

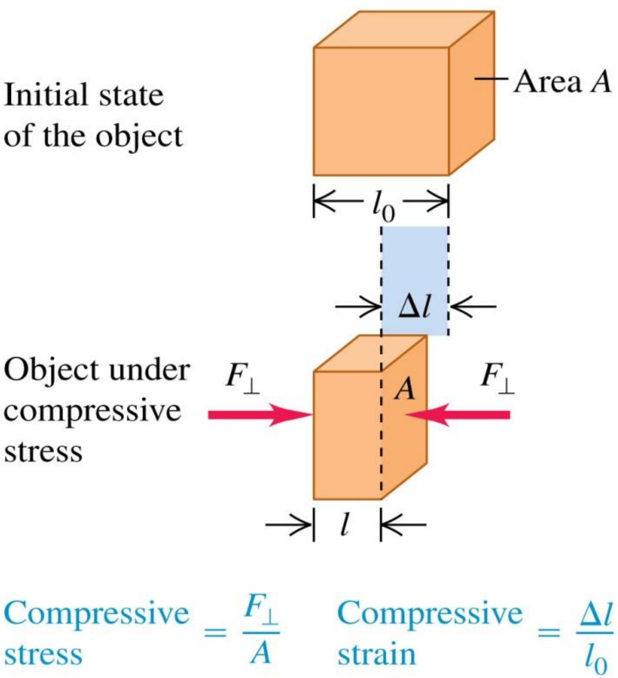
- Stretching, squeezing, and twisting a real body causes it to deform, as shown in Figure 11.12 below. We shall study the relationship between forces and the deformations they cause.
- *Stress* is the force per unit area and *strain* is the fractional deformation due to the stress. *Elastic modulus* is stress divided by strain.
- The proportionality of stress and strain is called *Hooke's law*.



Tensile and compressive stress and strain

- *Tensile stress* = $F_\perp /A$ and *tensile strain* = $\Delta l/l_0$. *Compressive stress* and *compressive strain* are defined in a similar way. (See Figures 11.13 and 11.14 below.)
- *Young's modulus* is tensile stress divided by tensile strain, and is given by $Y = (F_\perp/A)(l_0/\Delta l)$.





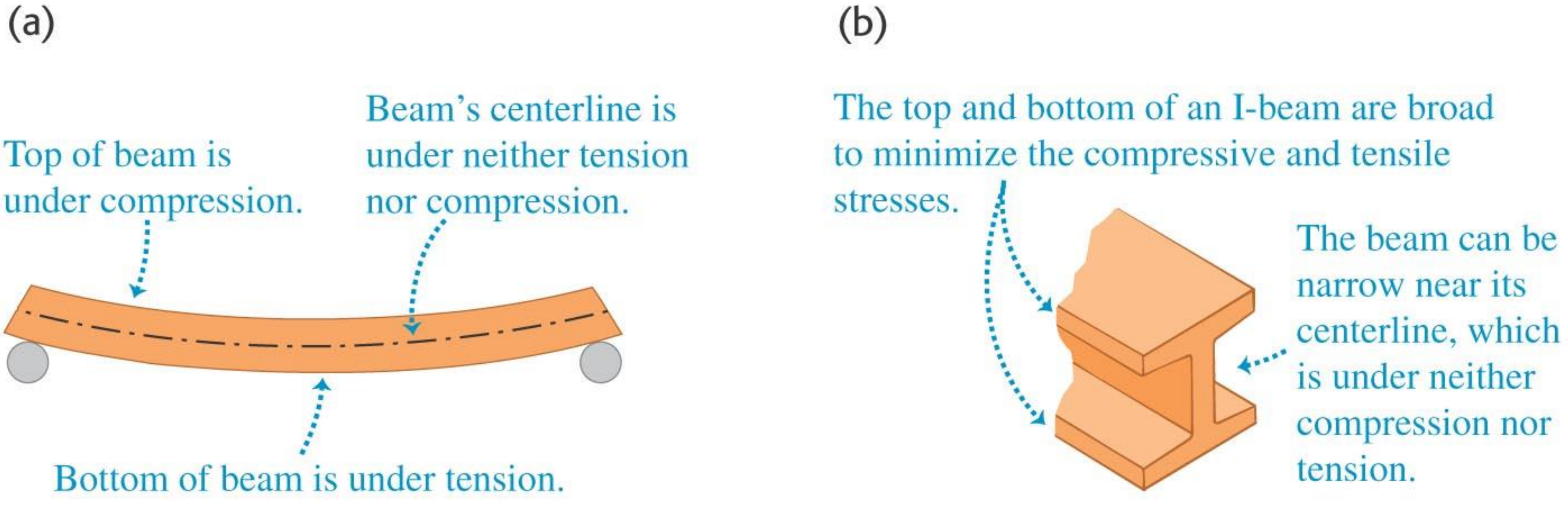
Some values of elastic moduli

Table 11.1 Approximate Elastic Moduli

Material	Young's Modulus, *Y* (Pa)	Bulk Modulus, *B* (Pa)	Shear Modulus, *S* (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Crown glass	6.0×10^{10}	5.0×10^{10}	2.5×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

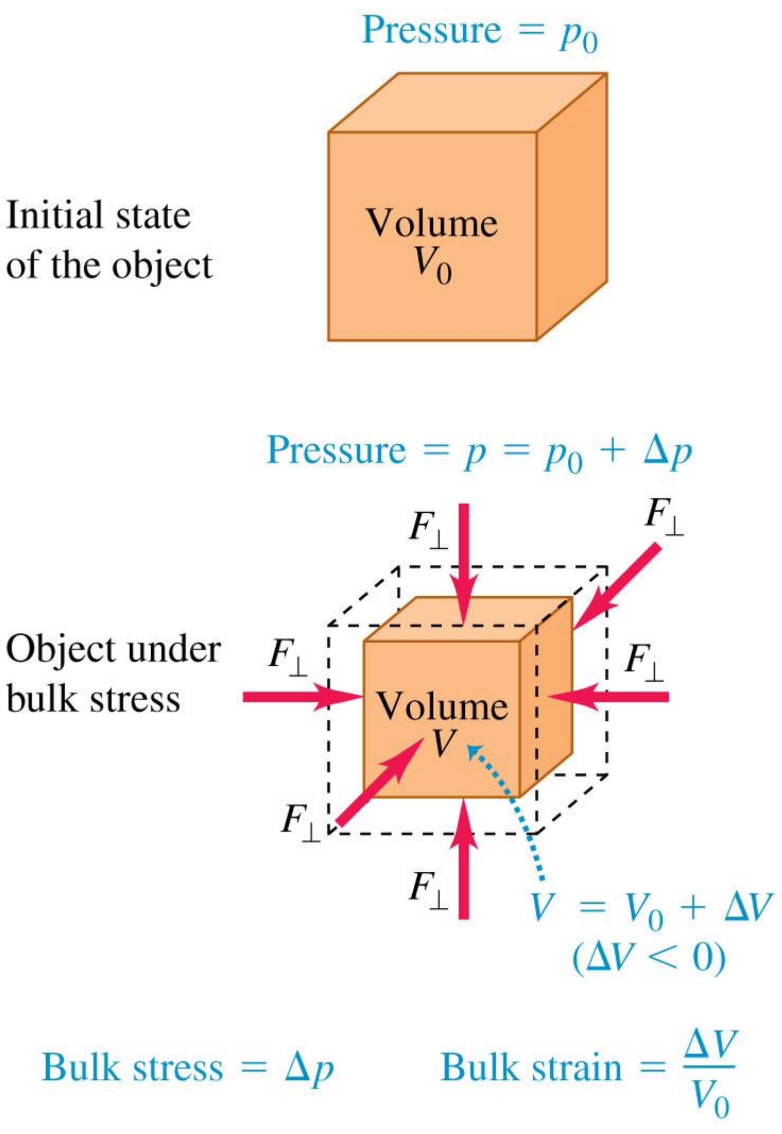
Tensile stress and strain

- In many cases, a body can experience both tensile and compressive stress at the same time, as shown in Figure 11.15 below.
- Follow Example 11.5.



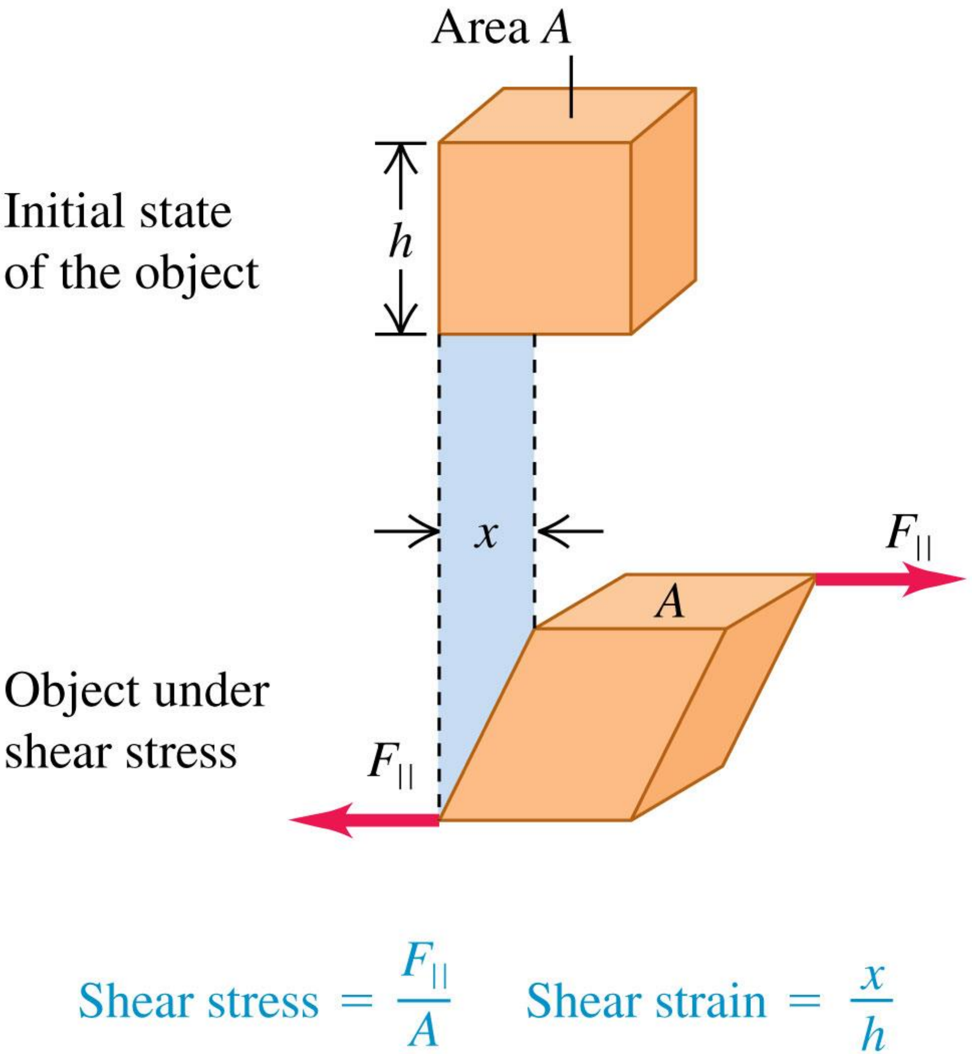
Bulk stress and strain

- Pressure in a fluid is force per unit area: $p = F_\perp/A$.
- *Bulk stress* is pressure change Δp and *bulk strain* is fractional volume change $\Delta V/V_0$. (See Figure 11.16.)
- *Bulk modulus* is bulk stress divided by bulk strain and is given by $B = -\Delta p/(\Delta V/V_0)$.
- Follow Example 11.6.



Sheer stress and strain

- *Sheer stress* is $F_{||}/A$ and *sheer strain* is x/h, as shown in Figure 11.17.
- *Sheer modulus* is sheer stress divided by sheer strain, and is given by $S = (F_{||}/A)(h/x)$.
- Follow Example 11.7.



Elasticity and plasticity

- Hooke's law applies up to point *a* in Figure 11.18 below.
- Table 11.3 shows some approximate breaking stresses.

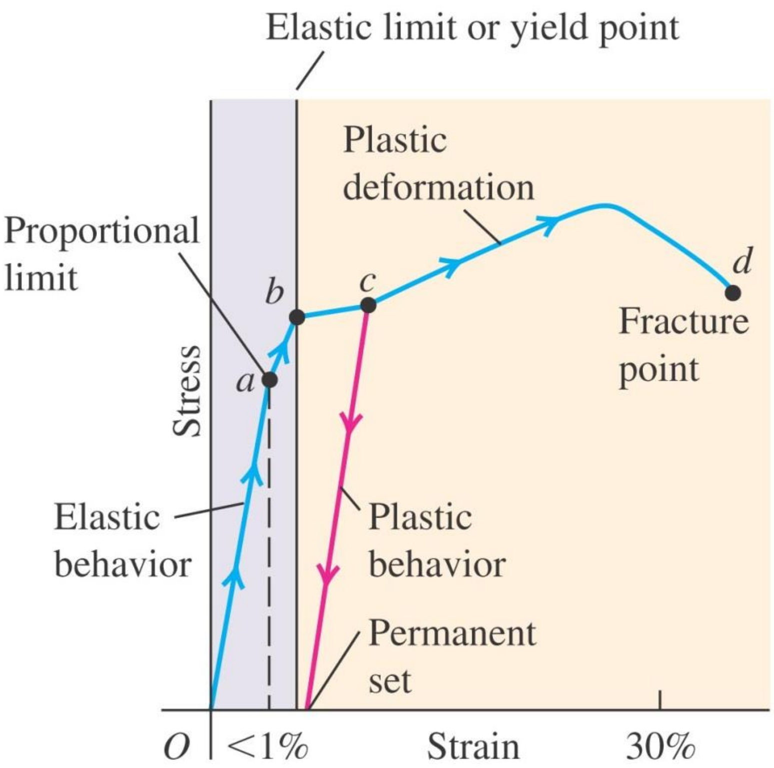


Table 11.3 Approximate Breaking Stresses

Material	Breaking Stress (Pa or N/m^2)
Aluminum	2.2×10^8
Brass	4.7×10^8
Glass	10×10^8
Iron	3.0×10^8
Phosphor bronze	5.6×10^8
Steel	$5 - 20 \times 10^8$