Chapter 3

Motion in Two or Three Dimensions

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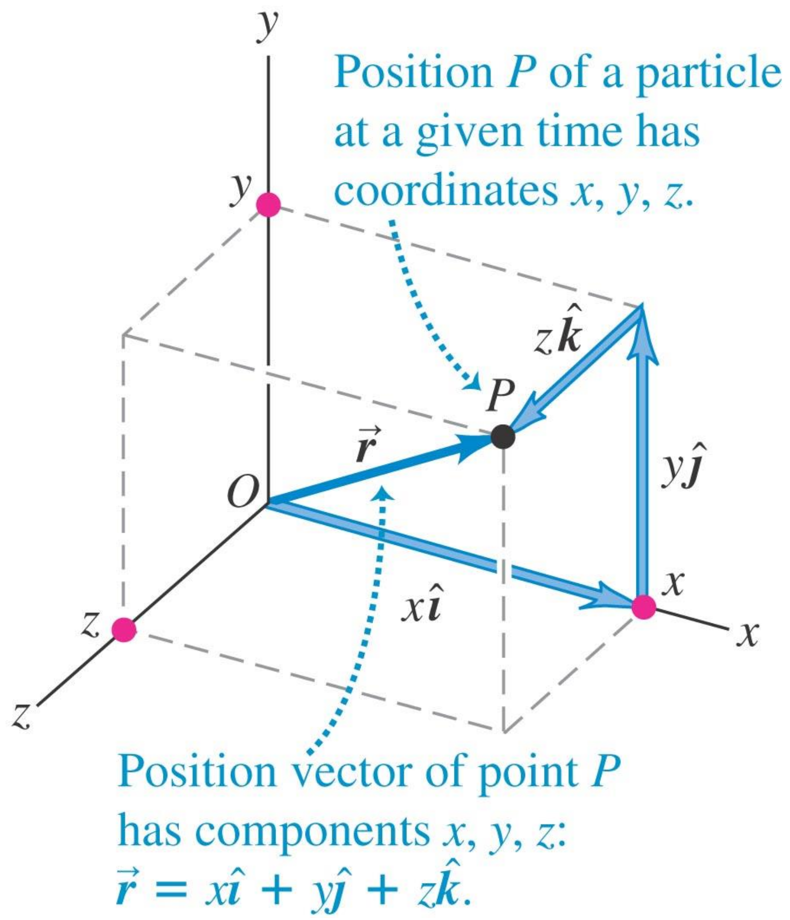
Lectures by Wayne Anderson

Goals for Chapter 3

- To use vectors to represent the position of a body
- To determine the velocity vector using the path of a body
- To investigate the acceleration vector of a body
- To describe the curved path of projectile
- To investigate circular motion

Position vector

- The position vector from the origin to point *P* has components *x*, *y*, and *z*.



Position and Displacement

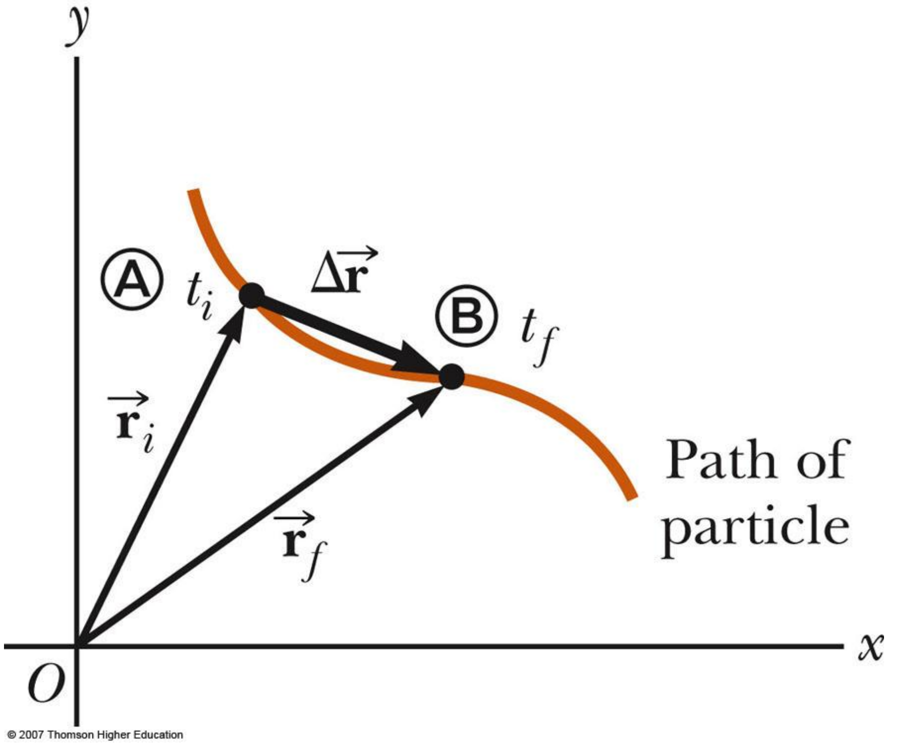
The position of an object is described by its position vector, $\vec{\mathbf{r}}$

The **displacement** of the object is defined as the ***change in its position***

$$\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

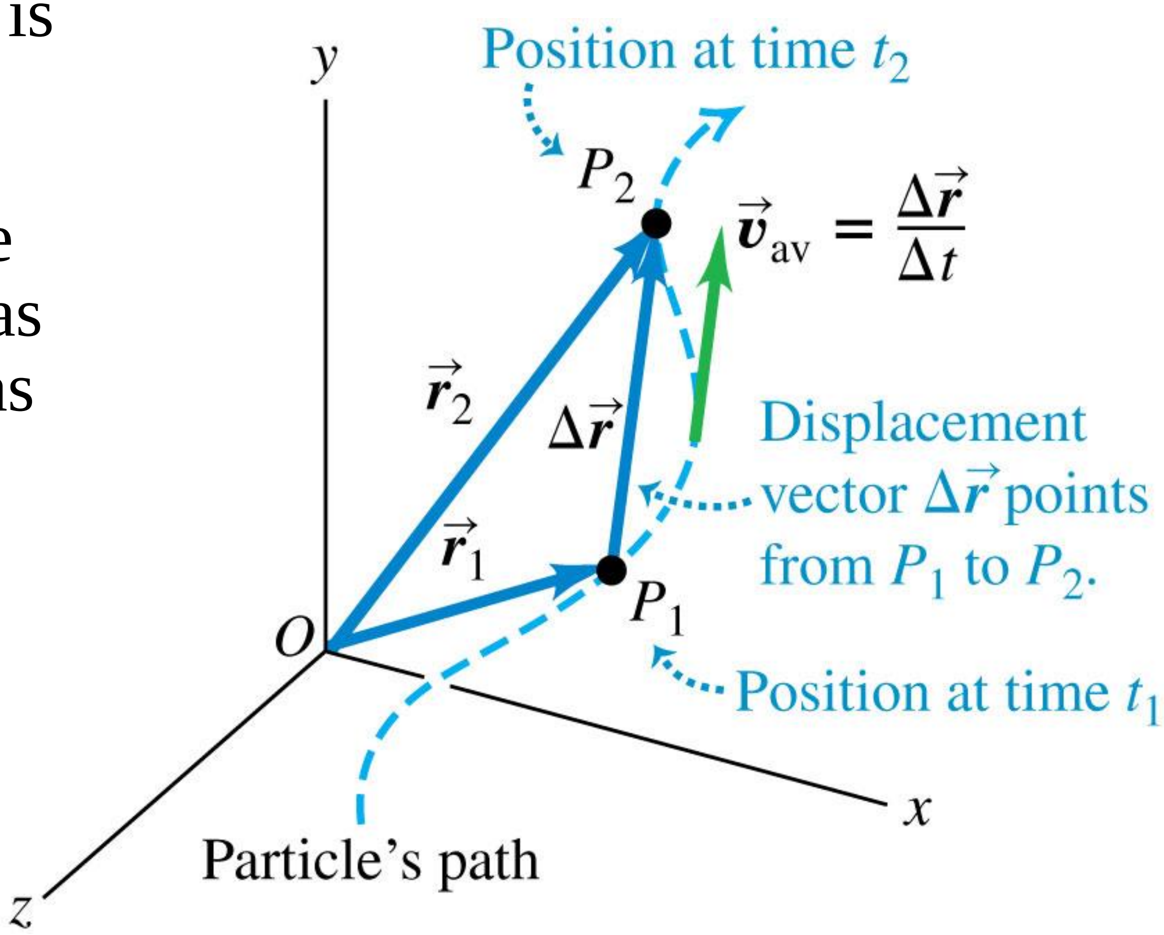
Where

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$



Average velocity

- The average velocity between two points is the displacement divided by the time interval between the two points, and it has the same direction as the displacement.



Average Velocity

The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{v}_{avg} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta \vec{x}}{\Delta t}\,\hat{i} + \frac{\Delta \vec{y}}{\Delta t}\,\hat{j}$$

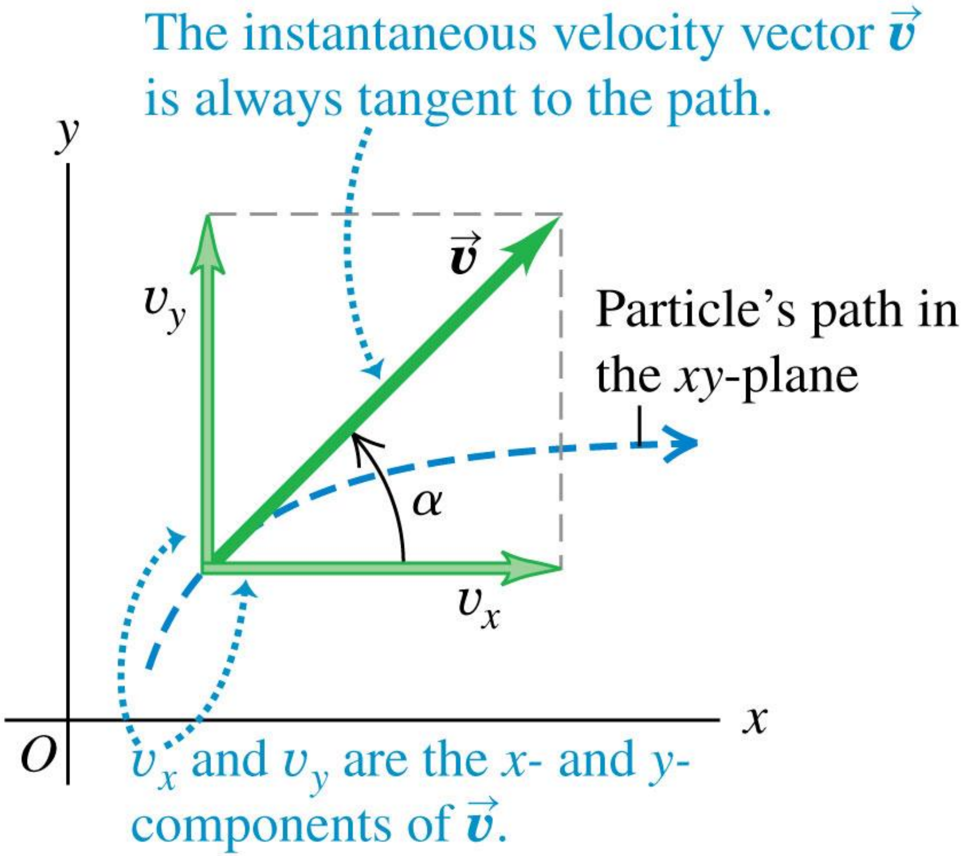
The direction of the average velocity is the direction of the displacement vector

The average velocity between points is *independent of the path* taken

- This is because it is dependent on the displacement, also independent of the path

Instantaneous velocity

- The *instantaneous velocity* is the instantaneous rate of change of position vector with respect to time.
- The components of the instantaneous velocity are $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$.
- The instantaneous velocity of a particle is always tangent to its path.

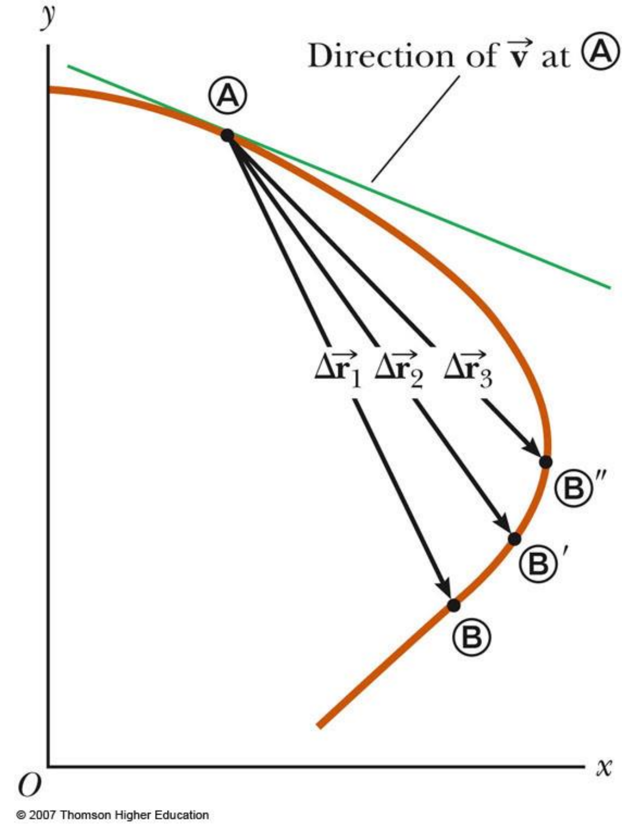


Instantaneous Velocity

The instantaneous velocity is the limit of the average velocity as Δt approaches zero

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

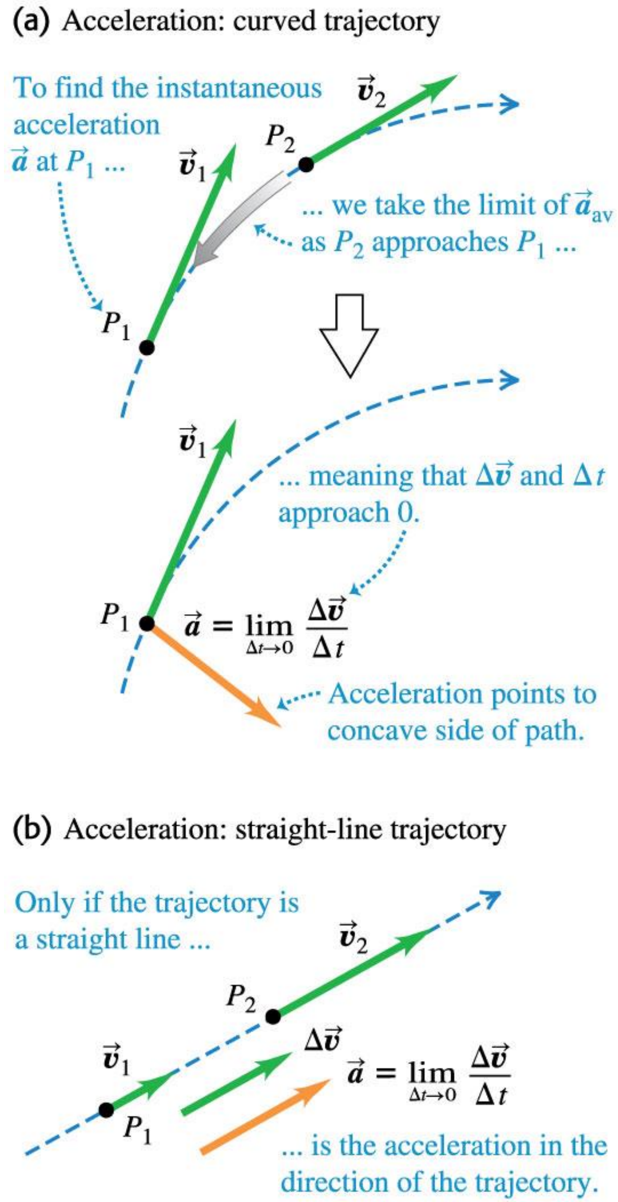
$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d\vec{x}}{dt}\hat{\mathbf{i}} + \frac{d\vec{y}}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$



- As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve

Instantaneous acceleration

- The *instantaneous acceleration* is the instantaneous rate of change of the velocity with respect to time.
- Any particle following a curved path is accelerating, even if it has constant speed.
- The components of the instantaneous acceleration are $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.



Instantaneous Acceleration

The instantaneous acceleration is the limiting value of the ratio $\Delta\vec{\mathbf{v}}/\Delta t$ as Δt approaches zero

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d\vec{v}_x}{dt}\hat{\mathbf{i}} + \frac{d\vec{v}_y}{dt}\hat{\mathbf{j}} = \mathbf{a}_x\hat{\mathbf{i}} + \mathbf{a}_y\hat{\mathbf{j}}$$

- The instantaneous acceleration equals the derivative of the velocity vector with respect to time

Kinematic Equations for Two-Dimensional Motion

When the two-dimensional motion has a constant acceleration, a series of equations can be developed that describe the motion

These equations will be similar to those of one-dimensional kinematics

Motion in two dimensions can be modeled as two *independent* motions in each of the two perpendicular directions associated with the x and y axes

- Any influence in the y direction does not affect the motion in the x direction

Kinematic Equations

Position vector for a particle moving in the xy plane

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

The velocity vector can be found from the position vector

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d\vec{x}}{dt}\hat{\mathbf{i}} + \frac{d\vec{y}}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

- Since acceleration is constant, we can also find an expression for the velocity and position as a function of time:
  - $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$ $\quad\quad$ $\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$

$$\mathbf{v_f = (v_{ix} + a_x\, t)\, \hat{i} + (v_{iy} + a_y t)\, \hat{j}}$$

$$\mathbf{r_f = (v_{ix}\, t + ½a_x\, t^2)\, \hat{i} + (v_{iy}\, t + ½a_y\, t^2)\, \hat{j}}$$

Projectile Motion: Special Case of Two-Dimensional Motion

Object is thrown with initial velocity v_0 at an angle θ.

Acceleration: $a_x = 0$ $a_y = -g$ It is directed downward

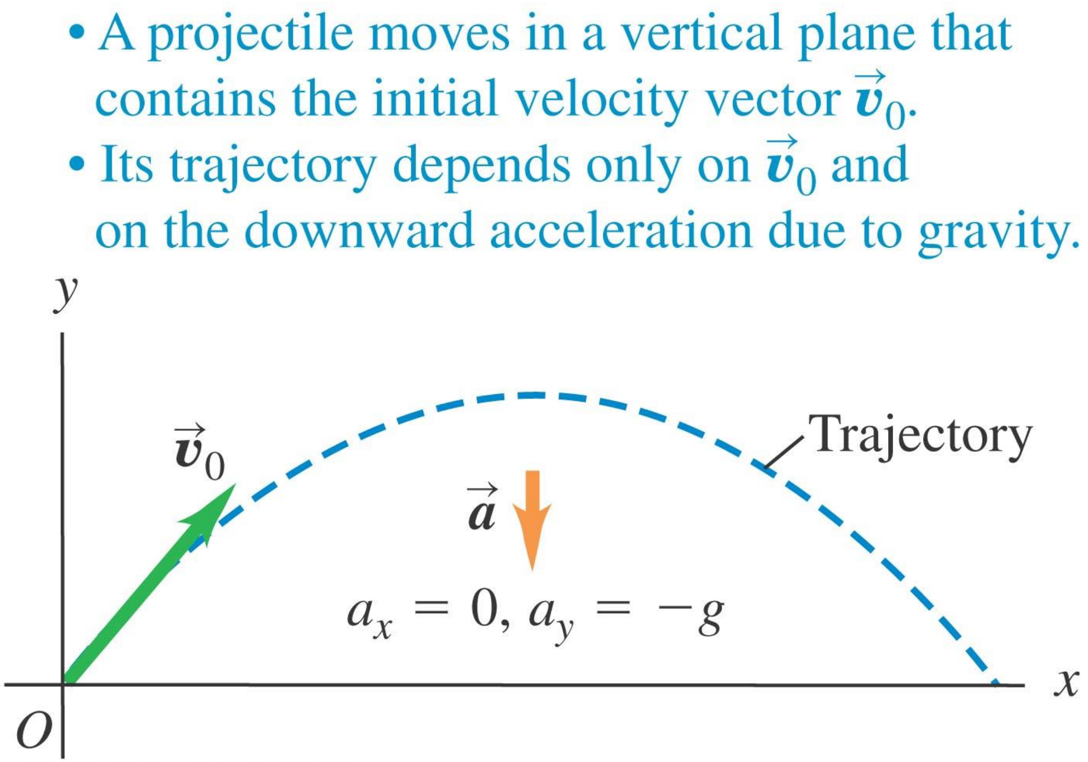
The effect of air friction is negligible

With these assumptions, an object in projectile motion will follow a parabolic path

- This path is called the ***trajectory***

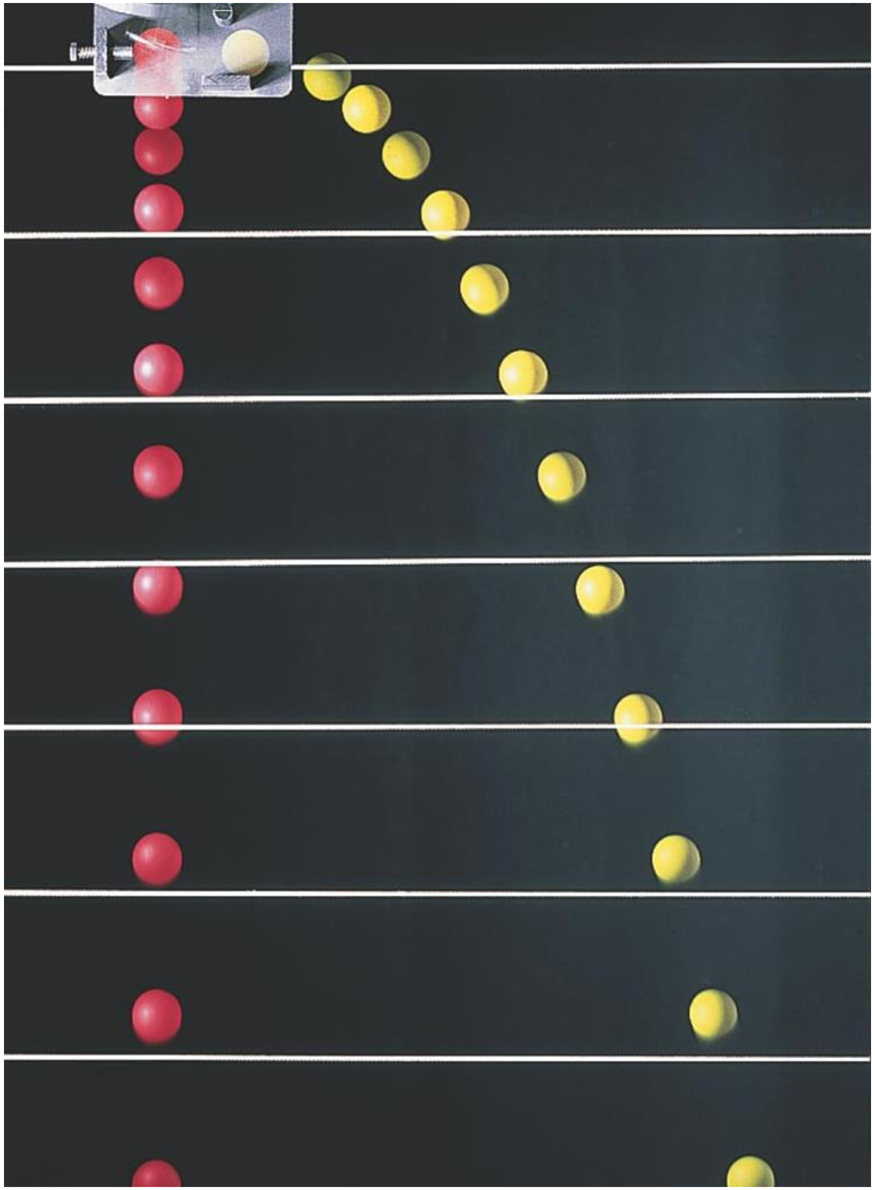
Projectile motion

- A projectile is any body given an initial velocity that then follows a path determined by the effects of gravity and air resistance.
- Begin by neglecting resistance and the curvature and rotation of the earth.



The *x* and *y* motion are separable—Figure 3.16

- The red ball is dropped at the same time that the yellow ball is fired horizontally.
- The strobe marks equal time intervals.
- We can analyze projectile motion as horizontal motion with constant velocity and vertical motion with constant acceleration: $a_x = 0$ and $a_y = -g$.



Analyzing Projectile Motion

Consider the motion as the superposition of the motions in the *x*- and *y*-directions

The actual position and velocity at any time is given by:

$$\mathbf{r_f} = (\mathbf{v_{ox}}\, \mathbf{t} + \tfrac{1}{2}\mathbf{a_x}\, \mathbf{t}^2)\, \hat{\mathbf{i}} + (\mathbf{v_{oy}}\, \mathbf{t} + \tfrac{1}{2}\mathbf{a_y}\, \mathbf{t}^2)\, \hat{\mathbf{j}}$$

$$\mathbf{v_f} = (\mathbf{v_{ix}} + \mathbf{a_x}\, \mathbf{t})\, \hat{\mathbf{i}} + (\mathbf{v_{iy}} + \mathbf{a_y} \mathbf{t})\, \hat{\mathbf{j}}$$

- The initial velocity can be expressed in terms of its components

$$\mathbf{v}_{xi} = \mathbf{v}_i \cos\theta \text{ and } \mathbf{v}_{yi} = \mathbf{v}_i \sin\theta$$

In *x*-direction the motion is motion with constant velocity

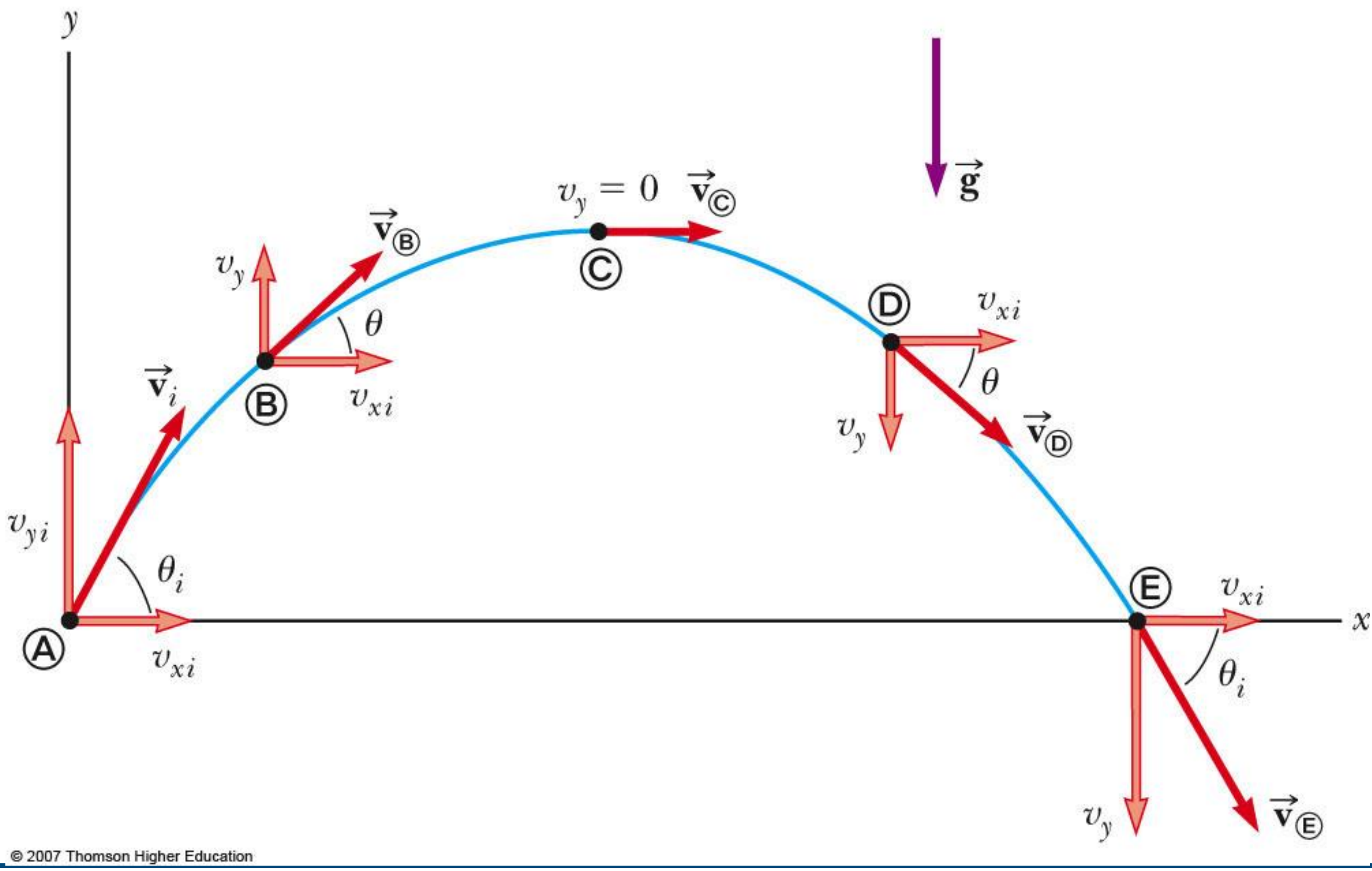
$$a_x = 0 \qquad \mathbf{x} = \mathbf{v_{ox}}\, \mathbf{t} \qquad \mathbf{v_x} = \mathbf{v_{ox}}$$

In y-direction the motion is motion with constant acceleration

$a_y = -g$

$$\mathbf{y} = (\mathbf{v_{oy}}\, \mathbf{t} - \tfrac{1}{2}\mathbf{g}\, \mathbf{t}^2) \qquad \mathbf{v_y} = \mathbf{v_{oy}} - \mathbf{g}\, \mathbf{t}$$

Projectile Motion Diagram



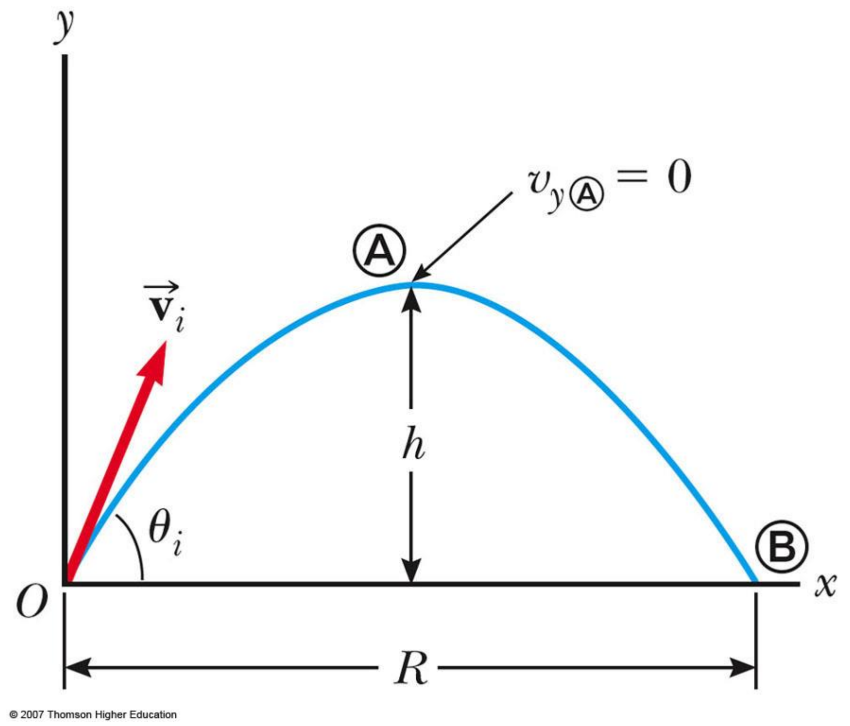
Range and Maximum Height of a Projectile

When analyzing projectile motion, three characteristics are of special interest

The range, *R*, is the horizontal distance of the projectile

The maximum height the projectile reaches is *h*

The total time the projectile spends in air t_{tot}



The range of a projectile can be expressed in terms of the initial velocity vector:

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

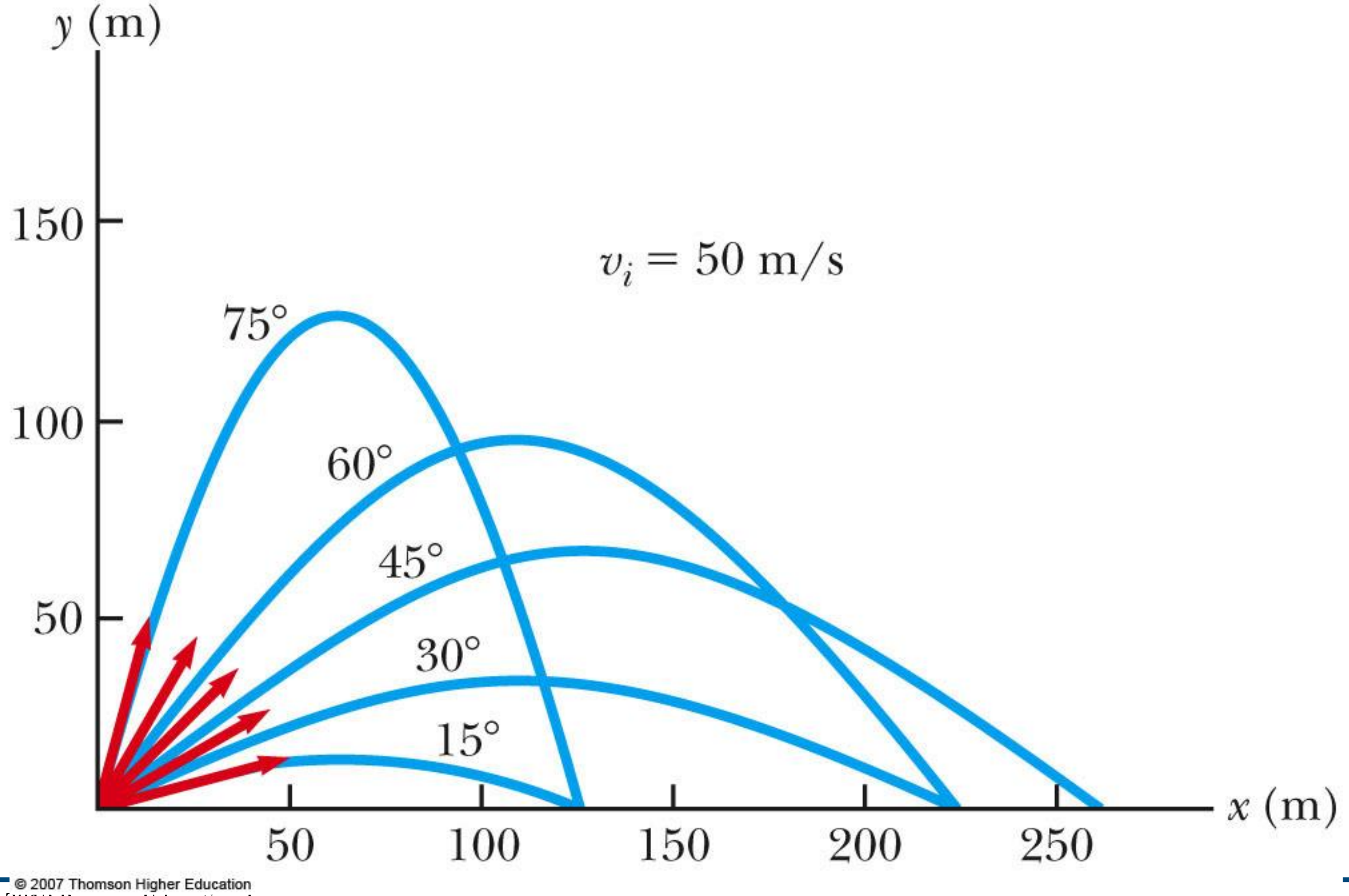
This is valid only for symmetric trajectory

The *y*-component of the velocity is zero at the maximum height of the trajectory

$$y_{max} = \frac{0 - v_{oy}^2}{-2g} = \frac{v_{oy}^2}{2g}$$

The total time is: $t_{tot} = \dfrac{2v_{oy}}{g}$

More About the Range of a Projectile



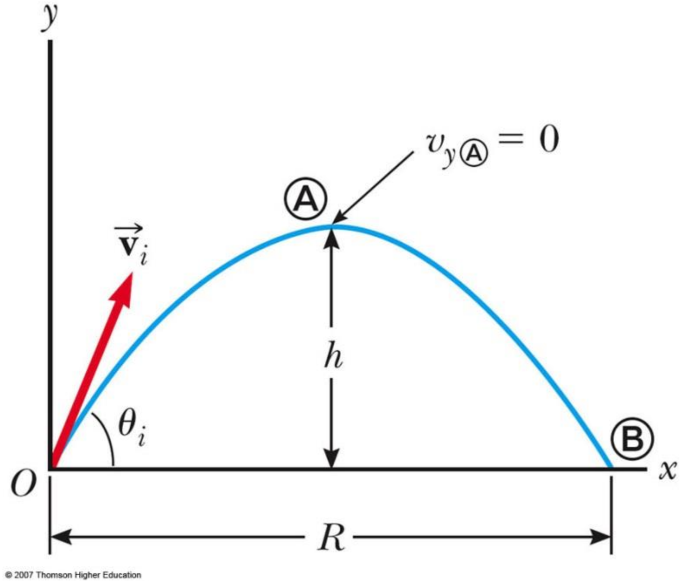
Trajectory Equation *y* vs *x*

$x = (v_o \cos\theta)t$ (1) $y = (v_o \sin\theta)t - ½gt^2$ (2)

By substituting value of *t* from

Equation 1 into Equation 2

the trajectory equation is obtained:



$$\mathbf{y = \tan\theta \cdot x - (g/2v_o^2 \cdot \cos^2\theta)x^2}$$

The equations for projectile motion

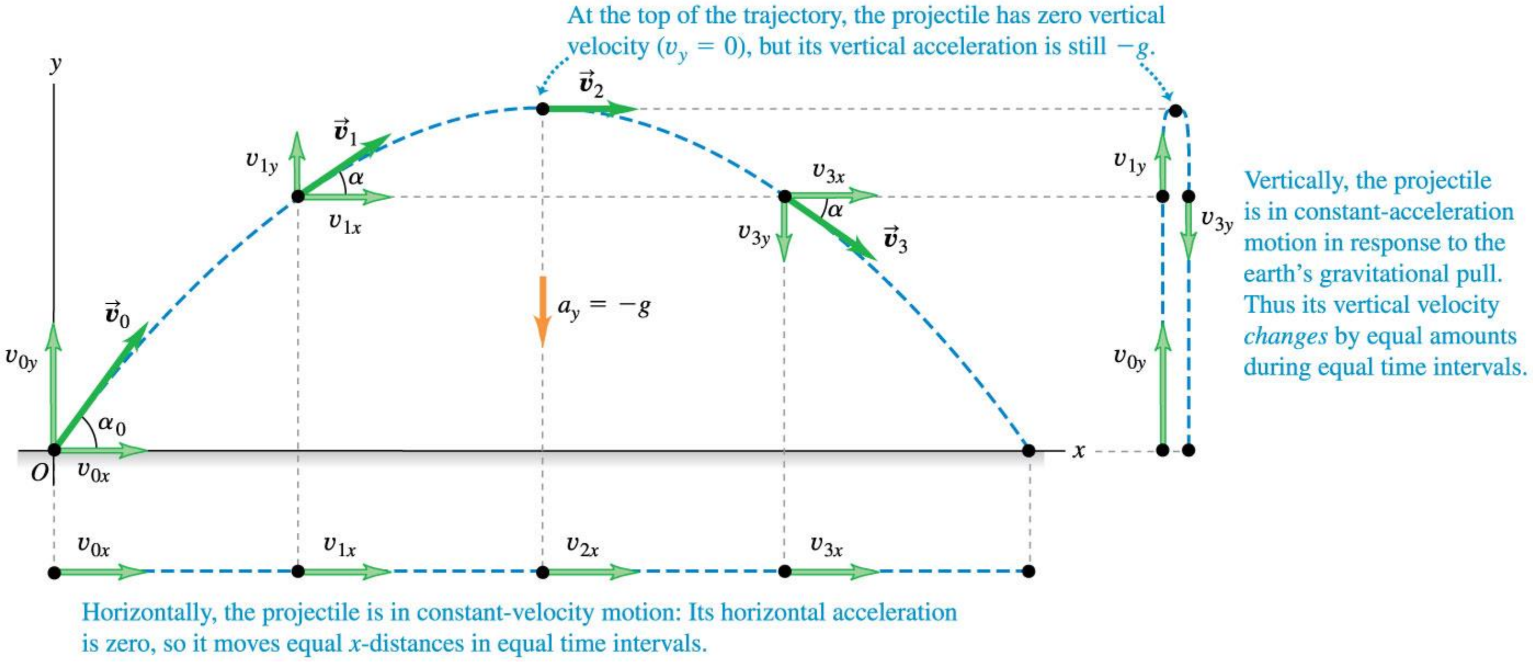
- If we set $x_0 = y_0 = 0$, the equations describing projectile motion are shown at the right.
- The trajectory is a parabola.

$$x = (v_0 \cos\alpha_0)t$$

$$y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\alpha_0$$

$$v_y = v_0 \sin\alpha_0 - gt$$



Analysis Model, Summary

$$v_{ox} = v_o \cos\theta \qquad v_{oy} = v_o \sin\theta$$

$$x = v_{ox}\, t \qquad y = v_{oy}\, t - \tfrac{1}{2} g t^2$$

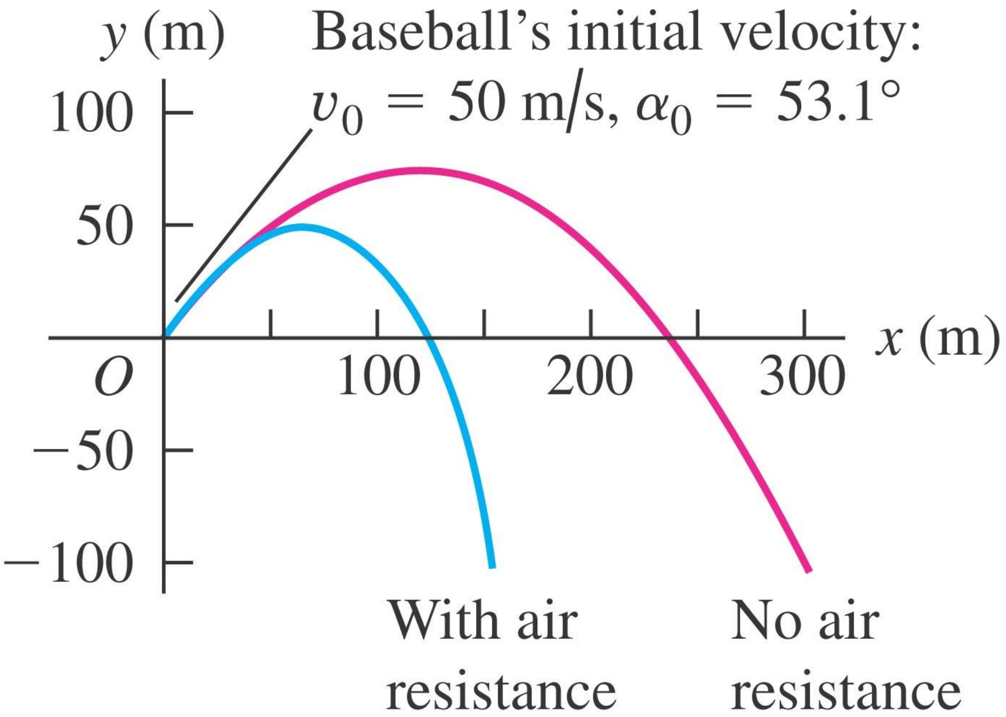
$$v_x = v_{ox} \qquad v_y = v_{oy} - gt$$

$$t_{tot} = \frac{2v_{oy}}{g} \qquad y = \frac{v_y^2 - v_{oy}^2}{-2g} \qquad y_{max} = \frac{v_{oy}^2}{2g}$$

$$R = \frac{v_o^2 \sin(2\theta)}{g} \qquad y = \tan\theta \cdot x - (g/2v_o^2 \cdot \cos^2\theta) x^2$$

The effects of air resistance—Figure 3.20

- Calculations become more complicated.
- Acceleration is not constant.
- Effects can be very large.
- Maximum height and range decrease.
- Trajectory is no longer a parabola.



Height and range of a projectile - Example

A batter hits a baseball so that leaves the bat at a speed of 37.0 m/s at an angle 53.10. (a) Find the position of the ball and its speed at t = 4 s. (b) Find the time where the ball reaches the highest point of its flight, and its height at this time. (c) find the range of the motion.

(a) v_{0x} =(37m/s)cos53.1^0=22.2 m/s

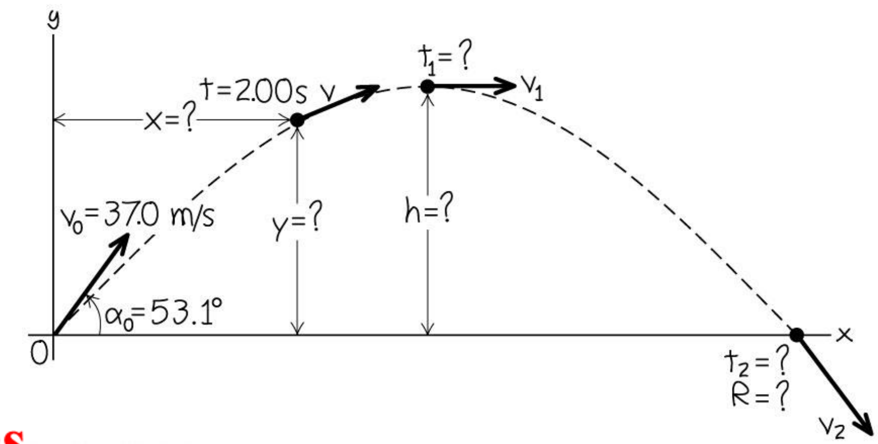
v_{0y} = (37m/s)sin53.1^0=29.6 m/s

x = v_{ox} t =22.2*4=88.8m

y=v_{oy} t - ½gt^2 = 29.6*4-4.9*4^2 =40m

v_y = v_{oy} – gt=29.6-9.8*4=-9.6m/s

v=$\sqrt{22.2^2+(-9.6)^2}$=24.2m/s



(b) v_y = v_{oy} – gt; v_y=0 t=29.6/9.8=3.0s

y_{max}=v_{oy} *3.0 - ½g3.0^2 = 44.7m

$$R = \frac{v_o^2 \sin(2\theta)}{g}$$

(c)

R=[37^2sin(53.1*2)^0]/9.8 =134.1m

or R = 22.2m/s*t_{tot}=22.2*(3.0*2) =133.2

A body projected horizontally - Example

A motorcycle rider rides off the edge of a cliff at horizontal velocity of 9 m/s. (a) Find the motorcycle's position and velocity 0.50 s after it leaves the edge of the cliff. (b) If the height of the cliff is 10 m, how long will it take for rider to reach the ground?

$V_{ox} = 9\ m/s \quad v_{0y} = 0 \quad t = 0.50s$

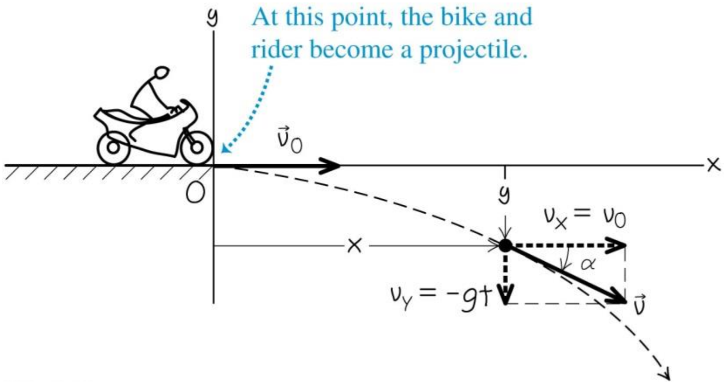
$y = y_0 - \frac{1}{2}gt^2 = 0 - \frac{1}{2}(9.8 m/s^2)(0.50s)^2 = -1.2\ m$

$X = v_{0x}t = (9m/s)(0.50s) = 4.5\ m$

$V_x = 9m/s \quad v_y = -gt = -(9.8\ m/s)(0.50s) = -4.9\ m/s$

$V = \sqrt{9^2 + 4.9^2} = 10.2\ m/s$

(b) $-y = -\frac{1}{2}gt^2 \quad t = \sqrt{\frac{2y}{g}} = 1.42\ s$



Different initial and final heights- Example

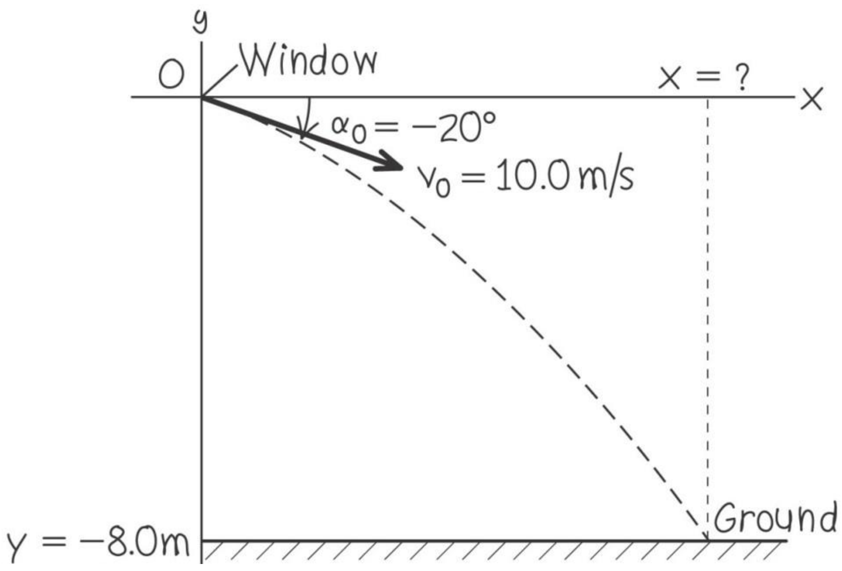
You throw a ball from your window 8.0 m above the ground with an initial velocity of 10 m/s at an angle of 20^0 below the horizontal. How far horizontally from your window will the ball hit the ground? Ignore air resistance.

v_{ox}=(10m/s)cos(-20^0)= 9.4m/s v_{oy}=(10m/s)sin(-20^0)=-3.42 m/s

Find time: $y=v_{oy}\,t - \frac{1}{2}gt^2$ $-8 = -3.43*t-4.9*t^2$ $4.9t^2+3.43t-8=0$

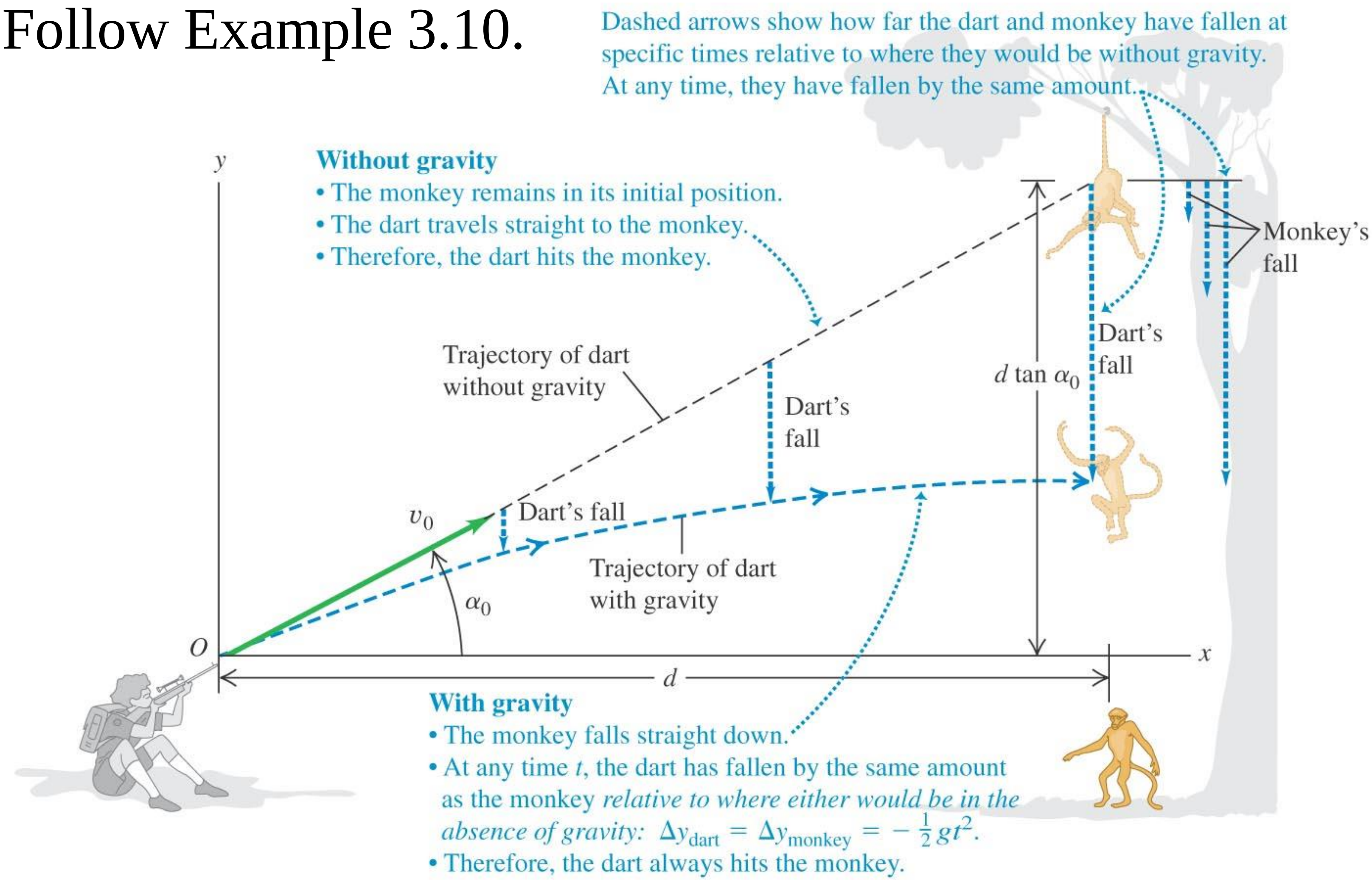
t=0.98s   then $x = v_{ox}\,t$

x=(9.4m/s)*0.98s=9.2 m



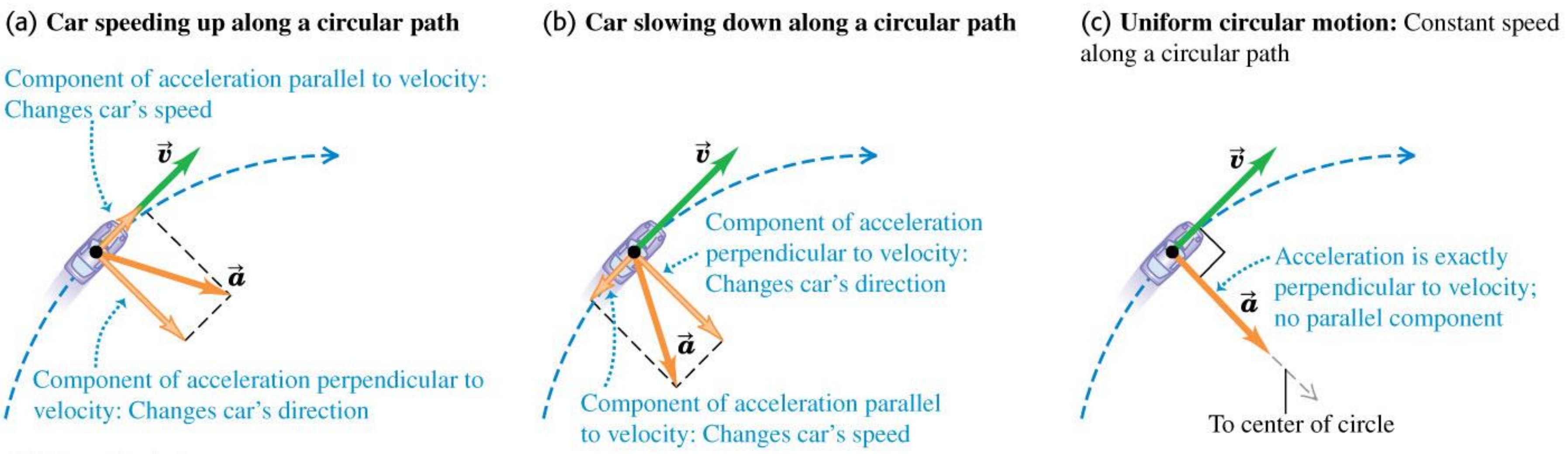
Tranquilizing a falling monkey

- Where should the zookeeper aim?
- Follow Example 3.10.



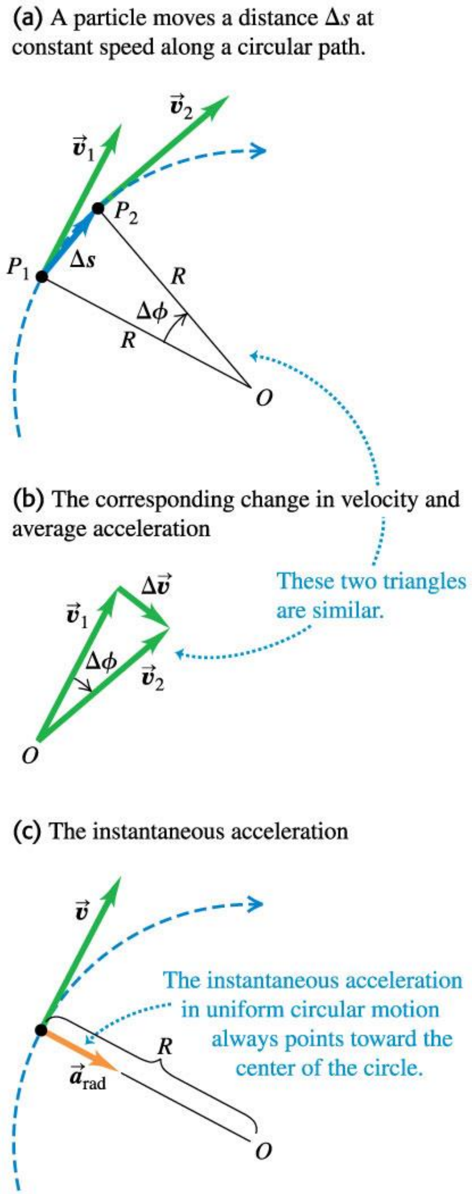
Uniform circular motion—Figure 3.27

- For *uniform circular motion*, the speed is constant and the acceleration is perpendicular to the velocity.



Acceleration for uniform circular motion

- For uniform circular motion, the instantaneous acceleration always points toward the center of the circle and is called the *centripetal acceleration*.
- The magnitude of the acceleration is $a_{rad} = v^2/R$.
- The *period T* is the time for one revolution, and $a_{rad} = 4\pi^2 R/T^2$.



Centripetal acceleration on a curved road

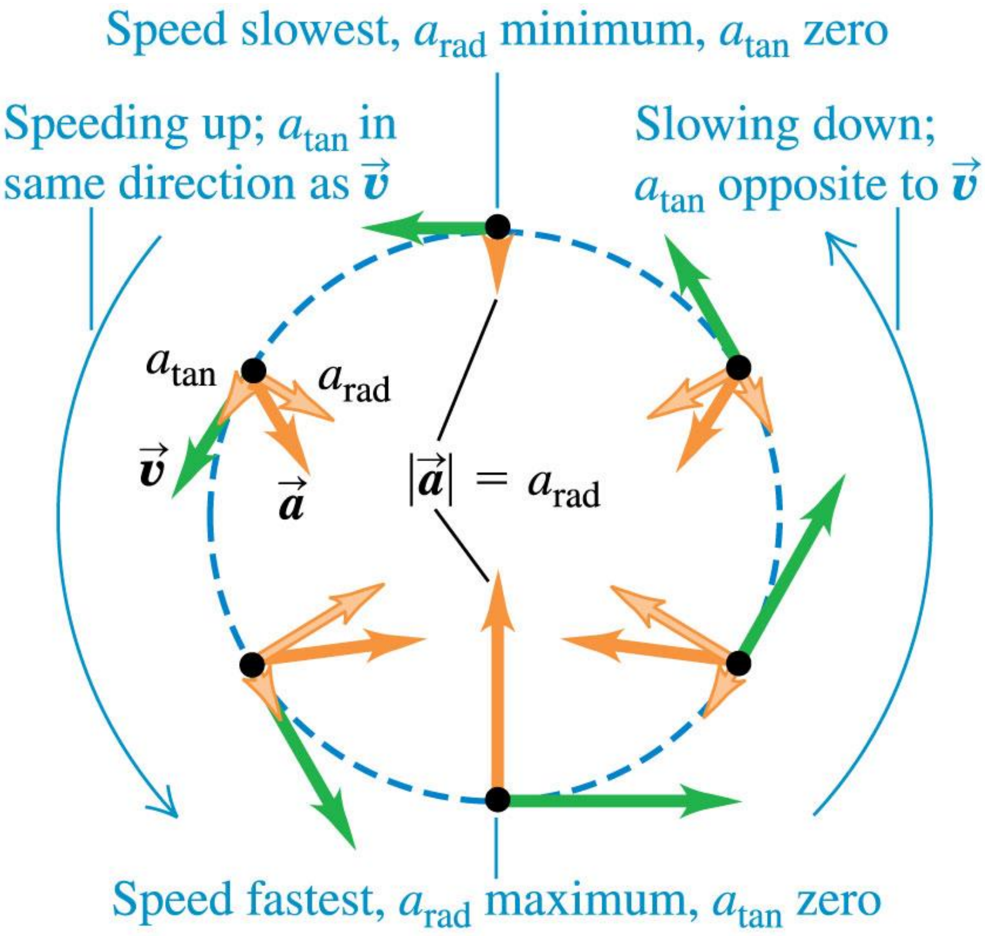
- A sports car has a lateral acceleration as its rounds a curve in the road.
- Follow Example 3.11.

Centripetal acceleration on a carnival ride

- Passengers move horizontally at constant speed with a known period.
- Follow Example 3.12.

Nonuniform circular motion—Figure 3.30

- If the speed varies, the motion is *nonuniform circular motion*.
- The radial acceleration component is still $a_{rad} = v^2/R$, but there is also a tangential acceleration component a_{tan} that is *parallel* to the instantaneous velocity.

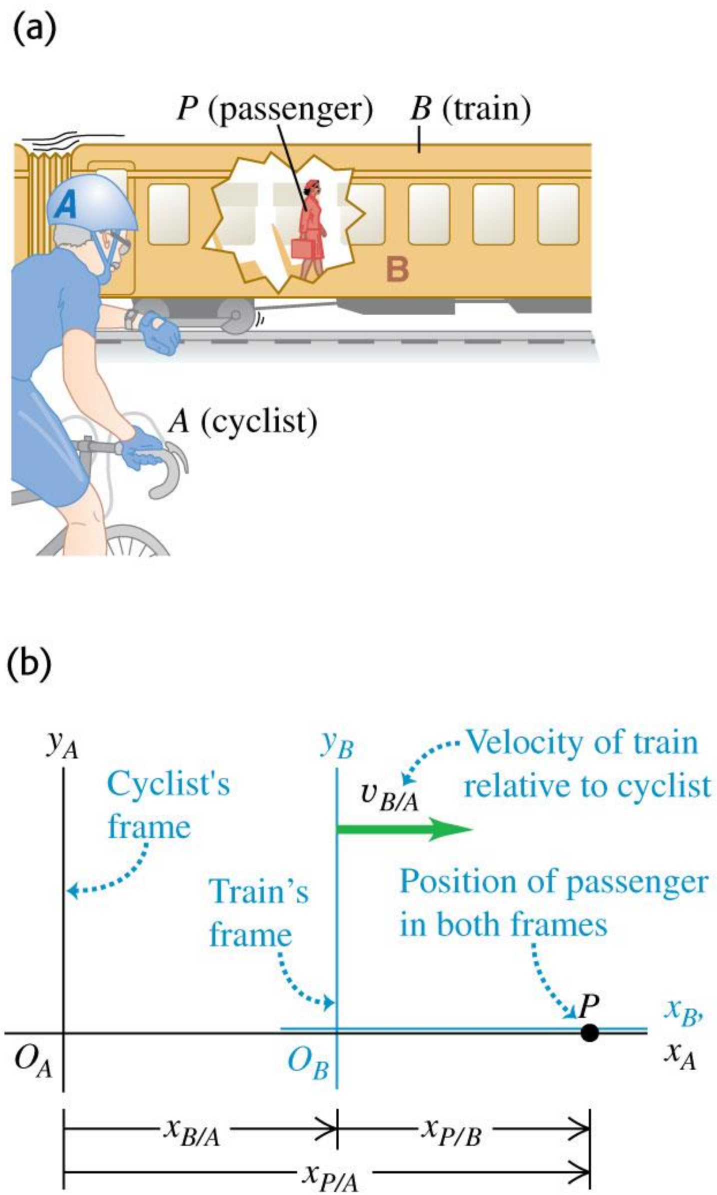


Relative velocity—Figures 3.31 and 3.32

- The velocity of a moving body seen by a particular observer is called the velocity *relative* to that observer, or simply the *relative velocity*.
- A *frame of reference* is a coordinate system plus a time scale.

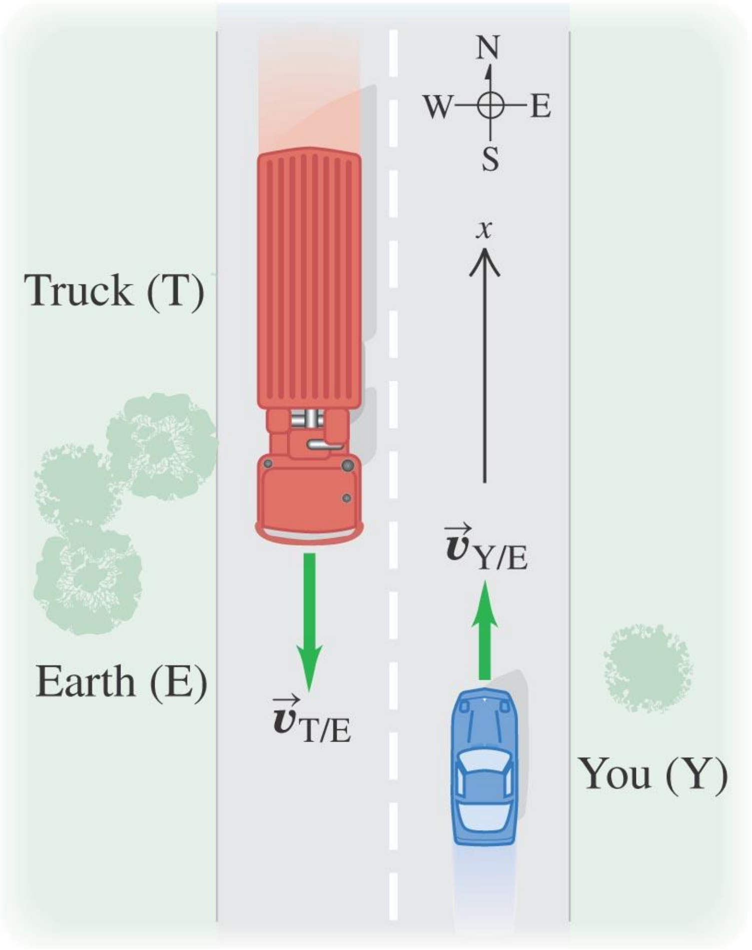
Relative velocity in one dimension

- If point P is moving relative to reference frame A, we denote the velocity of P relative to frame A as $v_{P/A}$.
- If P is moving relative to frame B and frame B is moving relative to frame A, then the x-velocity of P relative to frame A is $v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$.



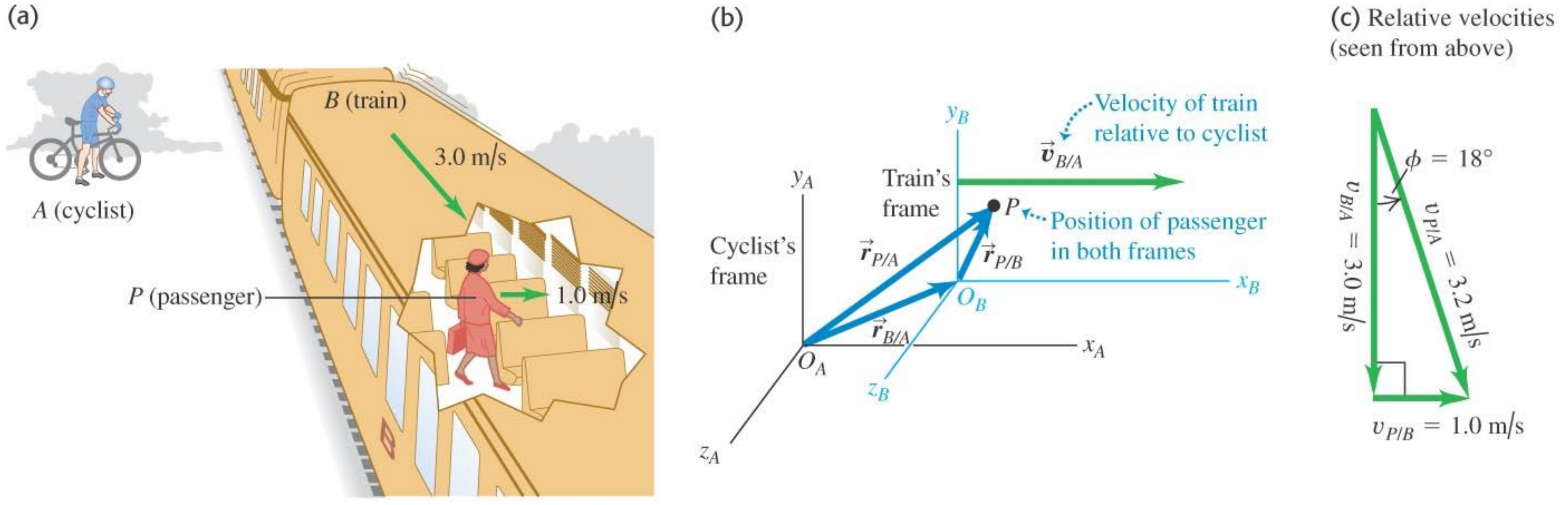
Relative velocity on a straight road

- Motion along a straight road is a case of one-dimensional motion.
- Follow Example 3.13 and Figure 3.33.
- Refer to Problem-Solving Strategy 3.2.



Relative velocity in two or three dimensions

- We extend relative velocity to two or three dimensions by using vector addition to combine velocities.
- In Figure 3.34, a passenger's motion is viewed in the frame of the train and the cyclist.



Flying in a crosswind

- A crosswind affects the motion of an airplane.
- Follow Examples 3.14 and 3.15.
- Refer to Figures 3.35 and 3.36.