Chapter 6

Work and Kinetic Energy

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 6

- To understand and calculate the work done by a force
- To understand the meaning of kinetic energy
- To learn how work changes the kinetic energy of a body and how to use this principle
- To relate work and kinetic energy when the forces are not constant or the body follows a curved path
- To solve problems involving power

Introduction

- In this chapter, the introduction of the new concepts of *work*, *energy*, and the *conservation of energy* will allow us to deal with such problems.

Work

The work, W, done on a system by a force exerted on the system is a **scalar (dot)** product of two vectors: the applied force $\vec{F}$ and the displacement of the point of application of the force $\vec{s}$. If the force has a constant value then work $W = \vec{F} \cdot \vec{s}$

$W = Fs\cos\phi$. or $W = F_x s_x + F_y s_y + F_z s_z$

More About Work

The sign of the work depends on the direction of the force relative to the displacement

- Work is positive when projection of $\vec{F}$ onto $\vec{s}$ is in the same direction as the displacement
- Work is negative when the projection is in the opposite direction

Work is a scalar quantity

The unit of work is a joule (J)

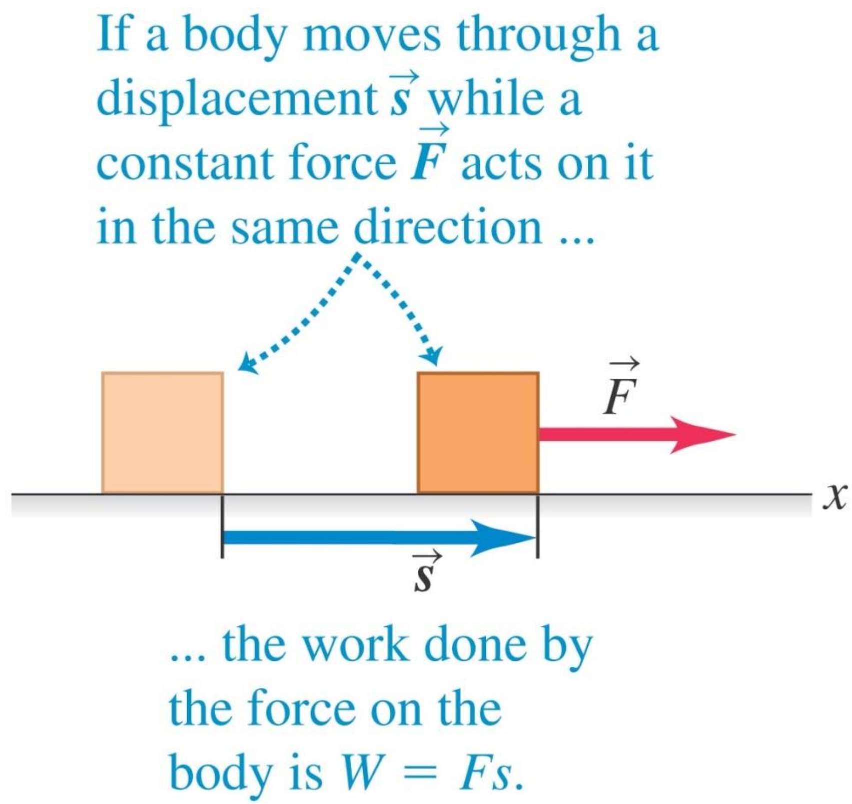
- **1 joule = 1 newton · 1 meter**
- **J = N · m**

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force

$$W_{total} = F_{net} \cdot \Delta r$$

Work - example

- A force on a body does *work* if the body undergoes a displacement.

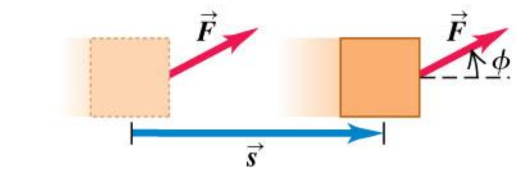


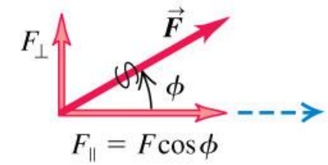
Positive, negative, and zero work

- A force can do positive, negative, or zero work depending on the angle between the force and the displacement.

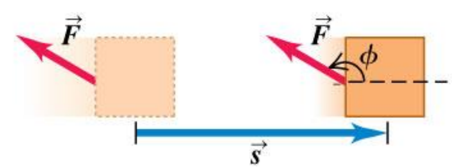
Direction of Force (or Force Component)	Situation	Force Diagram

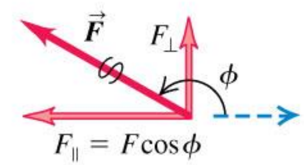
(a) **Force $\vec{F}$ has a component in direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *positive*.



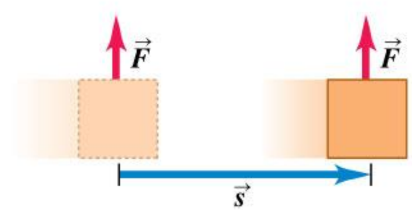


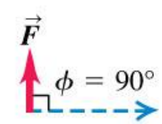
(b) **Force $\vec{F}$ has a component opposite to direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).





(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.



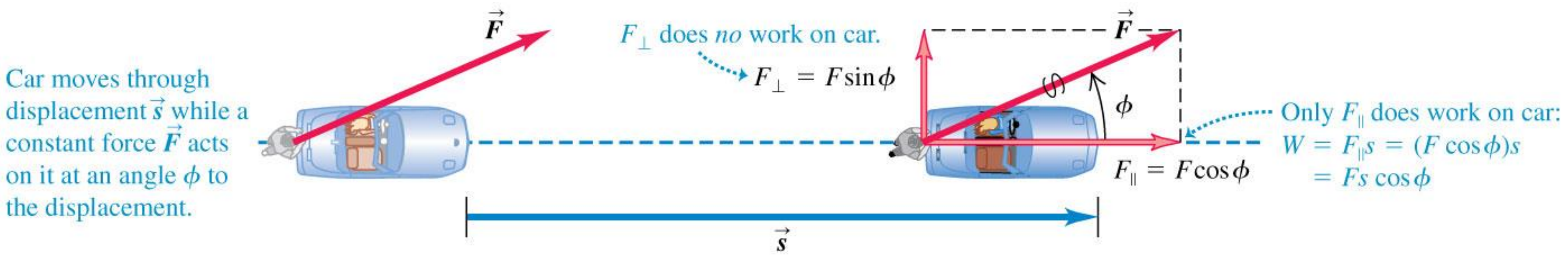


Work done by a constant force

- The work done by a constant force acting at an angle ϕ to the displacement is $W = Fs \cos \phi$.

- Example : Steve exerts a steady force of magnitude 210 N at an angle of 30^0 to the direction of motion. as he pushes it a distance 18m. Find work done by Steve on the car.

- $W = Fs\cos\Phi$ = 210N*18 m*$\cos 30^0$ = 3300 J



Example

A particle moving in *xy* plane undergoes displacement given by $\vec{s}$ = (15**i** -12**j**)m as a constant force $\vec{F}$=(5**i** +2**j**)N. Calculate the work done by the force on the particle.

$$W = F_x d_x + F_y d_y = 5N \cdot 15m - 2N \cdot 12m = 51\ J$$

Work done by gravitational force.

$W = mg \cdot \Delta x \cdot \cos 90^0 = 0$

$W = mg \cdot \Delta y \cdot \cos 180^0 = -mg \cdot \Delta y$

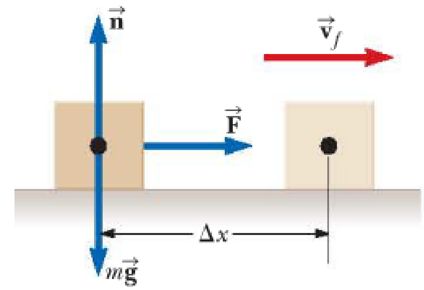
If $\Delta y = y_f - y_i$ then $W = -mg \cdot (y_f - y_i)$

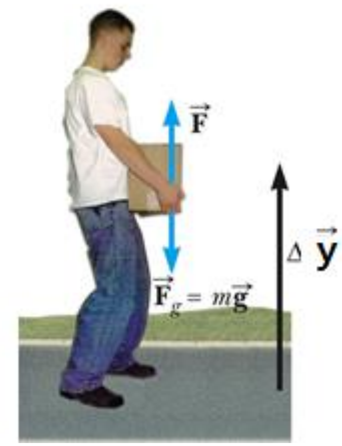
or $W = mg \cdot (y_i - y_f)$

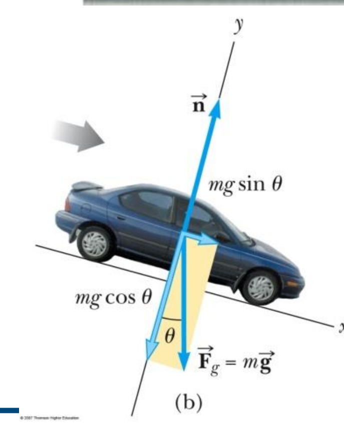
$W = mg \cdot \Delta x \cdot \cos(90^0 - \theta) =$

$mg \cdot \Delta x \cdot \sin\theta = mg \cdot (y_i - y_f)$

$$\mathbf{W = mg \cdot (y_i - y_f)}$$



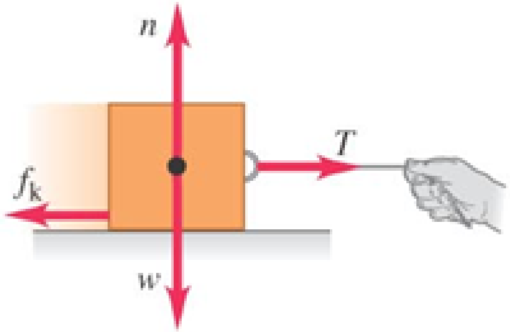




Work Done by Kinetic Friction Force

Kinetic friction force is always parallel to the surface and points in the direction opposite to the direction of motion. The angle between the displacement and the kinetic friction force is 180^0 . Work done by kinetic friction force is:

$$W_{fr} = F_k \cdot s \cdot \cos 180^0 = - F_k \cdot s$$



Work Done By Multiple Forces

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{net} = \left(\sum F_x\right)\Delta x = F_{net}\,\Delta x$$

The total work done on the body can be expressed also as a algebraic sum of the quantities of work done by individual forces

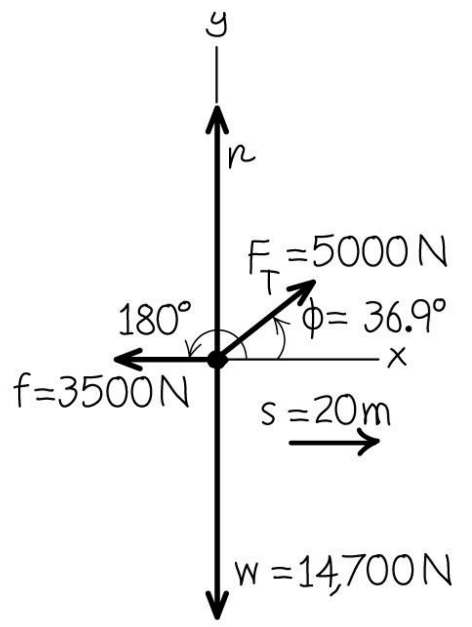
$$W_{net} = W_1 + W_2 + \ldots\ldots + W_n$$

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

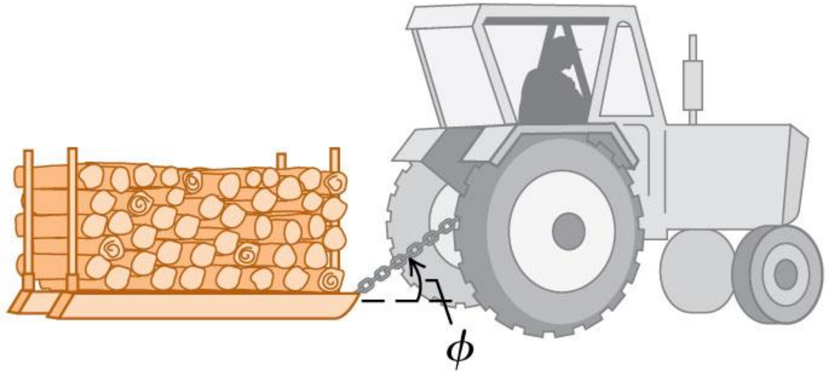
Work done by several forces - Example

A farmer hitches her tractor to a sled loaded with firewood and pulls it distance of 20 m along level ground. The total weight of sled and load is 14700 N. The tractor exerts constant 5000-N force at an angle of 36.9^0 above the horizontal. A 3500-N friction force opposes the sled's motion Find the total work done by all the forces

(b) Free-body diagram for sled



(a)



Solution

Method 1: $W_{tot} = (F_{net})_x \cdot s$

$F_{net} = F_T \cos\Phi - F_k$

$F_{net} = 5000\cos 36.9^0 - 3500\text{ N} = 500\text{ N}$

$W_{tot} = 500\text{ N}\cdot 20\text{ m} = 10000\text{J}$

Method 2: $W_{tot} = W_1 + W_2 + W_3 + W_4$

$W_T = 5000\text{N}\cdot 20\text{m}\cdot\cos 36.9^0 = 80000\text{J}$

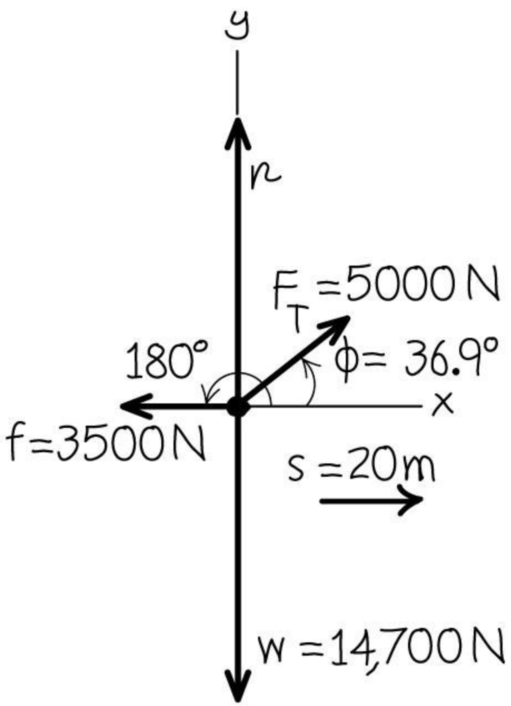
$W_{fr} = -3500\text{N}\cdot 20\text{m} = -70000\text{J}$

$W_w = 14700\text{N}\cdot 20\text{m}\cdot\cos 270^0 = 0$

$W_n = n\cdot 20\text{m}\cdot\cos 90^0 = 0$

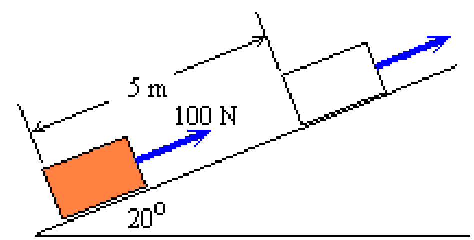
$W_{tot} = 80000\text{J} - 70000\text{J} + 0 + 0 = 10000\text{J}$

(b) Free-body diagram for sled



Example

A crate of mass 10 kg is pulled up a rough 20^0–incline a distance 5 m. The pulling force is parallel to the incline and has a magnitude of 100 N. The coefficient of kinetic friction is 0.25. (a) How much work is done by the gravitational force? b) How much work is done by friction? c) How much work is done by the 100-N force? (d) What is the total work done on the crate ?



$$W_g = w\cdot s\cdot \cos\Phi = 98\ N\cdot 5m\cdot \cos(90^0+20^0) = -168J$$

$$W_{fr} = F_k\cdot s\cdot \cos 180^0, \quad F_k = \mu_k n = 0.25\cdot 98N\cos 20^0 = 23N$$

$$W_{fr} = -23\ N\cdot 5m = -115\ J$$

$$W_F = F\cdot s\cdot \cos\Phi = 100N\cdot 5m\cos 0^0 = 500\ J$$

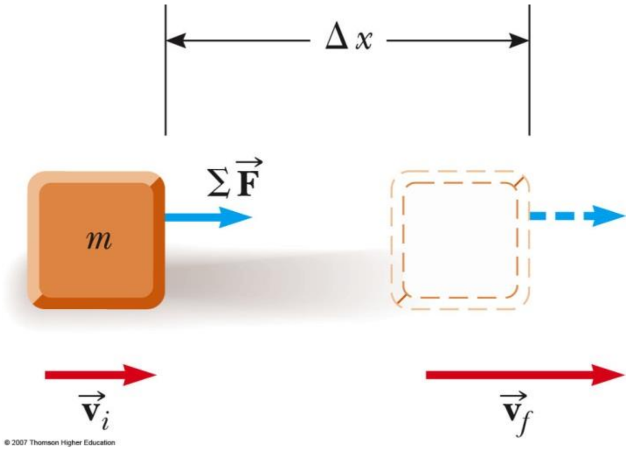
$$W_{tot} = 500J - 168\ J - 115J = 217\ J$$

Work - Kinetic Energy Theorem

For constant forces

$$W_{net} = F_{net} \cdot \Delta x = \frac{m \cdot a \, (v_f^2 - v_i^2)}{2a} =$$

$$\tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$



$K = \tfrac{1}{2} m v^2$ - kinetic energy

$$W_{net} = K_f - K_i$$

Kinetic Energy

Kinetic Energy is the energy of a particle due to its motion

- $K = \frac{1}{2} mv^2$
  - K is the kinetic energy
  - m is the mass of the particle
  - v is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into a system

Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states

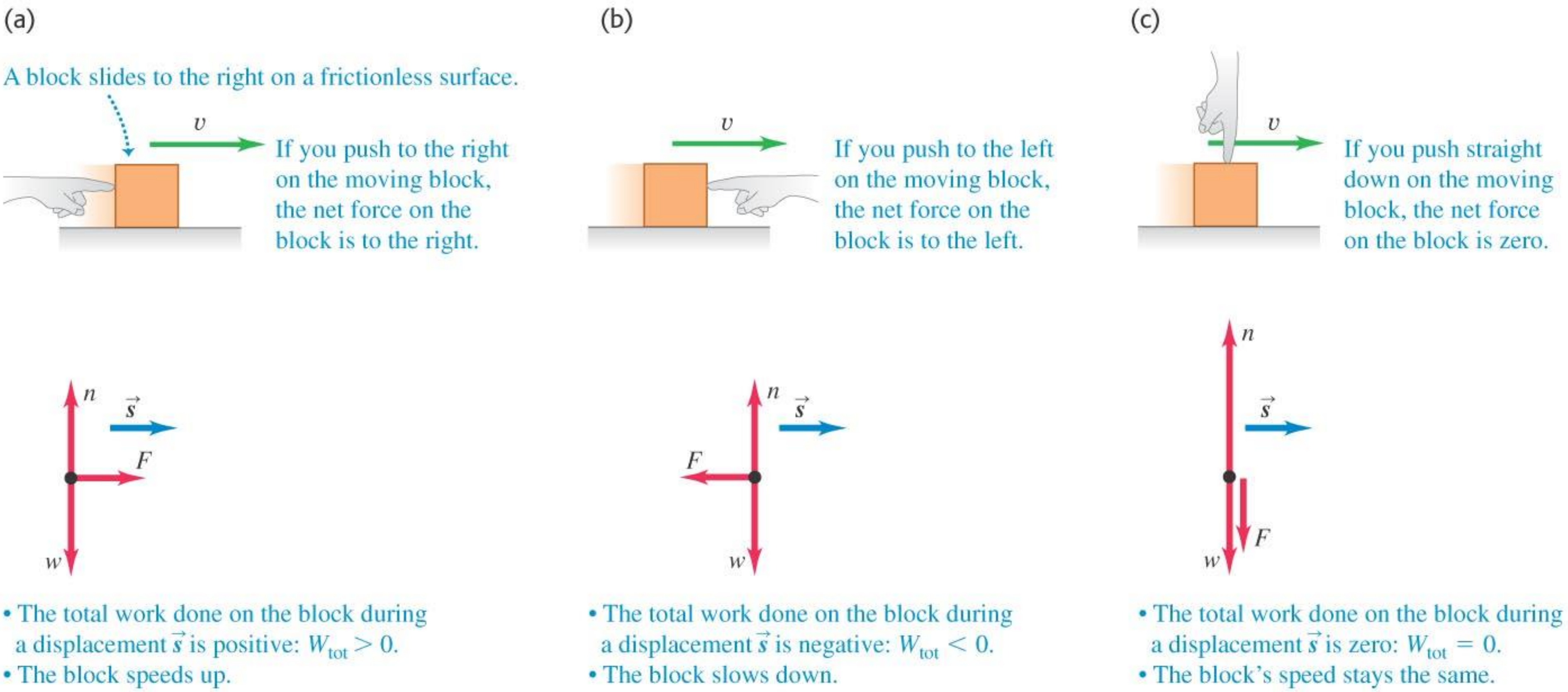
$$\Sigma W = K_f - K_i = \Delta K$$

The work done by the net force equals the change in kinetic energy of the system.

- The speed of the system increases if the net work done on it is positive
- The speed of the system decreases if the net work is negative
- Also valid for changes in rotational speed

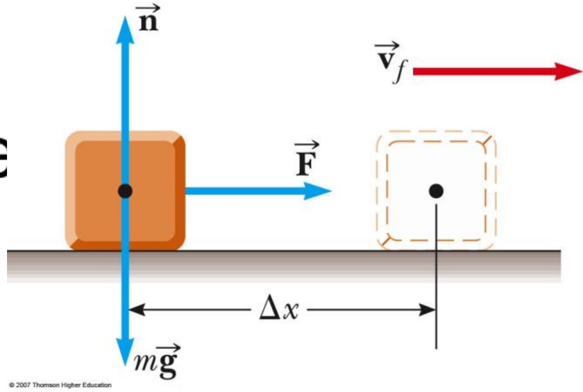
Kinetic energy

- The *kinetic energy* of a particle is $K = 1/2\ mv^2$.
- The net work on a body changes its speed and therefore its kinetic energy, as shown in Figure 6.8 below.



Work-Kinetic Energy Theorem, Example

A 2.0 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 8N. The coefficient of kinetic friction between the block and the surface is 0.25. Find the block's speed after is has moved 3m.

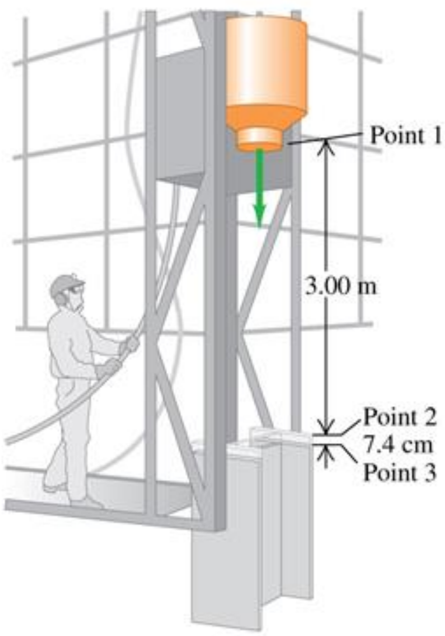


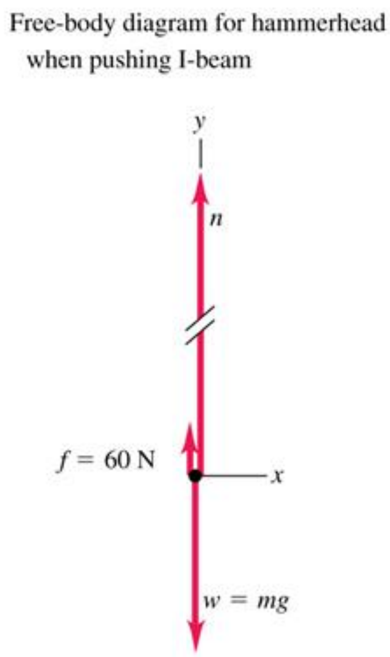
$(F_{net})_x = F - F_k = F - \mu_k n = F - \mu_k mg = 8N - 0.25 \cdot 2kg \cdot 9.8m/s^2 = 3.1\ N$ $\quad W_{tot} = 3.1N \cdot 3m = 9.3\ J$ $\quad W_{tot} = K_f - K_i = \frac{1}{2} m v_f^2 - 0$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 9.3J}{2kg}} = 3.05m/s$$

Forces on a hammerhead

The hammerhead of a pile driver is used to drive a beam into the ground. The 200-kg hammerhead of a pile driver is lifted 3.00m above the top of a vertical I-beam being driven into the ground. The hammerhead is then dropped, driving the I-beam 7.4 cm deeper into the ground. The vertical guide rails exert a 60-N friction force on the hammerhead. Find the average force the hammerhead exerts on the I-beam





Forces on a hammerhead

Solution:

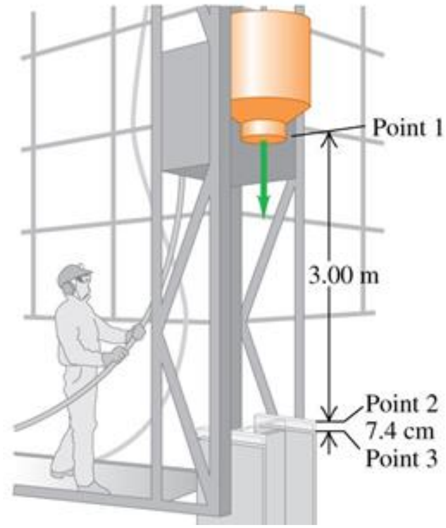
Total work: $W_{tot}=mg\cdot s_{13}\cos 0^0+f\cdot s_{23}\cos 180^0-n\cdot s_{23}\cos 180^0$

$W_{tot}=K_F-K_i=0$ $mg=200kg*9.8m/s^2 = 1960N$

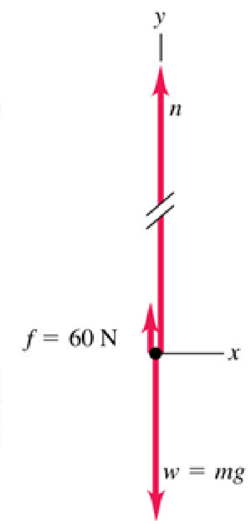
$S_{13} = 3.074m$ $s_{23}=0.074\ m$

$mg\cdot s_{13}^{-}\ f\cdot s_{13} - n\cdot s_{23}=0$

$$n=\frac{1960N\cdot 3.074m-60N\cdot 3.074m}{0.074m}=79000N$$

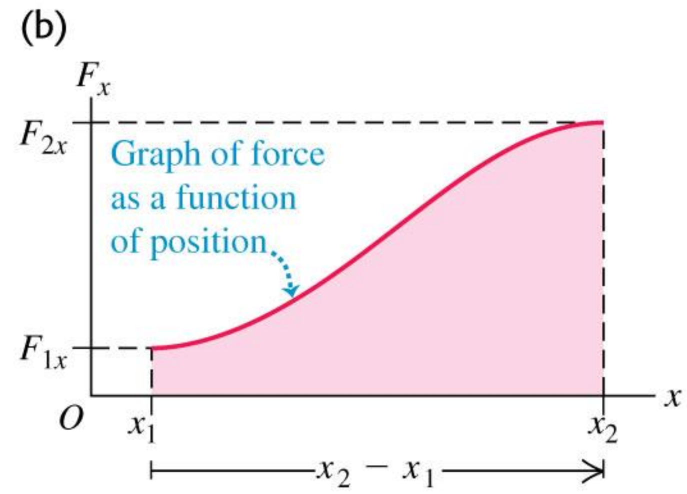


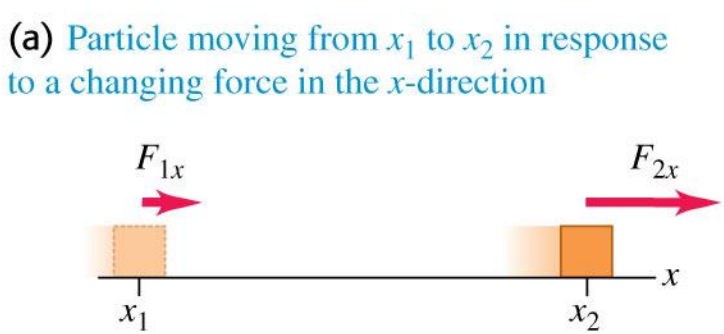
Free-body diagram for hammerhead when pushing I-beam

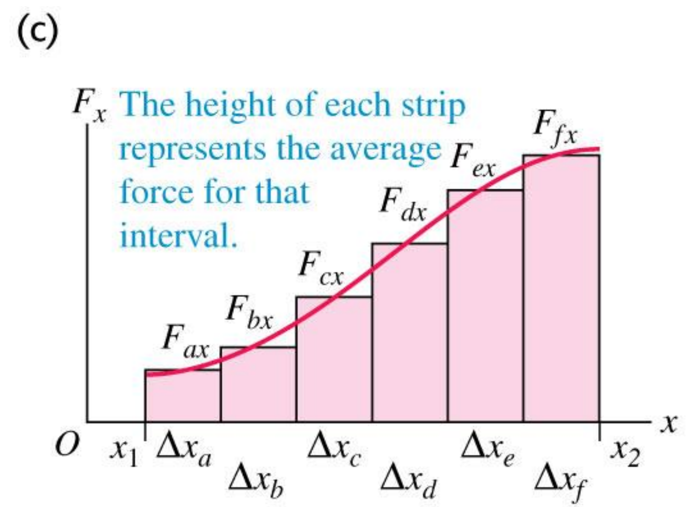


Work and energy with varying forces—

- Many forces, such as the force to stretch a spring, are not constant.
- We can approximate the work by dividing the total displacement into many small segments.



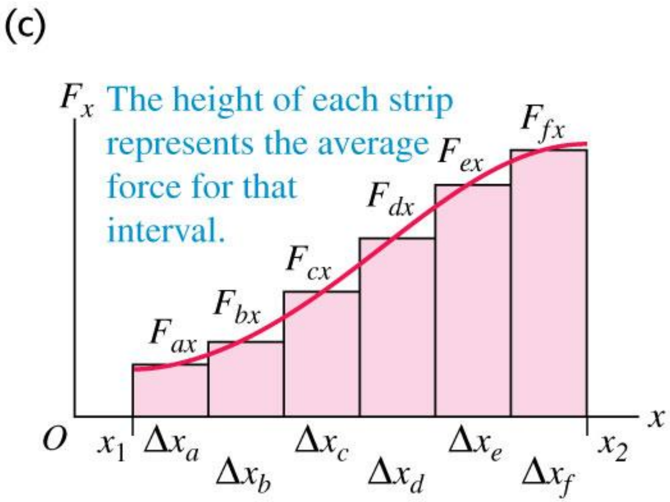




Work Done by a Varying Force, cont

The limit of sum of the area of rectangle is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$



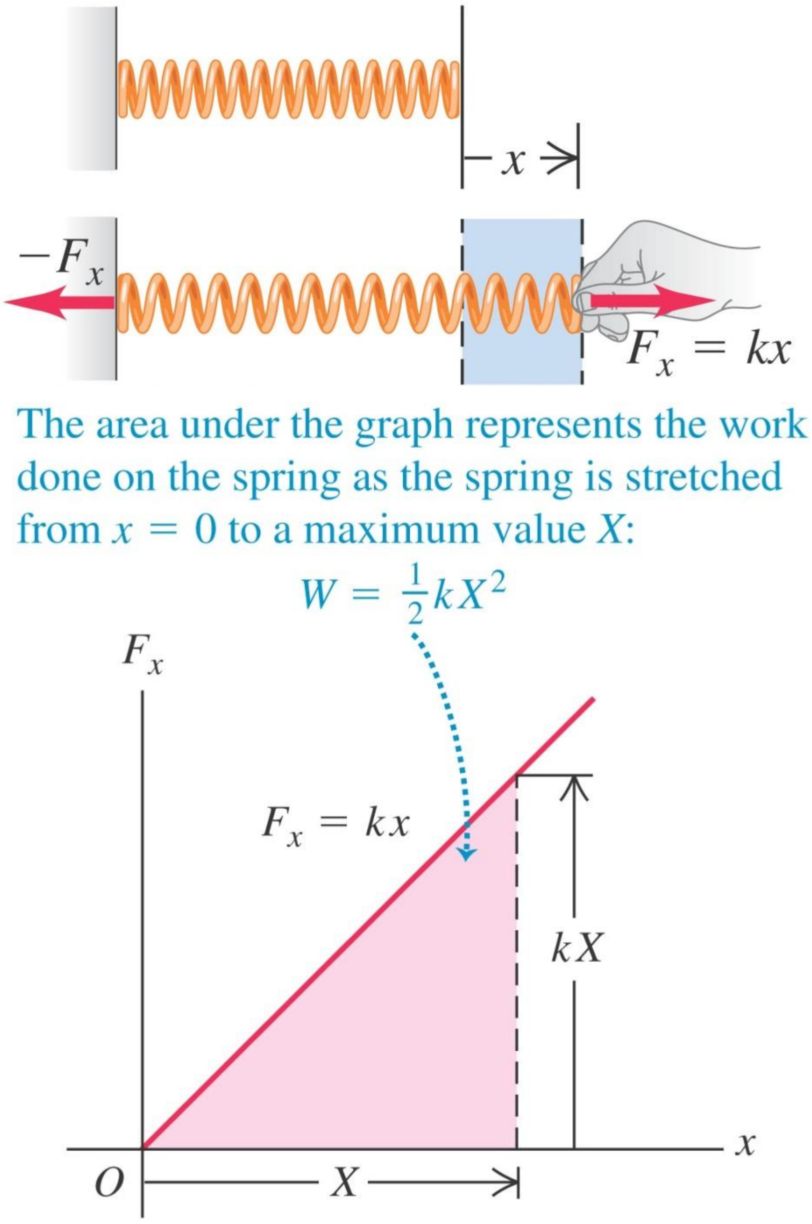
Therefore, $W = \int_{x_i}^{x_f} F_x dx$

The work done by variable force is equal to the area under the curve between x_i and x_f

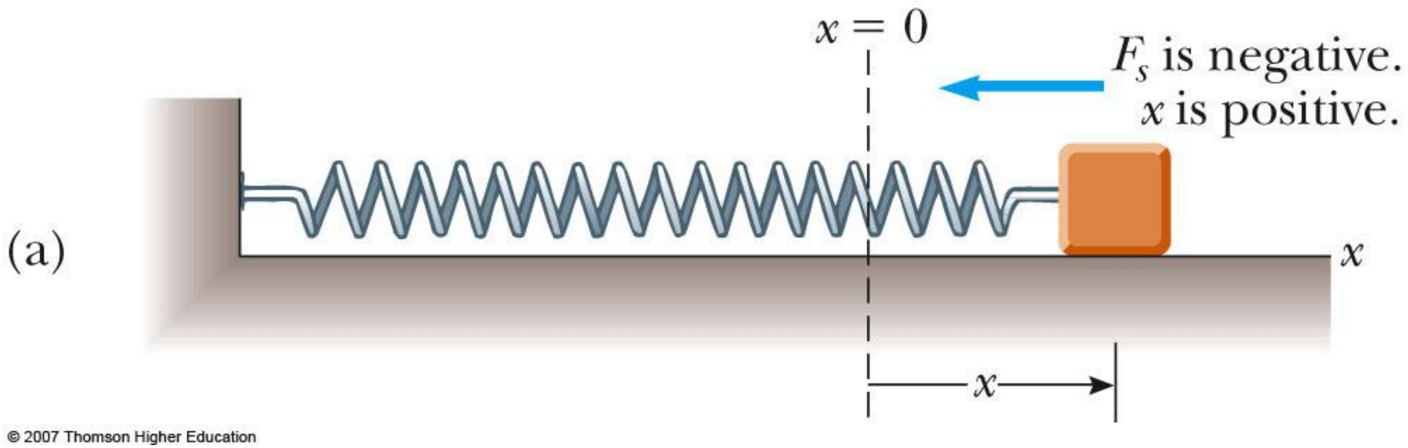
Stretching a spring

- The force required to stretch a spring a distance x is proportional to x: $F_x = kx$.
- k is the *force constant* (or *spring constant*) of the spring.
- The area under the graph represents the work done on the spring to stretch it a distance x:

$$W = \tfrac{1}{2}kx^2.$$



Hooke's Law



The force exerted by the spring is

$$F_s = - kx$$

- x is the position of the block with respect to the equilibrium position (x = 0)
- k is called the spring constant or force constant and measures the stiffness of the spring

This is called Hooke's Law

Work Done by a Spring, cont.

- Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$
- The work done by the spring on the block is

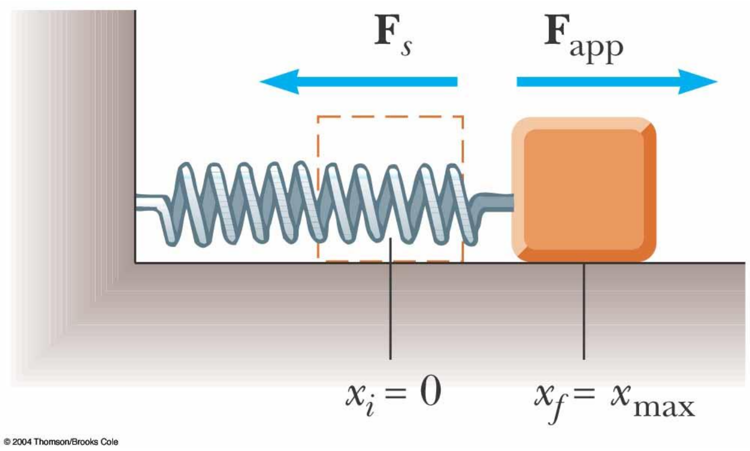
$$W_s = \int_{x_i}^{x_f} \left(-kx\right) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

  - If the motion ends where it begins, W = 0

Spring with an Applied Force

- Suppose an external agent, F_{app}, stretches the spring
- The applied force is equal and opposite to the spring force
- $F_{app} = -F_s = -(-kx) = kx$
- Work done by F_{app} is equal to $-\frac{1}{2} kx^2_{max}$
- The work done by the applied force is

$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

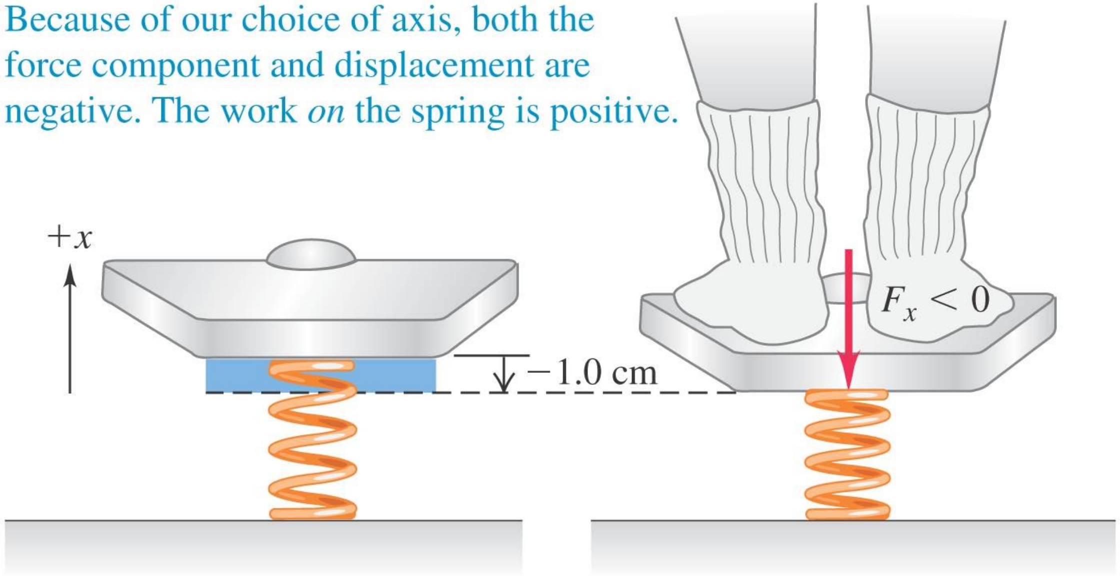


Work done on a spring scale

A woman weighing 600 N steps on a scale that contains a stiff spring. In equilibrium, the spring is compressed1.0 cm under her weight. Find the spring constant of the spring and the total work done on it during the compression.

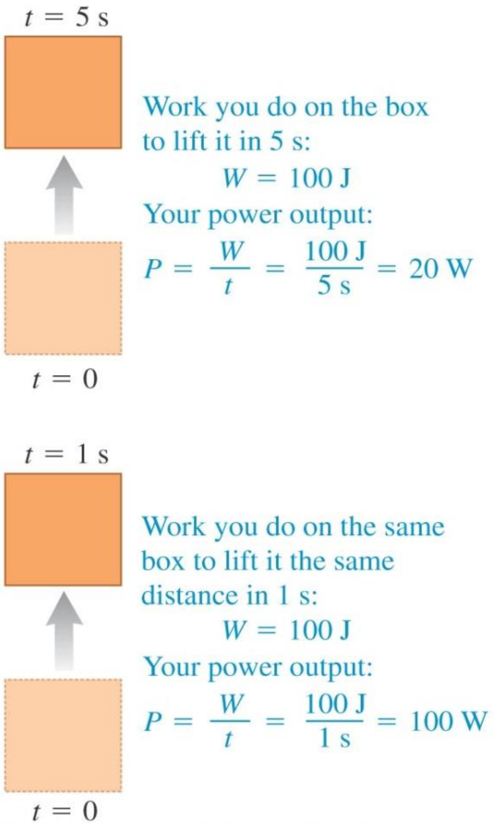
$$F = kx \quad k = \frac{F}{x} = \frac{-600N}{0.01m} = 60000N/m$$

work done on spring $W = \frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2} 6x10^4 N/m \cdot [(-0.01m)^2 - 0] = 3J$



Power

- *Power* is the rate at which work is done.
- *Average power* is $P_{av} = \Delta W/\Delta t$ and *instantaneous power* is $P = dW/dt$.
- The SI unit of power is the *watt* (1 W = 1 J/s), but other familiar units are the *horsepower* and the *kilowatt-hour*.
- 1 hp =746 W



Instantaneous Power and Average Power

The instantaneous power is the limiting value of the average power as Δt approaches zero

$$\bar{\wp} = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

The SI unit of power is called the watt

- 1 watt = 1 joule / second = 1 kg · m^2 / s^2

A unit of power in the US Customary system is horsepower

- 1 hp = 746 W

Units of power can also be used to express units of work or energy

- 1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J

Example

The loaded cab of the elevator has a mass of 1800 kg and moves 24 m up the shaft in 8 s . A constant friction force of 4000 N retards its motion. How much power must a motor deliver to lift the elevator cab?

$\Delta y = \frac{1}{2} a t^2$ $a = \frac{2 \cdot 24m}{8^2} = 0.75 m/s^2$ T-mg=ma

$T = 1800kg(9.8+0.75) = 18990N$

$P = \frac{W}{t} = \frac{T \cdot s\cos\theta}{t} = \frac{18990N \cdot 24m\cos 0^0}{8\ s} = 57kW$

Force and power

Each of the four jet engines develops a thrust of 322000 N. When the airplane is flying at 250 m/s, what horsepower does each engine develop?

$$P = F\cdot v\cos 0^0 = 322000N\cdot 250m/s\cos 0^0 = 8.05\text{x}10^7 W$$

$$1hp = 746W \quad P = 108000hp$$

(a)

(b)

Motion on a curved path—Example 6.8

- A child on a swing moves along a curved path.