# Chapter 7

# Potential Energy and Energy Conservation

PowerPoint® Lectures for
***University Physics, Thirteenth Edition***
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# Goals for Chapter 7

- To use gravitational potential energy in vertical motion
- To use elastic potential energy for a body attached to a spring
- To solve problems involving conservative and nonconservative forces
- To determine the properties of a conservative force from the corresponding potential-energy function

# Conservative Forces

- Conservative force is a such force for which the work done by this force on a particle moving between any two points is independent of the path taken by the particle
- The work done by a conservative force on a particle moving through any closed path is zero
  - A closed path is one in which the beginning and ending points are the same
- Examples of conservative forces:
  - Gravity
  - Spring force

# Nonconservative Forces

- A nonconservative force does not satisfy the conditions of conservative forces
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system
- Examples of nonconservative forces:
  - Friction
  - Tension

# Conservative and nonconservative forces

- A *conservative force* allows conversion between kinetic and potential energy. Gravity and the spring force are conservative.
- The work done between two points by any conservative force

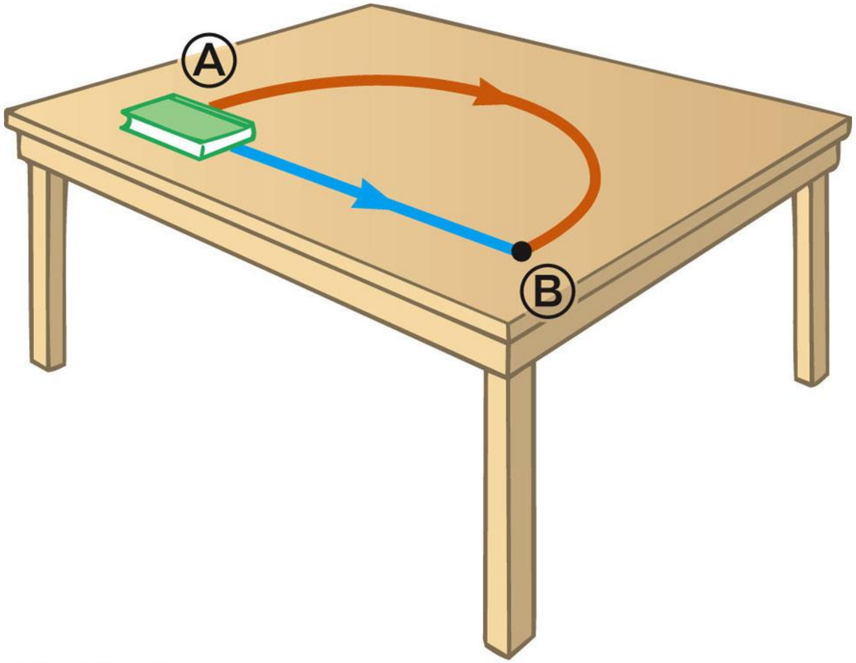
  a) is reversible.

  b) is independent of the path between the two points.

  c) is zero if the starting and ending points are the same.
- A force (such as friction) that is not conservative is called a *nonconservative force*, or a *dissipative force*.

# Nonconservative Forces, cont

- The value of work done by friction is greater along the brown path than along the blue path
- Because the work done depends on the path, friction is a nonconservative force



# Conservative Forces and Potential Energy

- Change in a potential energy of the system, $\Delta U$, is defined as being equal to the negative work done by a conservative force
- The work done by such a force, $F$, is

$$W_C = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

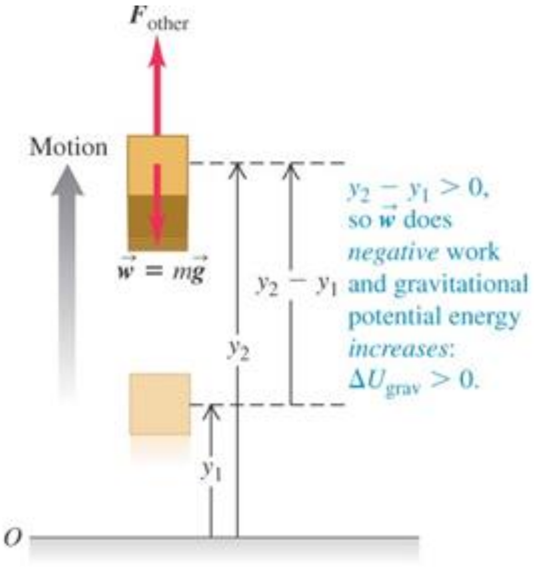
  - $\Delta U$ is negative when $F$ and $x$ are in the same direction

# Gravitational potential energy

Work done by gravitational force :

$W_g = (mg) \cdot [(y_2 - y_1)] \cos 180^0$

$= \; -mg(y_2 - y_1)$




The change of gravitational potential energy:

$$\Delta U_g = -W_g = mg(y_f - y_i)$$

# Gravitational Potential Energy

In general

$$W_g = (-mg\mathbf{j})\cdot[(x_f - x_i)\mathbf{i} + (y_f - y_i)\mathbf{j}] = -\,mg(y_f - y_i)$$

The change of gravitational potential energy:

$$\Delta U_g = -W_g \quad \text{so} \quad \Delta U_g = mg(y_f - y_i),$$

if $U_{gi} = 0$ for $y_i = 0$ then

$$U_g = mgy$$

# Gravitational Potential Energy, final

The quantity *mgy* is identified as the gravitational potential energy, $U_g$

- $U_g = mgy$

Units are joules (J)

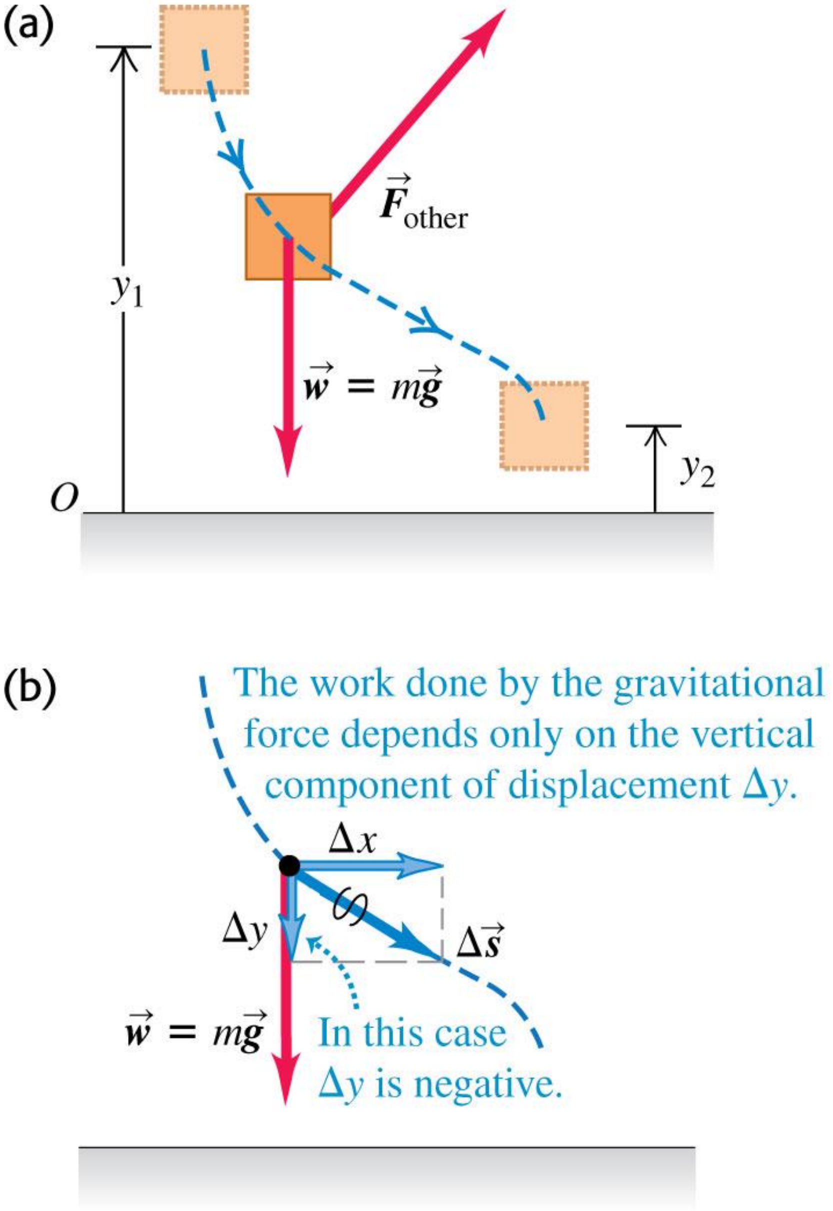
Is a scalar

Work done by gravitational force may change the gravitational potential energy of the system

# Work and energy along a curved path

We can use the same expression for gravitational potential energy $U_g = mgy$

whether the body's path is curved or straight.



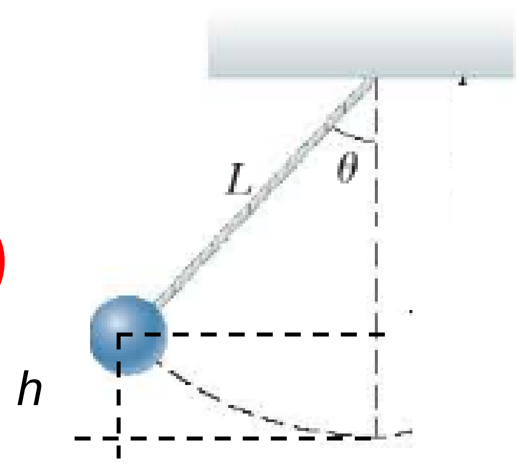
# Example 1

**A pendulum is made by letting a 2.0-kg object swing at the end of a string that has a length of 1.5 m. The maximum angle the string makes with the vertical as the pendulum swings is 30⁰. What is the maximum change of its potential energy.**

$L-h = L\cos\theta$

$L-L\cos\theta = h \quad h=L(1-\cos\theta)$

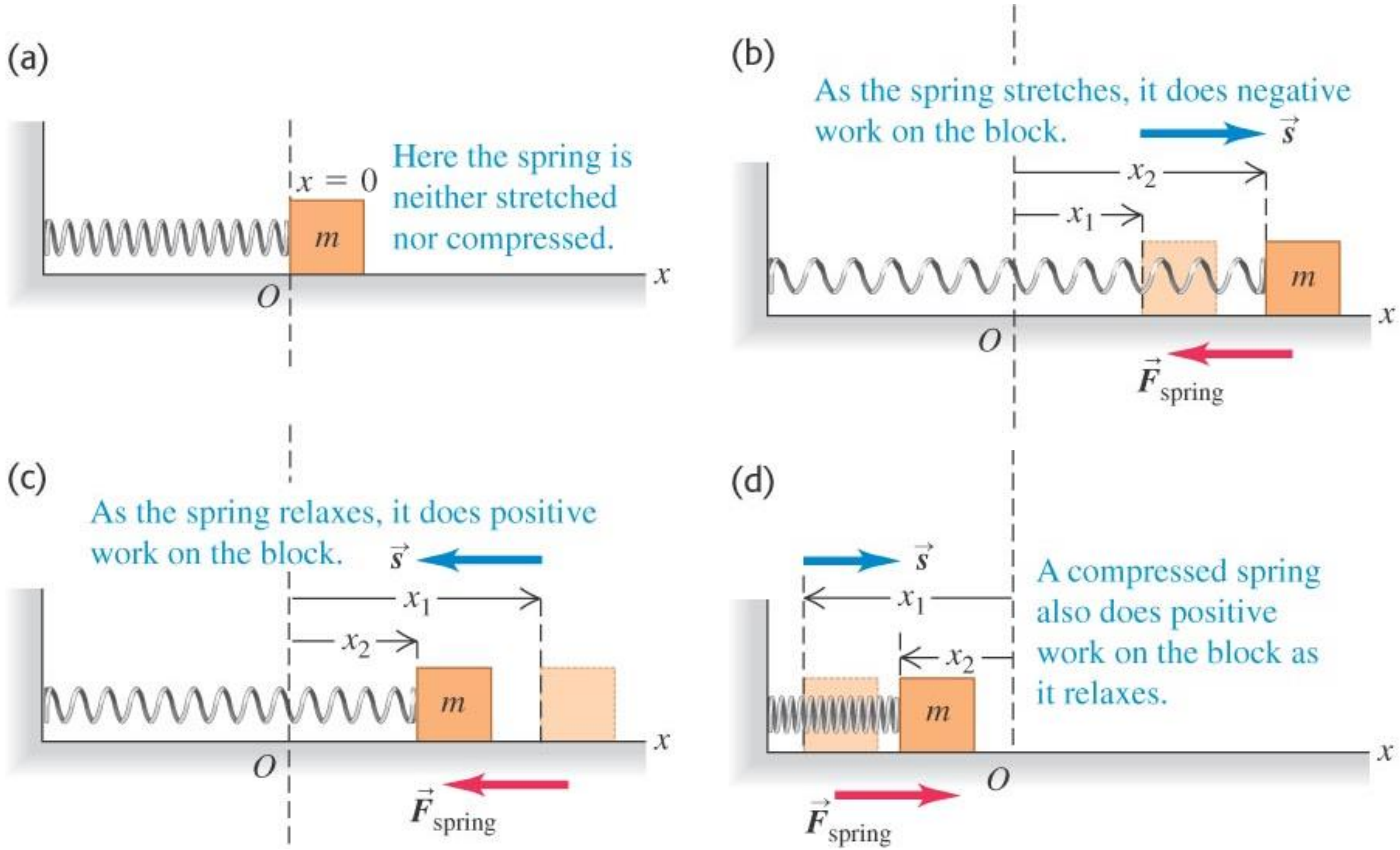
$\Delta U_g = mgh = mgL(1-\cos\theta)$




$\Delta U_g = 2kg \cdot 9.8m/s^2 \cdot 1.5m(1-\cos 30^0) = 3.94J$

# Work done by a spring

- Figure below shows how a spring does work on a block as it is stretched and compressed.



# Elastic Potential Energy

***Elastic Potential Energy*** is associated with a spring

The force the spring exerts (on a block, for example) is $F_s = -kx$

The work done by the spring force on a spring-block system is

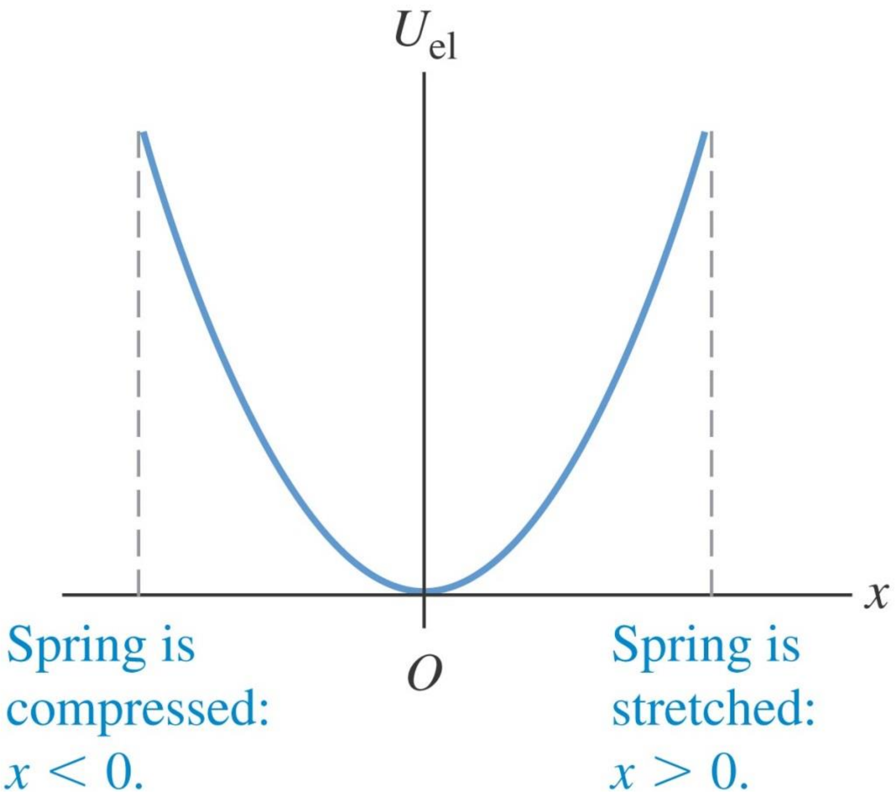
$$W_s = -(\tfrac{1}{2} kx_f^2 - \tfrac{1}{2} kx_i^2)$$

- $$\Delta U_s = -W_s = (\tfrac{1}{2} kx_f^2 - \tfrac{1}{2} kx_i^2)$$

if $U_s = 0$ for $x_i = 0$ then $U_s = \tfrac{1}{2} kx^2$

# Elastic potential energy

- A body is *elastic* if it returns to its original shape after being deformed.
- *Elastic potential energy* is the energy stored in an elastic body, such as a spring.
- The elastic potential energy stored in an ideal spring is $U_{el} = 1/2\ kx^2$.
- Figure at the right shows a graph of the elastic potential energy for an ideal spring.



# Energy Review

## Kinetic Energy, K

- Associated with movement of members of a system

## Potential Energy, U

- Determined by the configuration of the system
- Gravitational and Elastic

## Mechanical Energy, E

Kinetic energy plus potential energy

## Internal Energy

- Related to the temperature of the system

# Types of Systems

## Nonisolated systems

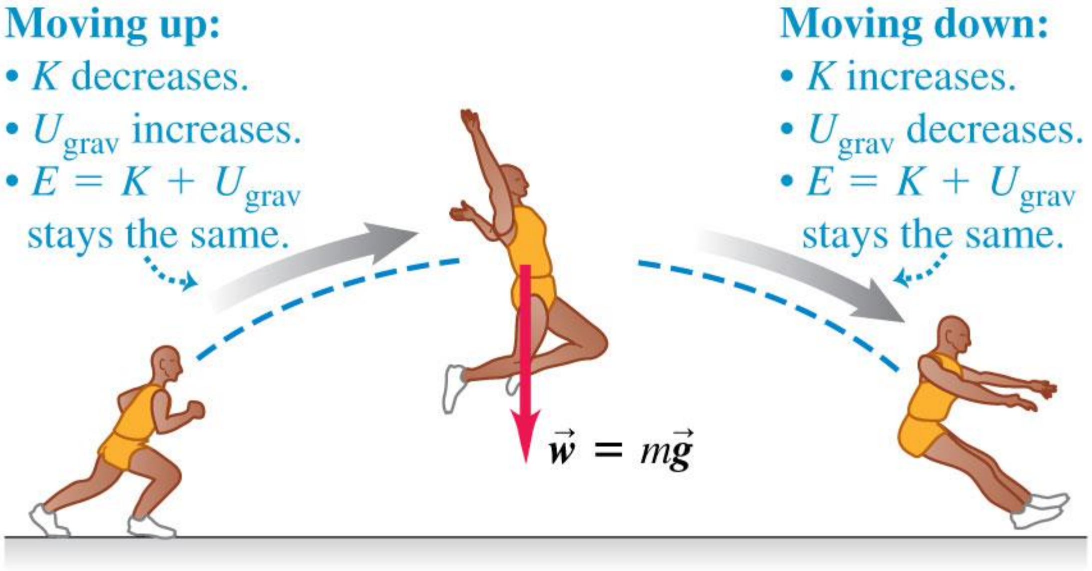
- Energy can cross the system boundary in a variety of ways
- Total energy of the system changes

## Isolated systems

- Energy does not cross the boundary of the system
- Total energy of the system is constant

# The conservation of mechanical energy

- The total *mechanical energy* of a system is the sum of its kinetic energy and potential energy.
- A quantity that always has the same value is called a *conserved* quantity.
- When only the force of gravity does work on a system, the total mechanical energy of that system is conserved. This is an example of the *conservation of mechanical energy*. Figure 7.3 below illustrates this principle.



# Conservation of Mechanical Energy

For an isolated system were only conservative forces are present:

$W_{total} = K_f - K_i$ , $W_{total} = W_c$ $W_{total} = -(U_f - U_i)$

$-(U_f - U_i) = K_f - K_i$ , $U_i + K_i = U_f + K_f$

*If* $E_{mech} = K + U$ *then* $\Delta E_{mech} = 0$

This is ***conservation of energy*** for an isolated system with no nonconservative forces acting

$$\mathbf{K_i + U_i = K_f + U_f} \quad \textbf{or} \ \Delta E_{mech} = 0$$

- Remember, this applies only to a system in which conservative forces act

If nonconservative forces are acting, some energy is transformed into internal energy

# Situations with both gravitational and elastic forces

- When a situation involves both gravitational and elastic forces, the total potential energy is the *sum* of the gravitational potential energy and the elastic potential energy: $U = U_{grav} + U_{el}$.
- Figure 7.15 below illustrates such a situation.

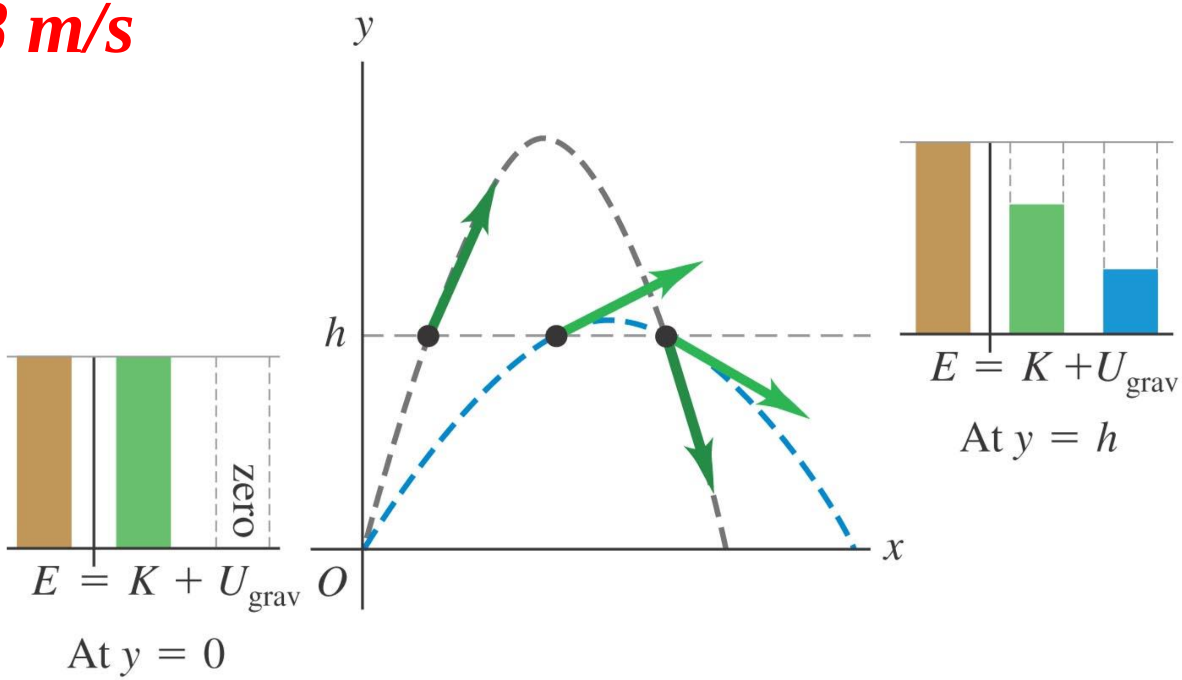
# Energy in projectile motion

Two identical balls leave from the ground with the same speed of 12 m/s but at $30^0$ and $65^0$ angles. Using conservation of energy law, find the velocity of the balls at height h=1.4 m if air resistance can be neglected.

$K_i+U_i = K_f+U_f \quad \frac{1}{2}mv_1^2 + mg0 = \frac{1}{2}mv_f^2 + mgh$

$mv_1^2 = mv_f^2 + 2mgh \quad v_1^2 = v_f^2 + 2gh \quad v_f^2 = v_1^2 - 2gh$

$V_f = 10.8$ m/s



# Example

**A 2.0-kg object is suspended from the ceiling at the end of a 2.0-m string. When pulled to the side and released, it has a speed of 4.0 m/s at the lowest point of its path. What maximum angle does the string make with the vertical as the object swings up?**

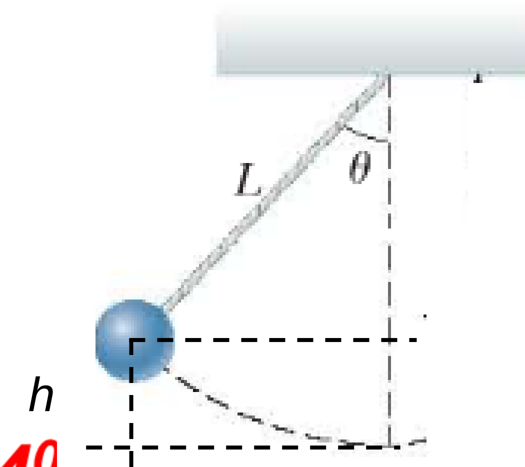
$U_i+K_i=U_f+K_f$ $U_i = mgh = mgL(1-\cos\theta)$

$K_i=0$ $U_f=0$ $K_f=\frac{1}{2}mv^2$

$mgL(1-\cos\theta)=\frac{1}{2}mv^2$

$2gL-2gL\cos\theta=v^2$

$\cos\theta=\frac{2gL-v^2}{2Lg}=0.59$ $\theta=\cos^{-1}0.59=54^0$



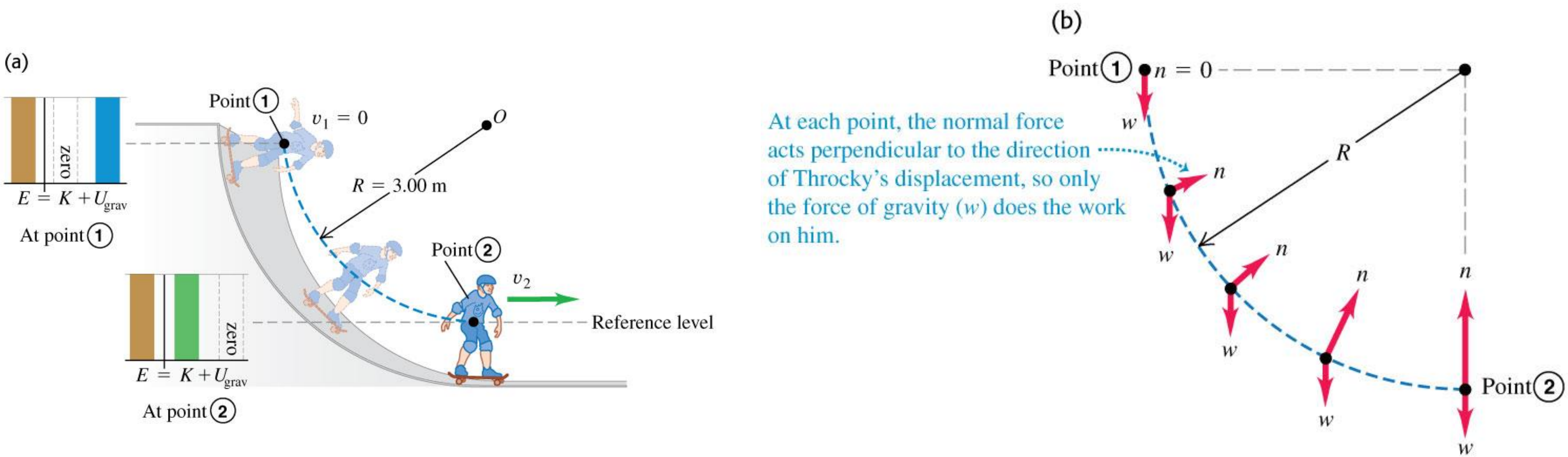
# Motion in a vertical circle with no friction

Bill skateboards from rest down a curved, frictionless ramp. He moves through a quarter-circle with radius r=3m. The total mass of Bill and skateboard is 85 kg. Find his speed at the bottom of the ramp and normal force that acts on him at the bottom of the ramp.

$E_i = mgh + 0 \quad E_f = 0 + \frac{1}{2}mv^2 \quad E_i = E_f \quad mgh = \frac{1}{2}mv^2 \; 2gh = v^2 \quad h = R$

$V^2 = 2 \cdot 9.8 m/s^2 \cdot 3m = 58.8 m^2/s^2 \quad v = 7.7 m/s^2$

$N - mg = m\frac{v^2}{R} \quad N = mg + \frac{v^2}{R} = 85kg \cdot 9.8m/s^2 + \frac{58.8}{3m} = 852.6N$



# Nonconservative and Conservative Forces are Present

$W_{total} = W_c + W_{nc}$ where $W_c$ – work done by conservative forces, $W_{nc}$ – work done by nonconservative forces

$W_c = -(U_f - U_i)$, $W_{tot} = K_f - K_i$

$K_f - K_i = -(U_f - U_i) + W_{nc}$

$$K_i + U_i + W_{nc} = K_f + U_f$$

If only nonconservative force acting in a system is the friction

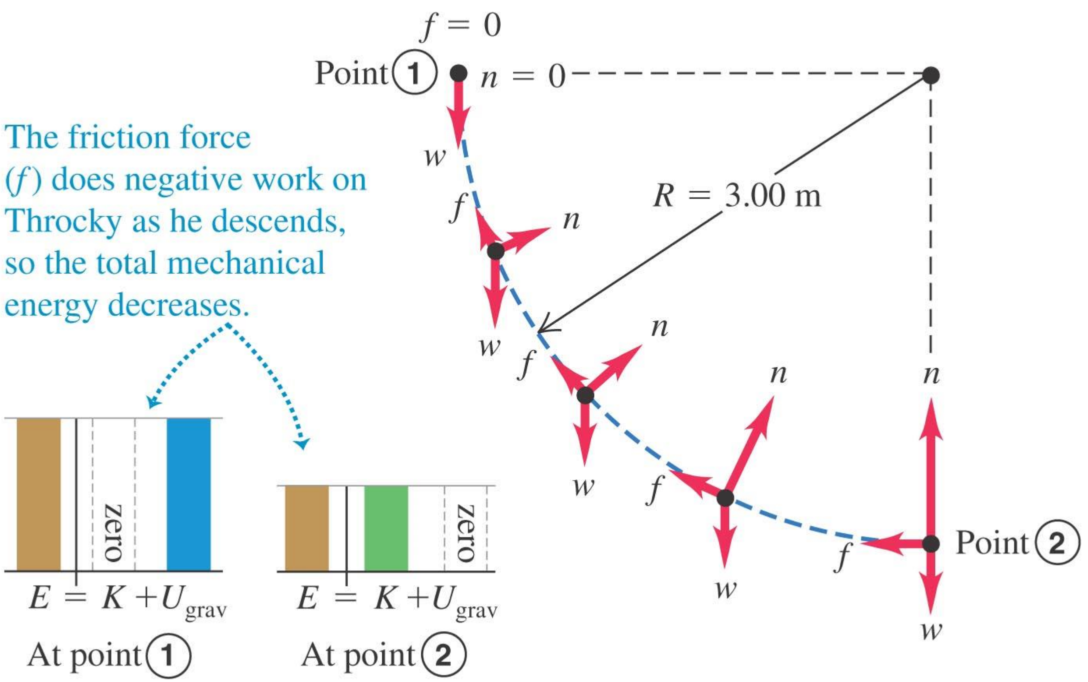
– $K_i + U_i - f_k d = K_f + U_f$

- Use this form when friction acts on an object
- If friction is zero, this equation becomes the same as for conservation of mechanical energy

# Motion in a vertical circle with friction

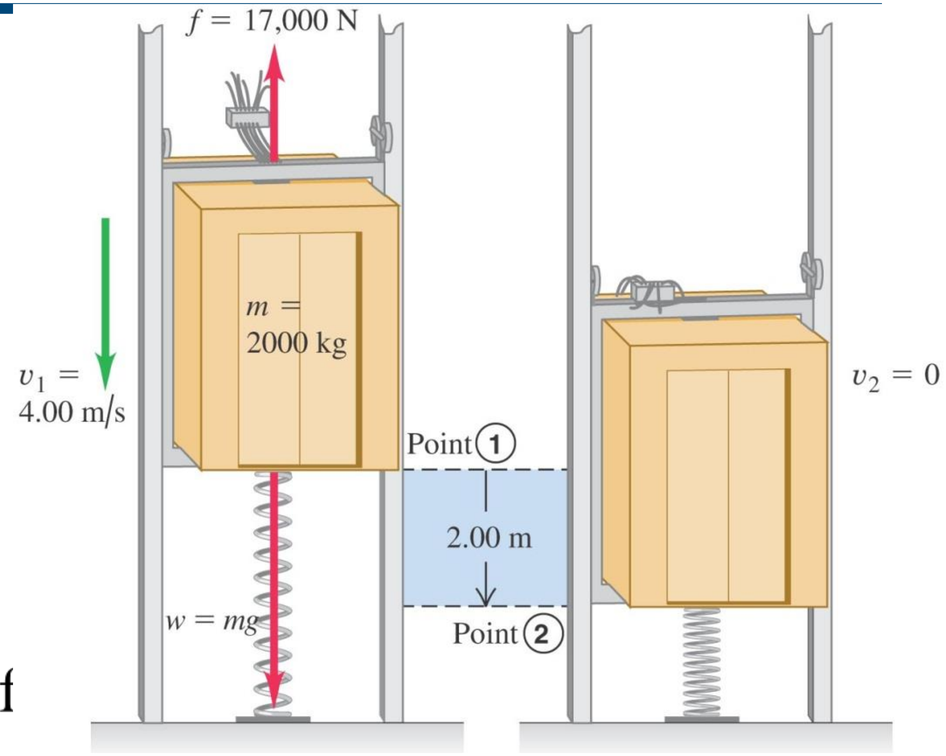
Revisit the same ramp as in the previous example, but this time with friction. Assume that now Bill's speed at the bottom the track is 6 m/s. What work was done on him by the friction force?

***$U_i+K_i+W_{fr}=U_f+K_f$  $W_{fr}=U_f+K_f-U_i+K_i=\frac{1}{2}mv^2-mgR=$***
***$\frac{1}{2}(85kg)(6m/s)^2-(85kg)(9.8m/s^2)(3m)=1530J-2499J=-969J$***



# A system having two potential energies and friction

In this example gravity, a spring, and friction all act on the elevator. A 2000kg elevator is falling at 4 m/s when it contacts a spring at the bottom of the shaft. The spring stops the elevator compressing 2m. During the motion a safety clamp applies 17000–N frictional force to the elevator. Find spring constant of the spring.




$$\tfrac{1}{2}mv^2 + F_k d\cos 180^0 = -mgh + \tfrac{1}{2}kx^2$$

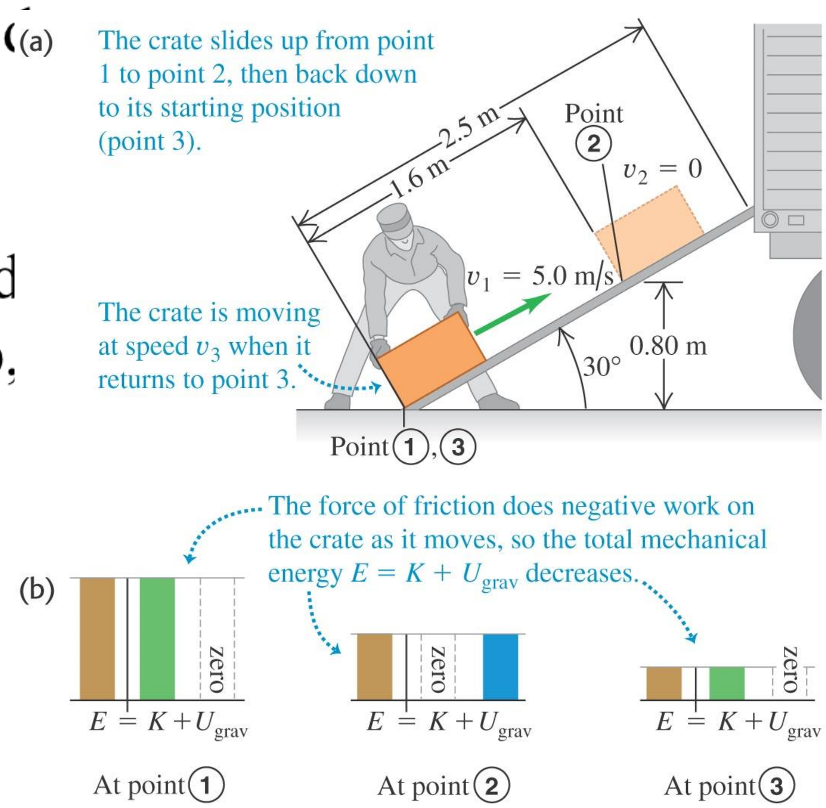
$$k = \frac{2(\tfrac{1}{2}mv^2 + mgh - F_k d)}{x^2} =$$

$1.06\text{x}10^4$ N/m

# Moving a crate on an inclined plane with friction

Notice that mechanical energy was lost ( friction.

A 12-kg crate is given a speed of 5 m/s to make it sliding up a long ramp inclined at $30^0$. The crate slides 1.6m up the ramp, stops, and slide back down. Find friction force acting on the crate.




$U_i = 0 \quad K_f = 0 \quad h = s \cdot \sin\theta$

$\frac{1}{2}mv^2 - f_k s = mgh = mg \cdot s \cdot \sin\theta$

$$f_k = \frac{\frac{1}{2}mv^2 - mg \cdot s \cdot \sin\theta}{s}$$

$$f_k = \frac{0.5(12kg)\left(\frac{5m}{s}\right)^2 - (117.6N)(1.6m)(\sin 30^0)}{1.6m} = 35N$$

# Conservation of energy - Summary

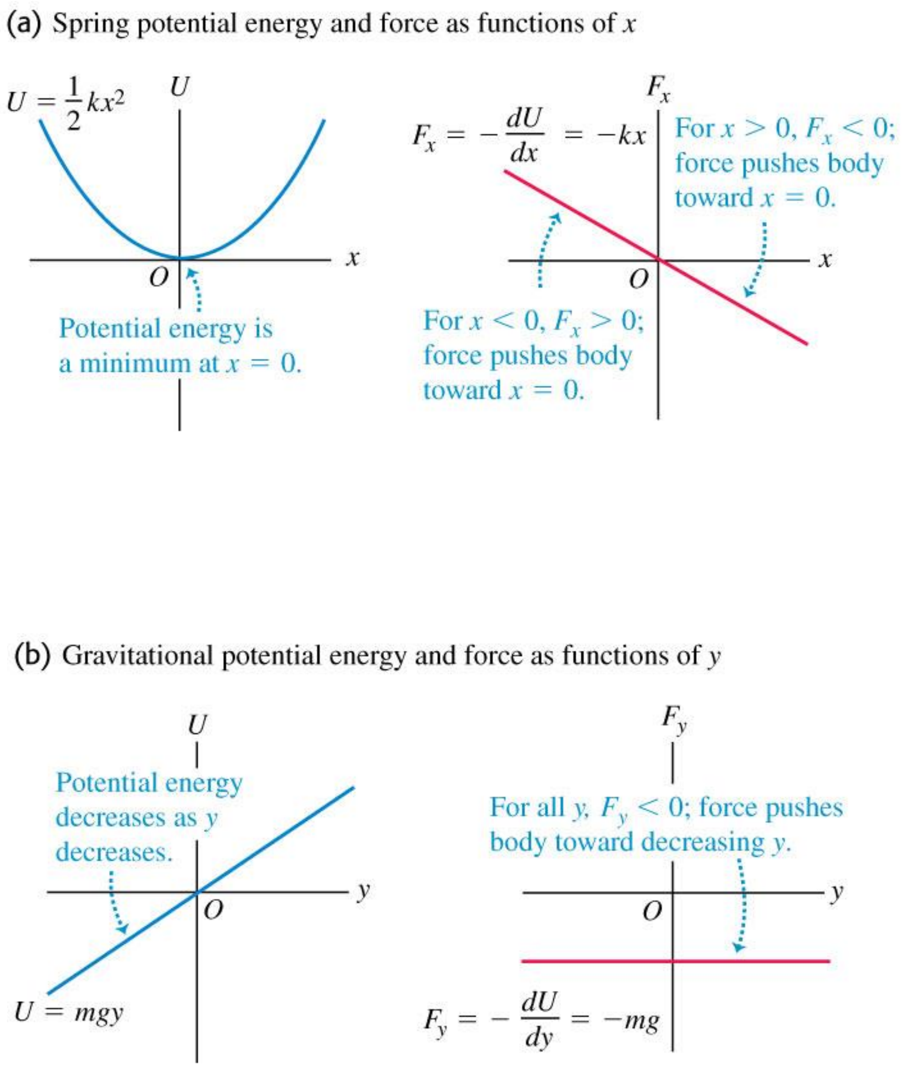
- Nonconservative forces do not store potential energy, but they do change the *internal energy* of a system.
- *The law of the conservation of energy* means that energy is never created or destroyed; it only changes form.
- This law can be expressed as $\Delta K + \Delta U + \Delta U_{int} = 0$.

# Force and potential energy in one dimension

- In one dimension, a conservative force can be obtained from its potential energy function using

$$F_x(x) = -dU(x)/dx$$

- Figure 7.22 at the right illustrates this point for spring and gravitational forces.
- Follow Example 7.13 for an electric force.



# Force and potential energy in two dimensions

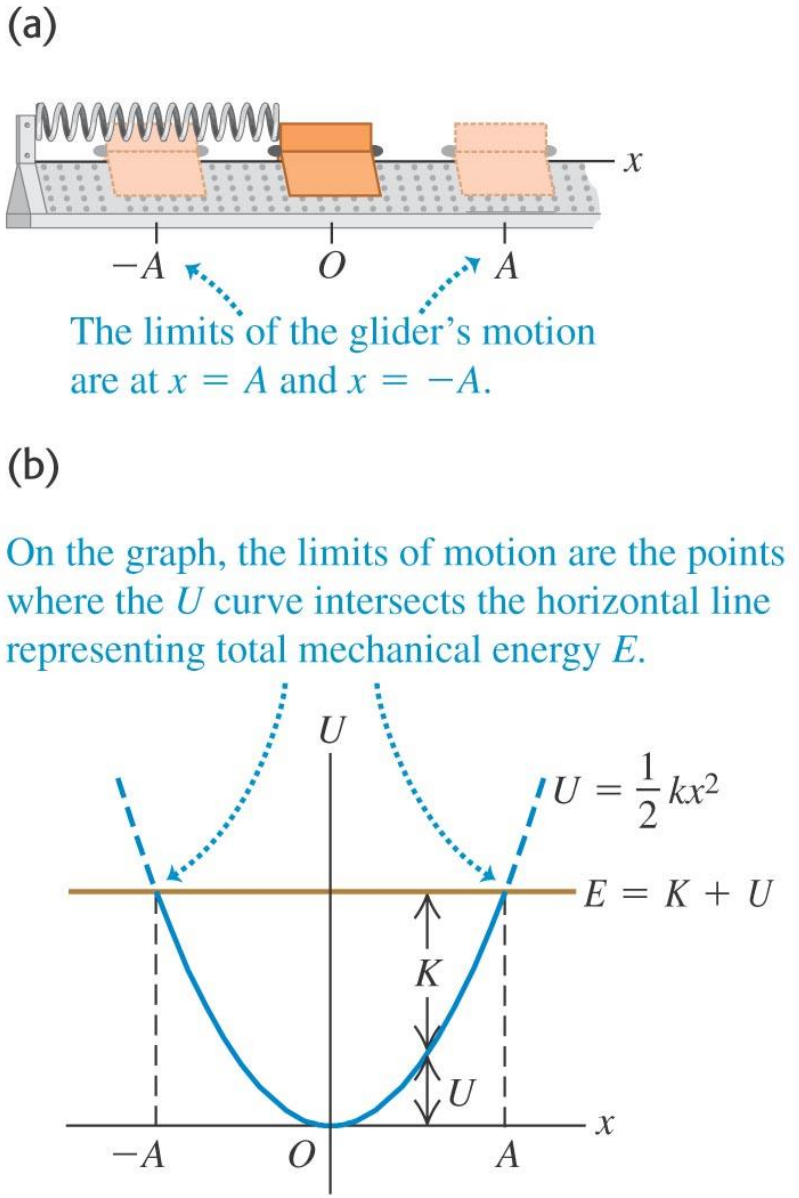
- In two dimension, the components of a conservative force can be obtained from its potential energy function using

$$F_x = -\partial U/dx \qquad \text{and} \qquad F_y = -\partial U/dy$$

- Follow Example 7.14 for a puck on a frictionless table.

# Energy diagrams

- An *energy diagram* is a graph that shows both the potential-energy function $U(x)$ and the total mechanical energy $E$.
- Figure 7.23 illustrates the energy diagram for a glider attached to a spring on an air track.



# Force and a graph of its potential-energy function

- Figure 7.24 below helps relate a force to a graph of its corresponding potential-energy function.