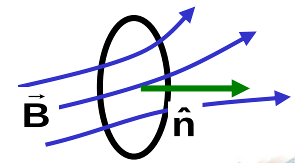
Physics 121 - Electricity and Magnetism
Lecture 12 - Inductance, RL Circuits
Y&F Chapter 30, Sect 1 - 4

- **Inductors and Inductance**
- **Self-Inductance**
- **RL Circuits – Current Growth**
- **RL Circuits – Current Decay**
- **Energy Stored in a Magnetic Field**
- **Energy Density of a Magnetic Field**
- **Mutual Inductance**
- **Summary**

Induction: basics

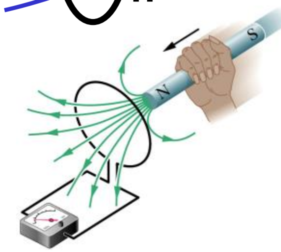
- **Magnetic Flux:**

$$d\Phi_B \equiv \vec{B} \circ d\vec{A} = \vec{B} \circ \hat{n} dA$$



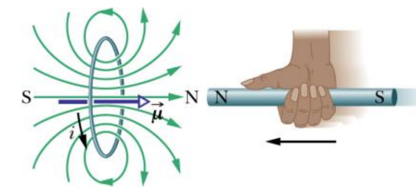
- **Faraday's Law: A changing magnetic flux through a coil of wire induces an EMF in the wire, proportional to the number of turns, *N*.**

$$\mathcal{E}_{ind} = -N \frac{d\Phi_B}{dt}$$



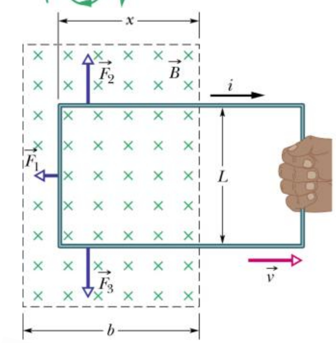
- **Lenz's Law: The current driven by an induced EMF creates an induced magnetic field that opposes the flux change.**

B_{ind} & i_{ind} oppose changes in Φ_B

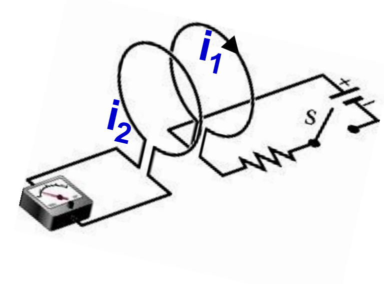


- **Induction and energy transfer: The forces on the loop oppose the motion of the loop, and the power required to sustain motion provides electrical power to the loop.**

$$P = \vec{F} \cdot \vec{v} = Fv \qquad P = i\varepsilon \qquad \varepsilon = -Blv$$



- **Transformer principle: changing current i_1 in primary induces EMF and current i_2 in secondary coil.**



- **A changing magnetic flux creates a non-conservative electric field.**

$$\varepsilon = \oint \vec{E} \cdot d\vec{s} = -N \frac{d\Phi_B}{dt}$$

Changing magnetic flux induces electric fields: trivial transformer

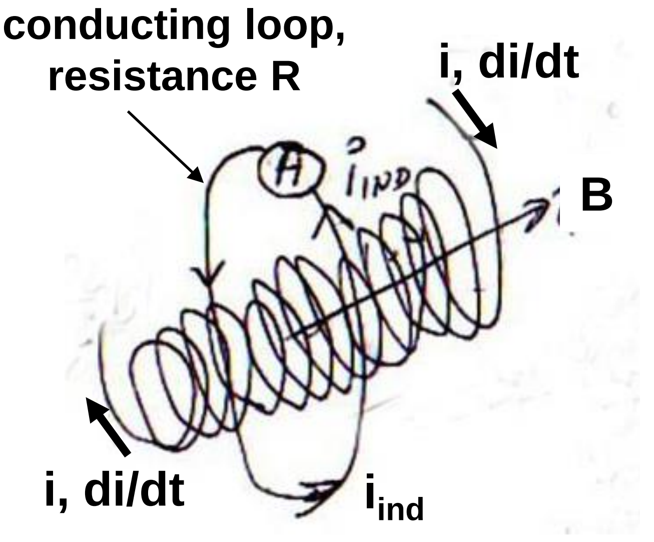
A thin solenoid, cross section A, n turns/unit length

- zero field outside solenoid
- inside solenoid:

$$B = \mu_0 i n$$

Flux through a conducting loop:

$$\Phi = BA = \mu_0 i n A$$



Current i varies with time, so flux varies and an EMF is induced in loop "A":

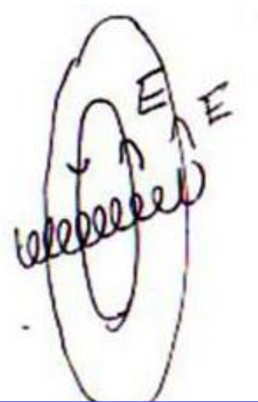
$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -\mu_0 n A \frac{di}{dt}$$

Current induced in the loop is: $i_{ind} = \dfrac{\mathcal{E}_{ind}}{R}$

If di/dt is positive, B is growing, then B_{ind} opposes change and i_{ind} is Counter-clockwise

What makes the current i_{ind} flow?

- B = 0 there so it's not the Lorentz force
- An induced electric field E_{ind} along the loop causes current to flow
- It is caused directly by dF/dt
- Electric field lines are loops that don't terminate on charge.
- E-field is there even without the conductor (no current flowing)
- E-field is non-conservative (not electrostatic) as the line integral around a closed path is not zero



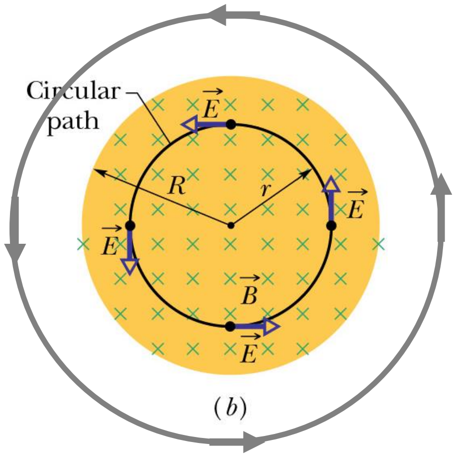
$$\therefore \mathcal{E}_{ind} = \oint_{loop} E_{ind} \circ d\vec{s} = -\frac{d\Phi_B}{dt}$$

Generalized Faradays' Law
Path must be constant

Example: Find the induced electric field

$$\mathcal{E}_{ind} = \oint_{loop} E_{ind} \circ d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \circ d\vec{s} = \mu_0 i_{enc}$$

In the right figure, dB/dt = constant, find the expression for the magnitude E of the induced electric field at points within and outside the magnetic field.



Due to symmetry: $\oint \vec{E} \cdot d\vec{s} = \oint E ds = E \oint ds = E(2\pi r)$

For r < R: $\Phi_B = BA = B(\pi r^2)$

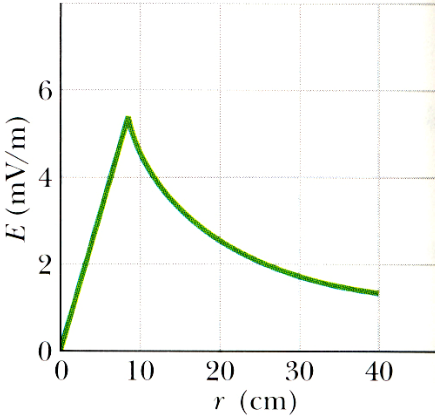
So $E(2\pi r) = \pi r^2 \frac{dB}{dt}$

$$E = \frac{r}{2} \frac{dB}{dt}$$

For r > R: $\Phi_B = BA = B(\pi R^2)$

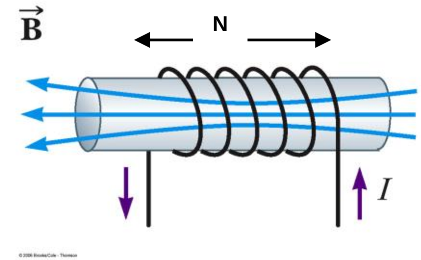
So $E(2\pi r) = \pi R^2 \frac{dB}{dt}$

$$E = \frac{R^2}{2r} \frac{dB}{dt}$$



The magnitude of induced electric field grows linearly with r, then falls off as 1/r for r>R

Self-Inductance: Analogous to inertia



ANY magnetic flux change is resisted.
Changing current in a single coil induces a "back EMF" $\mathcal{E}_{ind}$ in the same coil opposing the current change, an induced current i_{ind}, and a consistent induced field B_{ind}.

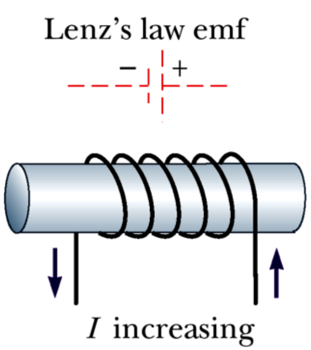
DISTINGUISH:

•**Mutual-induction**: di_1/dt in "transformer primary" also induces EMF and current i_2 in "linked" secondary coil (transformer principle).

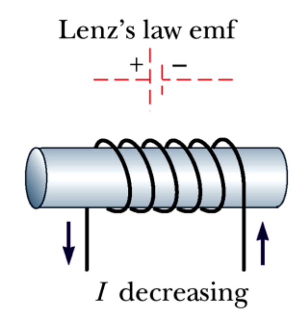
•**Self-induction** in a single Coil: di/dt produces "back EMF" due to Lenz & Faraday Laws: Φ_{ind} opposes $d\Phi/dt$ due to current change. $\mathcal{E}_{ind}$ opposes di/dt.

- Changing current in a single coil causes magnetic field and flux created by this current to change in the same sense
- Flux Change induces flux opposing the change, along with opposing EMF and current.
- This back emf limits the rate of current (flux) change in the circuit

- **For increasing current, back EMF limits the rate of increase**



- **For decreasing current, back EMF sustains the current**



Inductance measures oppositon to the rate of change of current

Definition of Self-inductance

Joseph Henry
1797 – 1878

Recall capacitance: depends only on geometry
It measures energy stored in the E field

$$C \equiv \frac{Q}{V}$$

Self-inductance depends only on coil geometry
It measures energy stored in the B field

$$L \equiv \frac{\text{linked flux}}{\text{unit current}}$$

number of turns →

self-inductance → $$L \equiv \frac{N\Phi_B}{i}$$

← flux through one turn depends on current & all N turns

← cancels current dependence in flux above

SI unit of inductance:

$$1\,\text{Henry} \equiv 1\text{ H.} \equiv 1\,\text{T.m}^2\,/\,\text{Ampere} = 1\,\text{Weber}\,/\,\text{Ampere}$$
$$= 1\,\text{Volt.sec}\,/\,\text{Ampere}\quad(\Omega\text{.sec})$$

Why choose this definition?

Cross-multiply

$$Li = N\Phi_B$$

Take time derivative

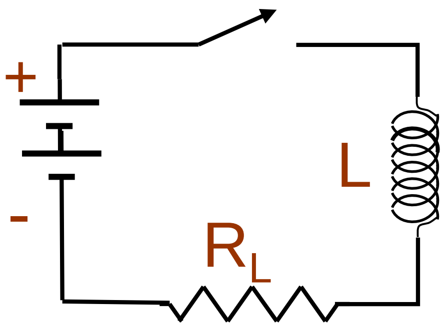
$$L\frac{di}{dt} = N\frac{d\Phi_B}{dt} = -\mathcal{E}_L$$

$$\mathcal{E}_L \equiv -L\frac{di}{dt}$$

- L contains all the geometry
- $\mathcal{E}_L$ is the "back EMF"

Another form of Faraday's Law!

Example: Find the Self-Inductance of a solenoid



Field: $B = \mu_0 i n$ where $n \equiv \dfrac{N}{\ell} = \dfrac{\#\text{ turns}}{\text{unit Length}}$

Flux in just one turn: $\Phi_B \equiv BA = \mu_0 i \dfrac{NA}{\ell}$

ALL N turns contribute to self-flux through ONE turn

Apply definition of self-inductance:

- N turns
- Area A
- Length ℓ
- Volume $V = A\ell$

$$\therefore\ L \equiv \frac{N\Phi_B}{i} = \mu_0 \frac{N^2 A}{\ell} = \mu_0 n^2 V$$

- Depends on geometry only, like capacitance.
- Proportional to N^2 !

Check: Same L if you start with Faraday's Law for F_B:

$$\mathcal{E}_{ind} = -N\frac{d\Phi_B}{dt}$$

for solenoid use Φ_B above

$$\mathcal{E}_{ind} = -N.\left(\mu_0 \frac{NA}{\ell}\frac{di}{dt}\right) = \frac{-\mu_0 N^2 A}{\ell}\cdot\frac{di}{dt} \equiv -L\frac{di}{dt}$$

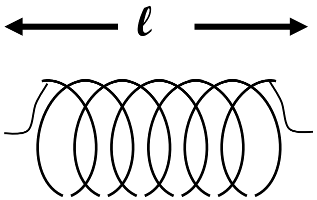
Note: Inductance per unit length has same dimensions as m_0

$$\frac{L}{\ell} = \mu_0 N^2 \frac{A}{\ell^2}$$

$$[\mu_0] = \frac{T.m}{A.} = \frac{H}{m}$$

Example: calculate self-inductance L for an ideal solenoid

$N = 1000$ turns, radius $r = 0.5$ m, length $\ell = 0.2$ m



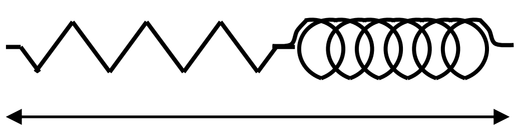
$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 10^{6} \times \pi \times (5 \times 10^{-2})^2}{0.2}$$

$$\therefore\ L = 49.4 \times 10^{-3}\ \text{Henrys} = 49.4\ \text{milli - Henrys}$$

Ideal inductor (abstraction):

- Internal resistance $r = 0$ (recall ideal battery)
- $B = 0$ outside
- $B = \mu_0 in$ inside (ideal solenoid)

Non-ideal inductors have internal resistance:



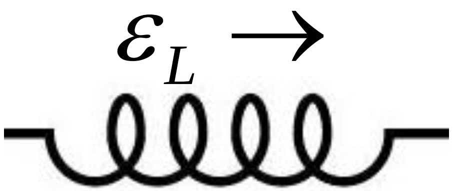
- $V_{IND} = \mathcal{E}_L - ir$ = measured voltage
- Direction of ir depends on current
- Direction of $\mathcal{E}_L$ depends on di/dt
- If current i is constant, then induced $\mathcal{E}_L = 0$

  Inductor behaves like a wire with resistance r

Induced EMF in an Inductor

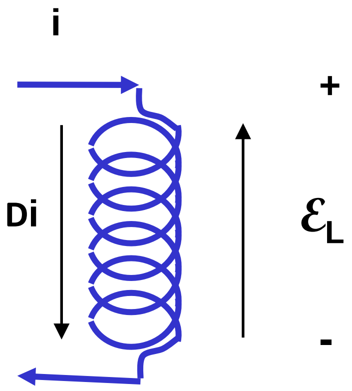
12 – 1: Which statement describes the current through the inductor below, if the induced EMF is as shown?

A. Rightward and constant.
B. Leftward and constant.
C. Rightward and increasing.
D. Leftward and decreasing.
E. Leftward and increasing.



$$\mathcal{E}_L \equiv -L\frac{di}{dt}$$

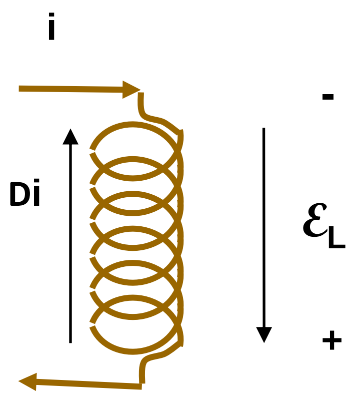
Lenz's Law applied to Back EMF



If i is increasing: $\frac{d\Phi_B}{dt} > 0$

$\therefore$ $\mathcal{E}_L$ opposes increase in i

Power is being stored in B field of inductor



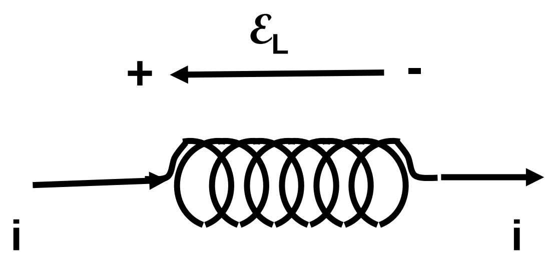
If i is decreasing: $\frac{d\Phi_B}{dt} < 0$

$\therefore$ $\mathcal{E}_L$ opposes decrease in i

Power is being tapped from B field of inductor

What if CURRENT i is constant?

Example: Current I increases uniformly from 0 to 1 A. in 0.1 seconds. Find the induced voltage (back EMF) across a 50 mH (milli-Henry) inductance.



i defines positive direction

$\frac{di}{dt} > 0$ means that current i is increasing and toward the right

Apply: $$\mathcal{E}_L = -L\frac{di}{dt}$$

Substitute: $$\frac{\Delta i}{\Delta t} = \frac{+1 \text{ Amp}}{0.1 \text{ sec}} = 10\frac{\text{Amp}}{\text{sec}}$$

$$\mathcal{E}_L = -50\,\text{mH} \cdot 10\frac{\text{Amp}}{\text{sec}} = -0.5 \text{ Volts}$$

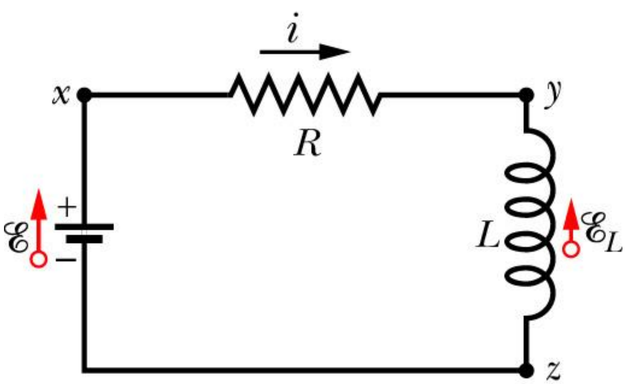
Negative result means that induced EMF is opposed to both di/dt and i.

Inductors in Circuits—The RL Circuit

- Inductors, sometimes called "coils", are common circuit components.
- Insulated wire is wrapped around a core.
- They are mainly used in AC filters and tuned (resonant) circuits.

Analysis of series RL circuits:

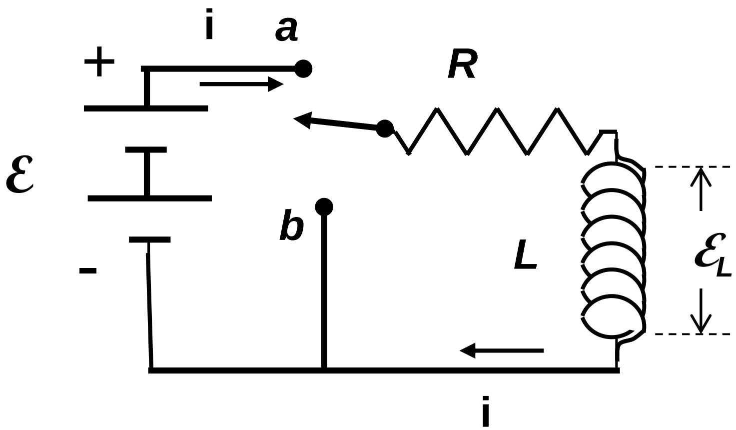
- A battery with EMF $\mathcal{E}$ drives a current around the loop, producing a back EMF $\mathcal{E}_L$ in the inductor.
- Derive circuit equations: apply Kirchoff's loop rule, convert to differential equations (as for RC circuits) and solve.



New rule: when traversing an inductor in the <u>same</u> direction as the assumed current, insert:

$$\mathcal{E}_L \equiv -L\frac{di}{dt}$$

Series LR circuits



- Inductance & resistance + EMF
- Find time dependent behavior
- Use Loop Rule & Junction Rule
- Treat $\mathcal{E}_L$ as an EMF along current

$$\mathcal{E}_L \equiv -L\frac{di}{dt}$$ ALWAYS

Given $\mathcal{E}$, R, L: Find i, $\mathcal{E}_L$, U_L for inductor as functions of time

Growth phase, switch to "a". Loop equation:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

- i through R is clockwise and growing: $\mathcal{E}_L$ opposes $\mathcal{E}$
- At t = 0, rapidly growing current but i = 0, $\mathcal{E}_L = \mathcal{E}$
  L acts like a broken wire
- As t → infinity, large stable current, di/dt → 0
  Back EMF $\mathcal{E}_L$ → 0, i → $\mathcal{E}$ / R,
  L acts like an ordinary wire
- Energy is stored in L & dissipated in R

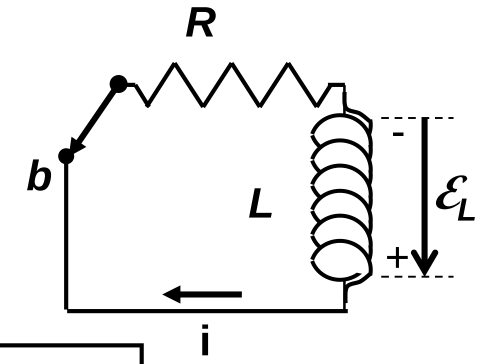
Decay phase, switch to "b", exclude $\mathcal{E}$, Loop equation:

$$-iR - L\frac{di}{dt} = 0$$

- Energy stored in L now dissipated in R
- Current through R is still clockwise, but collapsing
- $\mathcal{E}_L$ now acts like a battery maintaining current
- Current i at t = 0 equals $\mathcal{E}$ / R
- Current → 0 as t → infinity – energy depleted

LR circuit: decay phase solution

- After growth phase equilibrium, switch from a to b, battery out
- Current $i_0 = \mathcal{E} / R$ initially still flowing CW through R
- Inductance tries to maintain current using stored energy
- Polarity of $\mathcal{E}_L$ reverses versus growth. Eventually $\mathcal{E}_L \rightarrow 0$



Loop Equation is : $-iR + \mathcal{E}_L = 0$

Substitute : $\mathcal{E}_L(t) = -L\dfrac{di}{dt}$

Circuit Equation:

$$\frac{di}{dt} = -\frac{R}{L} i$$

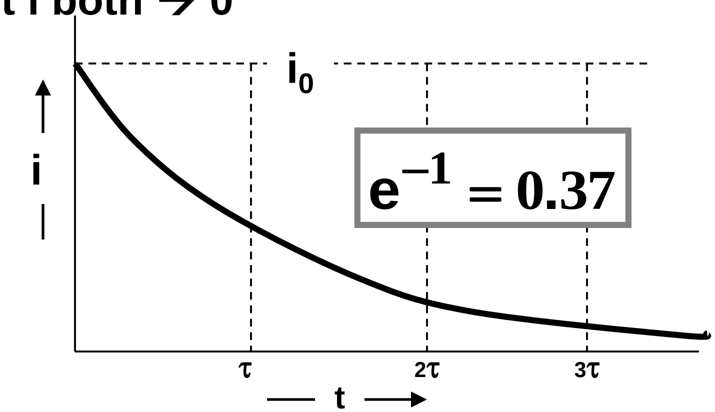
di/dt <0 during decay, opposite to current

- First order differential equation with simple exponential solution
- At t = 0: large current implies large di / dt, so $\mathcal{E}_L$ is large
- As t → infinity: current stabilizes, di / dt and current i both → 0

Current decays exponentially:

$$i(t) = i_0 e^{-t/\tau_L} \qquad i_0 \equiv \frac{\mathcal{E}}{R}$$

$\tau_L \equiv L/R \equiv$ inductive time constant



Back EMF $\mathcal{E}_L$ and V_R decay exponentially:

$$\frac{di}{dt} = -\frac{\mathcal{E}}{R\tau_L} e^{-t/\tau_L} \Rightarrow$$

$$\mathcal{E}_L = +\mathcal{E} e^{-t/\tau_L}$$

$$V_R = -\mathcal{E}_L = -iR$$

Compare to RC circuit, decay

$$Q(t) = C\mathcal{E}e^{-t/RC}$$

$RC \equiv$ capacitive time constant

LR circuit: growth phase solution

Loop Equation is : $\mathcal{E} - iR + \mathcal{E}_L = 0$

Substitute : $\mathcal{E}_L(t) = -L\frac{di}{dt}$

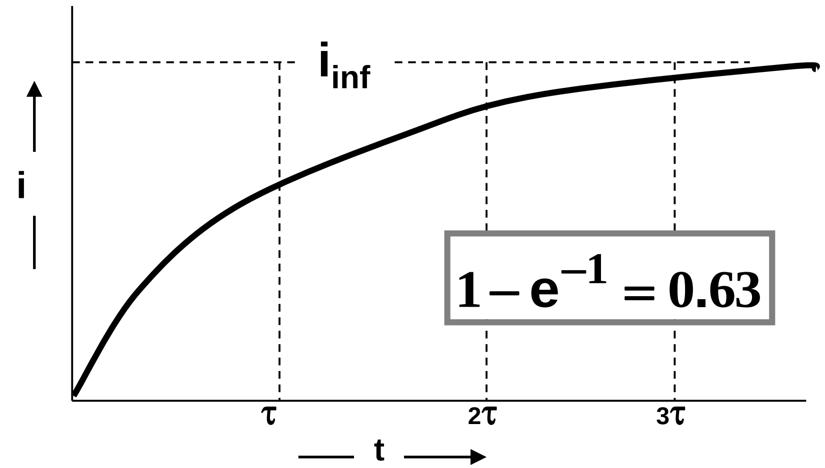
Circuit Equation: $i + \frac{L}{R}\frac{di}{dt} = \frac{\mathcal{E}}{R}$

- First order differential equation again - saturating exponential solutions
- As t → infinity, di / dt approaches zero, current stabilizes at $i_{inf} = \mathcal{E} / R$
- At t = 0: current is small, di / dt is large, back EMF opposes battery.

Current starts from zero, grows as a saturating exponential.

$$i(t) = i_{inf}\left(1 - e^{-t/\tau_L}\right) \qquad i_{inf} \equiv \frac{\mathcal{E}}{R}$$

$\tau_L \equiv L/R \equiv$ inductive time constant



- i = 0 at t = 0 in above equation → $di/dt = \mathcal{E}/L$ fastest rate of change, largest back EMF

Back EMF $\mathcal{E}_L$ decays exponentially

$$\frac{di}{dt} = \frac{\mathcal{E}}{R\,\tau_L}e^{-t/\tau_L} \Rightarrow \mathcal{E}_L = -\mathcal{E}\,e^{-t/\tau_L}$$

Voltage drop across resistor $V_R = -iR$

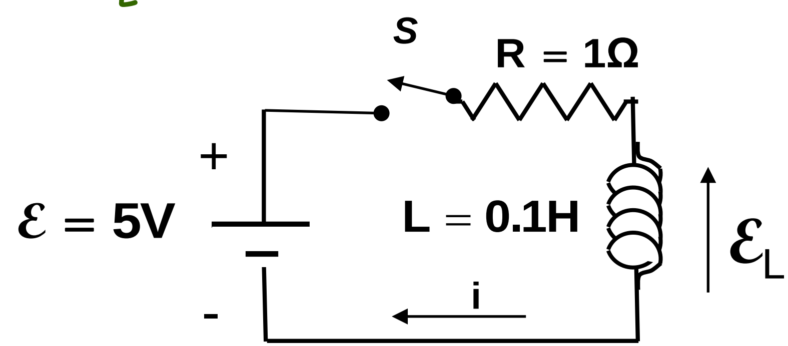
Compare to RC circuit, charging

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

RC ≡ capacitive time constant

Example: For growth phase find back EMF $\mathcal{E}_L$ as a function of time

Use growth phase solution $i(t) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L})$



At t = 0: current = 0

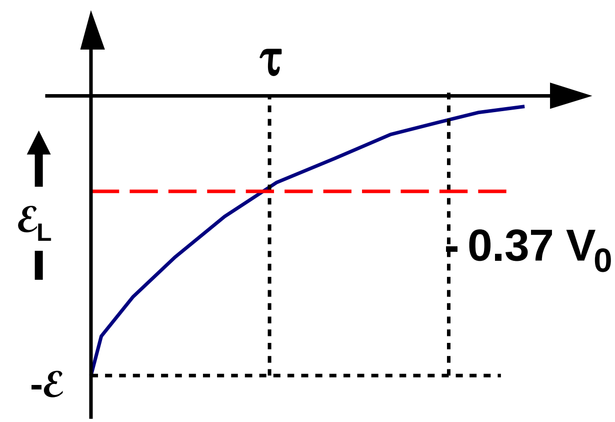
$$i(t=0) = \frac{\mathcal{E}}{R}(1 - e^{0}) = 0$$

Back EMF is ~ to rate of change of current

Derivative: $\frac{di}{dt} = \frac{\mathcal{E}}{R}\frac{(-)(-)}{\tau_L}e^{-t/\tau_L}$ where $\tau_L = \frac{L}{R}$

$$\tau_L = \frac{L}{R} = \frac{0.1H}{1\Omega} = 0.1 \text{ sec}$$

$$\mathcal{E}_L = -L\frac{di}{dt} = -\mathcal{E}e^{-t/\tau_L}; \quad \text{At } t = 0: \; \mathcal{E}_L = -\mathcal{E}$$



- Back EMF $\mathcal{E}_L$ equals the battery potential causing current i to be 0 at t = 0
- iR drop across R = 0
- L acts like a broken wire at t = 0

After a very long (infinite) time:
- Current stabilizes, back EMF=0

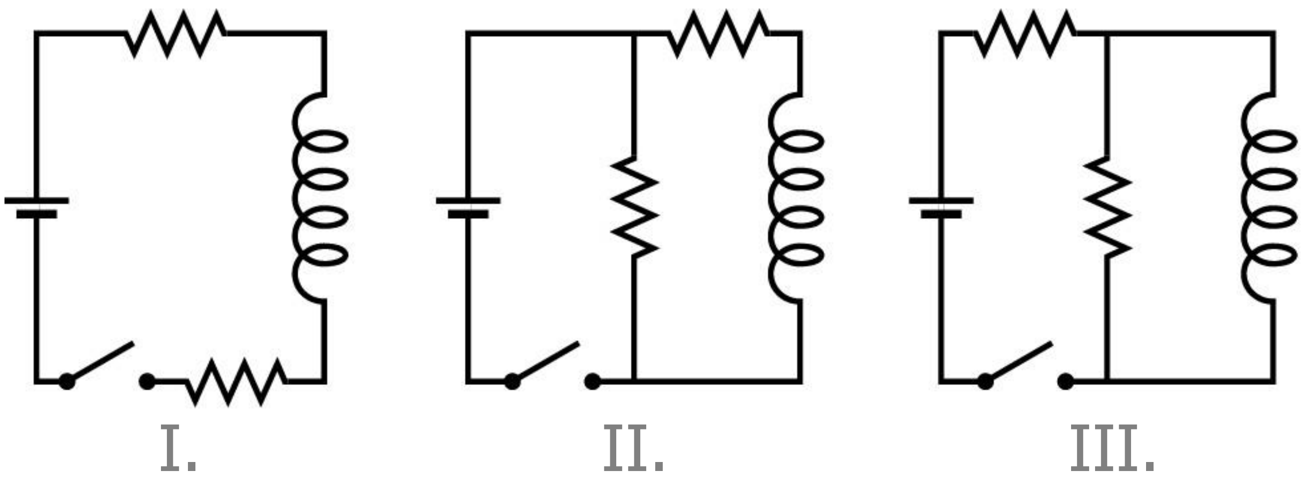
$$i_\infty = \frac{\mathcal{E}}{R} = 5A$$

- L acts like an ordinary wire at t = infinity

Current through the battery - 1

12 – 2: The three loops below have identical inductors, resistors, and batteries. Rank them in terms of current through the battery just after the switch is closed, greatest first.

A. I, II, III.
B. II, I, III.
C. III, I, II.
D. III, II, I.
E. II, III, I.



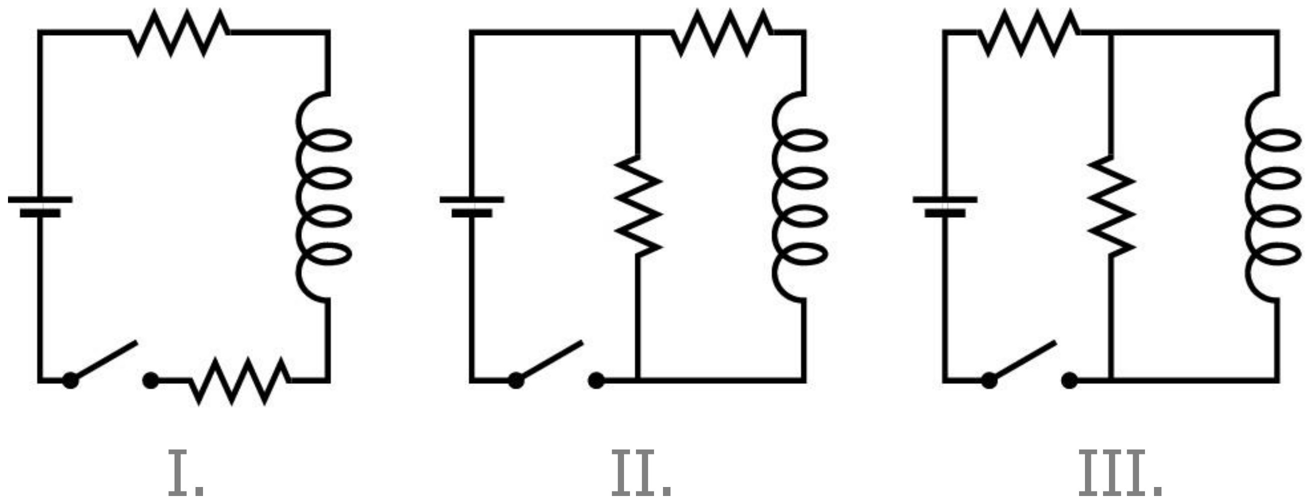
Hint: what kind of wire does L act like?

$$i(t) = i_{inf}\left(1 - e^{-t/\tau_L}\right) \qquad i_{inf} \equiv \frac{\mathcal{E}}{R_{eq}} \qquad \tau_L \equiv L/R_{eq} \equiv \text{inductive time constant}$$

Current through the battery - 2

12 – 3: The three loops below have identical inductors, resistors, and batteries. Rank them in terms of current through the battery a long time after the switch is closed, greatest first.

A. I, II, III.
B. II, I, III.
C. III, I, II.
D. III, II, I.
E. II, III, I.

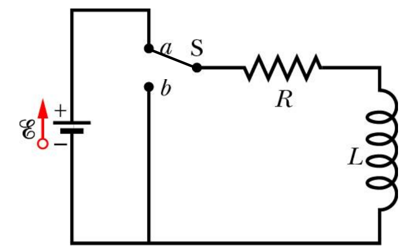


Hint: what kind of wire does L act like?

$$i(t) = i_{inf}\left(1 - e^{-t/\tau_L}\right) \qquad i_{inf} \equiv \frac{\mathcal{E}}{R_{eq}} \qquad \tau_L \equiv L/R_{eq} \equiv \text{inductive time constant}$$

Summarizing RL circuits growth phase

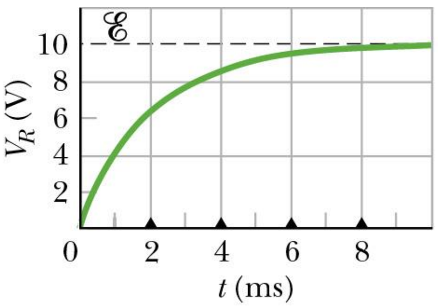
- When t is large: $i = \frac{\varepsilon}{R}$ Inductor acts like a wire. $i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$
- When t is small (zero), $i = 0$. Inductor acts like an open circuit.



- The current starts from zero and increases up to a maximum of $i = \varepsilon / R$ with a time constant given by

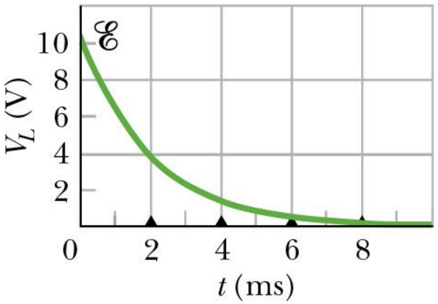
$$\tau_L = \frac{L}{R}$$ Inductive time constant

Compare: $\tau_C = RC$ Capacitive time constant



- The voltage across the resistor is

$$V_R = -iR = -\varepsilon(1 - e^{-Rt/L})$$

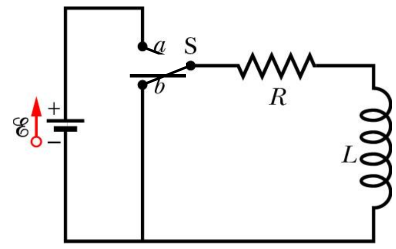


- The voltage across the inductor is

$$V_L = -\varepsilon - V_R = -\varepsilon + \varepsilon(1 - e^{-Rt/L}) = -\varepsilon\, e^{-Rt/L}$$

Summarizing RL circuits decay phase

The switch is thrown from *a* to *b*

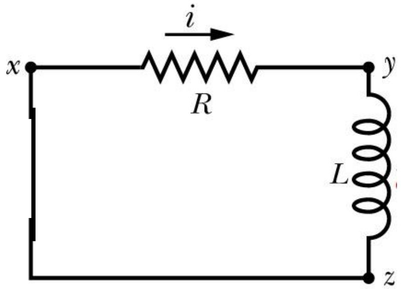


- Kirchoff's Loop Rule for growth was:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

- Now it is:

$$iR + L\frac{di}{dt} = 0$$

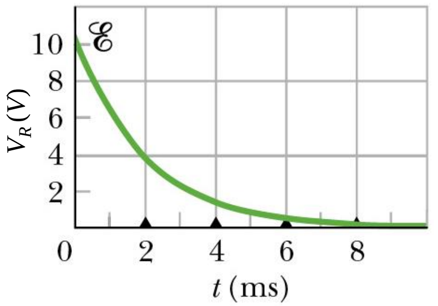


- The current decays exponentially:

$$i = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

- Voltage across resistor also decays:

$$V_R = -iR = -\mathcal{E}\,e^{-Rt/L}$$



- Voltage across inductor:

$$V_L = +L\frac{di}{dt} = +L\frac{\mathcal{E}}{R}\frac{d}{dt}e^{-Rt/L} = -\mathcal{E}\,e^{-Rt/L}$$

Energy stored in inductors

Recall: Capacitors store energy in their electric fields

$U_E \equiv$ electrostatic potential energy

$$U_E \equiv \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$u_E \equiv$ electrostatic energy density

$$u_E \equiv \frac{U_E}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

derived using p-p capacitor

Inductors also store energy, but in their magnetic fields

Magnetic PE Derivation – consider power into or from inductor

$$\text{Power} \equiv \frac{dU_B}{dt} \equiv \mathcal{E}_L i = Li\frac{di}{dt} \quad \Rightarrow \quad U_B = \int dU_B = L\int i\,di = \frac{1}{2}Li^2$$

$U_B \equiv$ magnetic potential energy

$$U_B = \frac{1}{2}Li^2$$

$u_B \equiv$ magnetic energy density

$$u_B \equiv \frac{U_B}{\text{Volume}} = \frac{B^2}{2\mu_0}$$

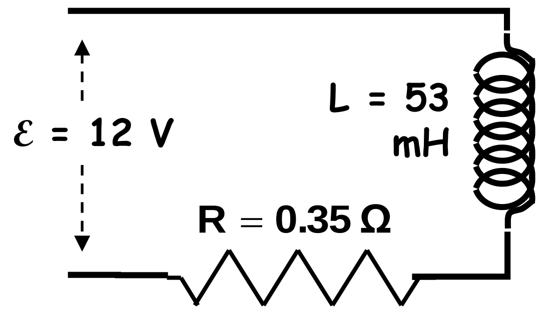
derived using solenoid

- U_B grows as current increases, absorbing energy
- When current is stable, U_B and u_B are constant
- U_B diminishes when current decreases. It powers the persistent EMF during the decay phase for the inductor

Sample problem: energy storage in magnetic field of an inductor during growth phase

a) At equilibrium (infinite time) how much energy is stored in the coil?

$$i_\infty = \frac{\mathcal{E}}{R} \text{ (Coil acts like a wire) } = \frac{12}{0.35} = 34.3 \text{ A}$$



$$U_\infty = \frac{1}{2} L \cdot i_\infty^2 = \frac{1}{2} \text{ x } 53 \text{ x } 10^{-3} \text{ x } (34.3)^2$$

$$\boxed{U_\infty = 31 \text{ J}}$$

b) How long ($t_{1/2}$) does it take to store half of this energy?

$$\text{At } t_{1/2}: \quad U_B = \frac{1}{2} U_\infty \quad \Rightarrow \quad \frac{1}{2} L i_{1/2}^2 = \frac{1}{2} \cdot \frac{1}{2} \cdot L \cdot i_\infty^2 \quad \Rightarrow \quad i_{1/2} = \frac{i_\infty}{\sqrt{2}}$$

$$i_{1/2} = \frac{i_\infty}{\sqrt{2}} = i_\infty (1 - e^{-t_{1/2}/\tau_L})$$

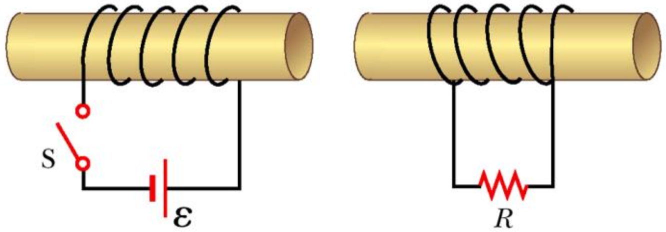
$$e^{-t_{1/2}/\tau_L} = \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 1 - 1/\sqrt{2}$$

take natural log of both sides

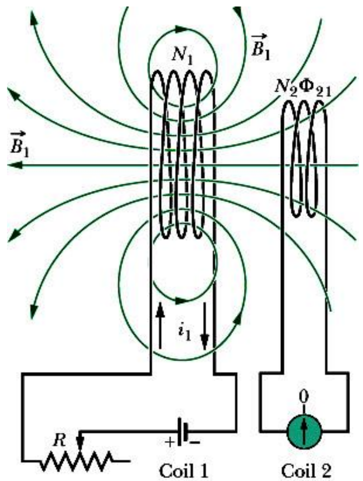
$$\boxed{t_{1/2} = -\tau_L \ln(1 - 1/\sqrt{2}) = 1.23\,\tau}$$

$$\tau_L \equiv L/R = 53 \text{ x } 10^{-3} / 0.35 = 0.15 \text{ sec.}$$

Mutual Inductance



- Example: a pair of co-axial coils
- di/dt in the first coil induces current in the second coil, in addition to self-induced effects.
- M_{21} depends on *geometry only*, as did L and C
- *Changing* current in primary (i_1) creates varying flux through coil 2 → induced EMF in coil 2



Definition:

$$M_{21} \equiv \frac{N_2\, \Phi_{21}}{i_1}$$

- number of turns in coil 2 → N_2
- flux through one turn of coil 2 due to all N_1 turns of coil 1 → Φ_{21}
- current in coil 1 → i_1
- mutual inductance → M_{21}

cross-multiply

$$M_{21}\, i_1 = N_2\, \Phi_{21}$$

time derivative

$$M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{21}}{dt}$$

$$\mathcal{E}_2 \equiv -M_{21}\frac{di_1}{dt}$$

- M_{21} contains all the geometry
- $\mathcal{E}_2$ is EMF" induced in 2 by 1

Another form of Faraday's Law!

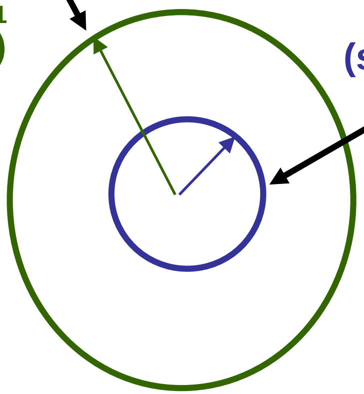
The smaller coil radius determines how much flux is linked, so…..

$$\mathcal{E}_1 \equiv -M_{12}\frac{di_2}{dt} \qquad M_{12} = M_{21} \equiv M$$

proof not obvious

Calculating the mutual inductance M

Let coil 1 (outer) be a short loop of N_1 turns, not a long Solenoid



$B_1 \equiv$ Field inside loop 1 near center

$$= \frac{\mu_0 i_1 N_1}{2R_1} \quad \text{(assume uniform)}$$

Flux through Loop 2 – depends on area A_2 & B_1

$$\Phi_{21} = B_1 A_2 = \frac{\mu_0 i_1 N_1}{2R_1}\pi R_2^2 \quad \text{(for each loop in coil 2)}$$

If current in Loop 1 is changing:

$$\mathcal{E}_2 \equiv \text{induced voltage in loop 2} = -N_2\frac{d\Phi_{21}}{dt} = -\frac{\mu_0 N_1 N_2 \pi R_2^2}{2R_1}\frac{di_1}{dt} \equiv -M_{21}\frac{di_1}{dt}$$

$$\therefore \quad M_{21} \equiv M = \frac{\mu_0 N_1 N_2}{2R_1}\pi R_2^2$$

smaller radius (R_2) determines the linkage

Summarizing results for mutual inductance:

$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt} \qquad M_{21} = \frac{N_2\Phi_{21}}{i_1}$$

$$M_{21} = M = \frac{N_2\Phi_{21}}{i_1} = \frac{N_1\Phi_{12}}{i_2}$$

Summary: Lecture 12 Chapter 30 – Induction II – LR Circuits

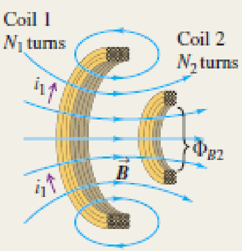
CHAPTER 30 SUMMARY

Mutual inductance: When a changing current i_1 in one circuit causes a changing magnetic flux in a second circuit, an emf $\mathcal{E}_2$ is induced in the second circuit. Likewise, a changing current i_2 in the second circuit induces an emf $\mathcal{E}_1$ in the first circuit. If the circuits are coils of wire with N_1 and N_2 turns, the mutual inductance M can be expressed in terms of the average flux Φ_{B2} through each turn of coil 2 caused by the current i_1 in coil 1, or in terms of the average flux Φ_{B1} through each turn of coil 1 caused by the current i_2 in coil 2. The SI unit of mutual inductance is the henry, abbreviated H. (See Examples 30.1 and 30.2.)

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and}$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt} \qquad (30.4)$$

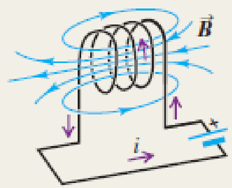
$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2} \qquad (30.5)$$



Self-inductance: A changing current i in any circuit causes a self-induced emf $\mathcal{E}$. The inductance (or self-inductance) L depends on the geometry of the circuit and the material surrounding it. The inductance of a coil of N turns is related to the average flux Φ_B through each turn caused by the current i in the coil. An inductor is a circuit device, usually including a coil of wire, intended to have a substantial inductance. (See Examples 30.3 and 30.4.)

$$\mathcal{E} = -L\frac{di}{dt} \qquad (30.7)$$

$$L = \frac{N\Phi_B}{i} \qquad (30.6)$$

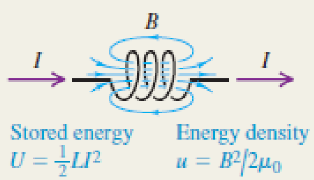


Magnetic-field energy: An inductor with inductance L carrying current I has energy U associated with the inductor's magnetic field. The magnetic energy density u (energy per unit volume) is proportional to the square of the magnetic field magnitude. (See Example 30.5.)

$$U = \tfrac{1}{2}LI^2 \qquad (30.9)$$

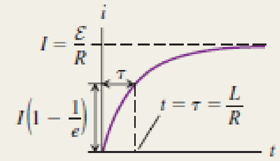
$$u = \frac{B^2}{2\mu_0} \quad \text{(in vacuum)} \qquad (30.10)$$

$$u = \frac{B^2}{2\mu} \quad \text{(in a material with magnetic permeability } \mu\text{)} \qquad (30.11)$$



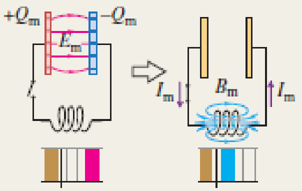
R-L circuits: In a circuit containing a resistor R, an inductor L, and a source of emf, the growth and decay of current are exponential. The time constant τ is the time required for the current to approach within a fraction $1/e$ of its final value. (See Examples 30.6 and 30.7.)

$$\tau = \frac{L}{R} \qquad (30.16)$$



L-C circuits: A circuit that contains inductance L and capacitance C undergoes electrical oscillations with an angular frequency ω that depends on L and C. This is analogous to a mechanical harmonic oscillator, with inductance L analogous to mass m, the reciprocal of capacitance $1/C$ to force constant k, charge q to displacement x, and current i to velocity v_x. (See Examples 30.8 and 30.9.)

$$\omega = \sqrt{\frac{1}{LC}} \qquad (30.22)$$



L-R-C series circuits: A circuit that contains inductance, resistance, and capacitance undergoes damped oscillations for sufficiently small resistance. The frequency ω' of damped oscillations depends on the values of L, R, and C. As R increases, the damping increases; if R is greater than a certain value, the behavior becomes overdamped and no longer oscillates. (See Example 30.10.)

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad (30.29)$$