Physics 121 - Electricity and Magnetism
Lecture 14 - AC Circuits, Resonance
Y&F Chapter 31, Sec. 3 - 8

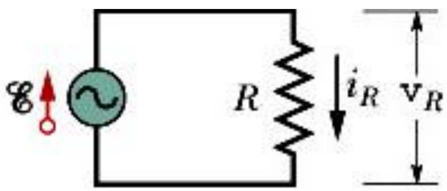
- **The Series RLC Circuit. Amplitude and Phase Relations**
- **Phasor Diagrams for Voltage and Current**
- **Impedance and Phasors for Impedance**
- **Resonance**
- **Power in AC Circuits, Power Factor**
- **Examples**
- **Transformers**
- **Summaries**

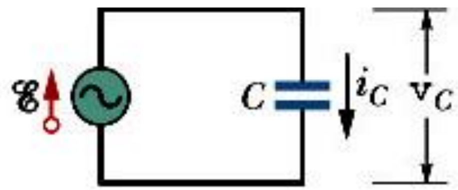
Current & voltage phases in pure R, C, and L circuits

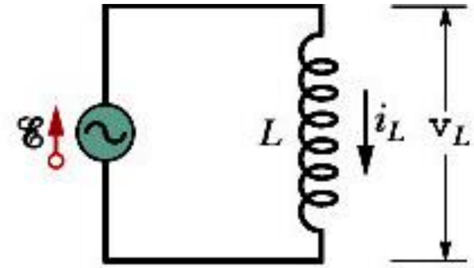
Current is the same everywhere in a single branch (including phase)

Phases of voltages in elements are referenced to the current phasor

- Apply sinusoidal voltage $\mathcal{E}(t) = \mathcal{E}_m \mathrm{Cos}(\omega_D t)$
- For pure R, L, or C loads, phase angles are 0, $+\pi/2$, $-\pi/2$
- Reactance" means ratio of peak voltage to peak current (generalized resistances).







V_R & i_R in phase
Resistance

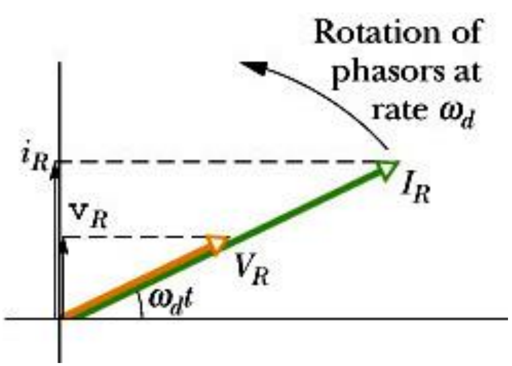
$$V_{max} / i_R \equiv R$$

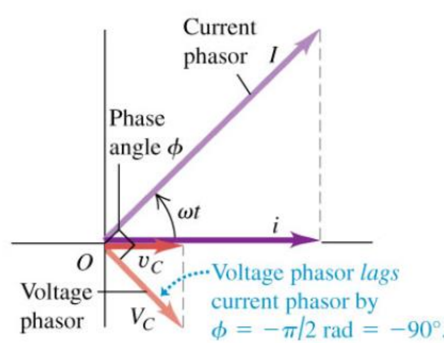
V_C lags i_C by $\pi/2$
Capacitive Reactance

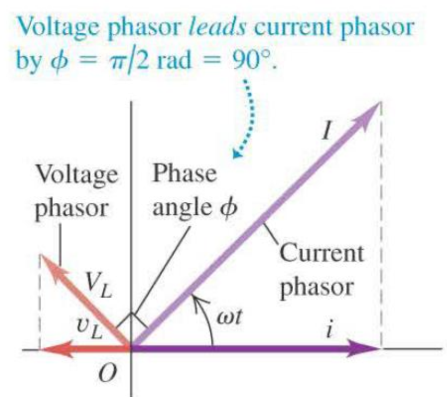
$$V_{max} / i_C \equiv X_C = \frac{1}{\omega_D C}$$

V_L leads i_L by $\pi/2$
Inductive Reactance

$$V_{max} / i_L \equiv X_L = \omega_D L$$







The *impedance* is the ratio of peak EMF to peak current

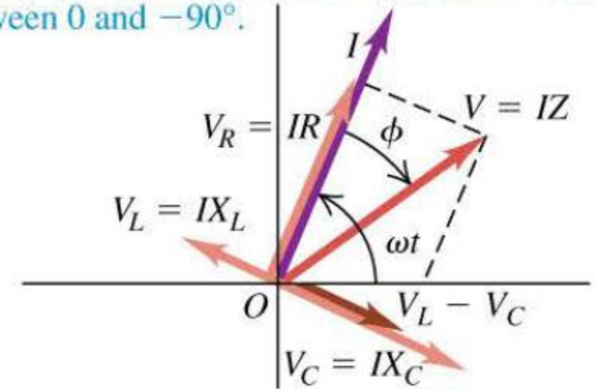
$$\vec{Z} \equiv \frac{\vec{\mathcal{E}}_m}{i_m}$$

$\vec{\mathcal{E}}_m$ ← peak applied voltage

i_m ← peak current that flows

$[Z] = \text{ohms}$

If $X_L < X_C$, the source voltage phasor lags the current phasor, $X < 0$, and ϕ is a negative angle between 0 and −90°.



Magnitude of $\mathcal{E}_m$: $\mathcal{E}_m^2 = V_R^2 + (V_L - V_C)^2$

Reactances: $X_L = \omega_D L \qquad X_C = \dfrac{1}{\omega_D C}$

$X_L \equiv V_L / i_L \qquad X_C \equiv V_C / i_C \qquad R \equiv V_R / i_R$

$i_m = i_{R,max} = i_{L,max} = i_{C,max}$

For series LRC circuit, divide $\mathcal{E}_m$ by peak current

Magnitude of Z: $Z \equiv [R^2 + (X_L - X_C)^2]^{1/2}$

Applies to a single branch with L, C, R

Phase angle Φ: $\tan(\Phi) \equiv \dfrac{V_L - V_C}{V_R} = \dfrac{X_L - X_C}{R}$

see diagram

Φ measures the power absorbed by the circuit: $P \propto \vec{\mathcal{E}}_m \circ \vec{i}_m = \mathcal{E}_m i_m \cos(\Phi)$

- R ~ 0 → tiny losses, no power absorbed → i_m normal to $\mathcal{E}_m$ → Φ ~ +/- $\pi/2$
- $X_L = X_C$ → i_m parallel to $\mathcal{E}_m$ → $\Phi = 0$ → Z=R → maximum current (resonance)

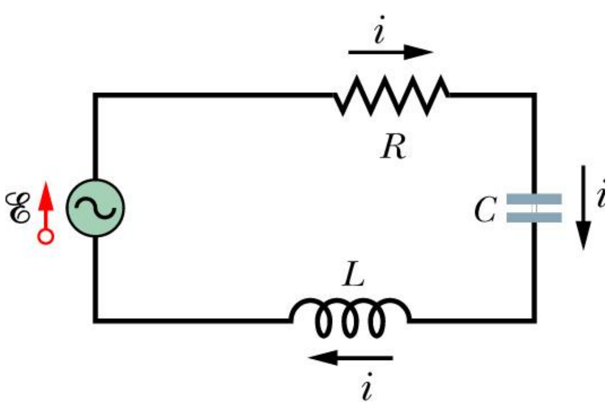
Table 31.1 Circuit Elements with Alternating Current

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

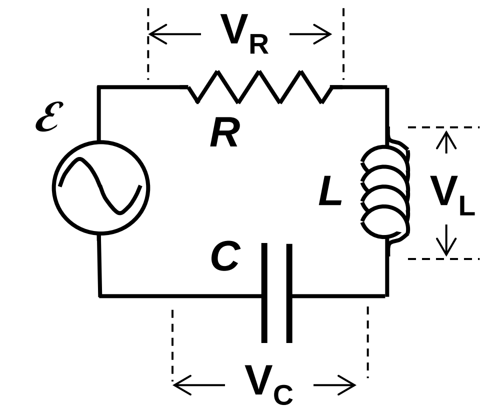
Example 1: Analyzing a series RLC circuit

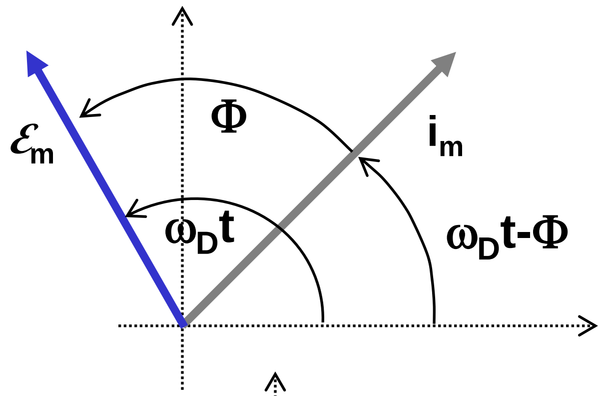
A series RLC circuit has R = 425 Ω, L = 1.25 H, C = 3.50 μF.
It is connected to an AC source with f = 60.0 Hz and ε_m= 150 V.

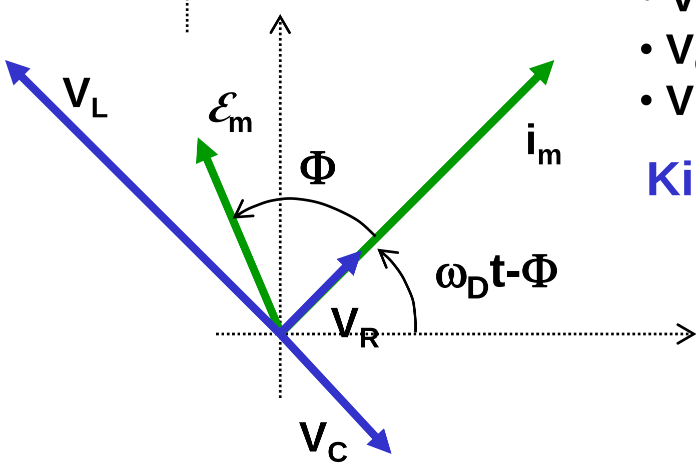
(A) Determine the impedance of the circuit.
(B) Find the amplitude of the current (peak value).
(C) Find the phase angle between the current and voltage.
(D) Find the instantaneous current across the RLC circuit.
(E) Find the peak and instantaneous voltages across each circuit element.



Series LCR circuit driven by an external AC voltage







Apply EMF:

$\mathcal{E}(t) = \mathcal{E}_m \mathrm{Cos}(\omega_D t)$ ω_D is the driving frequency

The current i(t) is the same everywhere in the circuit

$i(t) = \mathcal{E}_m \mathrm{Cos}(\omega_D t + \Phi)$

- Same frequency dependance as $\mathcal{E}(t)$ but...
- Current leads or lags $\mathcal{E}(t)$ by a constant phase angle Φ
- Same phase for the current in $\mathcal{E}$, R, L, & C

Phasors all rotate CCW at frequency ω_D

- Lengths of phasors are the peak values (amplitudes)
- The "y" components are the measured values.

Plot voltages in components with phases relative to current phasor i_m :

- V_R has same phase as i_m $\qquad V_R = i_m R$
- V_C lags i_m by $\pi/2$ $\qquad V_C = i_m X_C$
- V_L leads i_m by $\pi/2$ $\qquad V_L = i_m X_L$

Kirchoff Loop rule for potentials (measured along y)

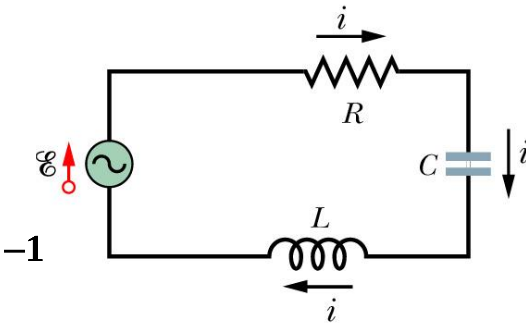
$$\vec{\mathcal{E}}(t) - \vec{V}_R(t) - \vec{V}_L(t) - \vec{V}_C(t) = 0$$

$$\vec{\mathcal{E}}_m = \vec{V}_R + (\vec{V}_L + \vec{V}_C) \qquad \vec{V}_C \text{ lags } \vec{V}_L \text{ by } 180^0$$

along i_m ↗ ↖ perpendicular to i_m

Example 1: Analyzing a series RLC circuit

A series RLC circuit has R = 425 Ω, L = 1.25 H, C = 3.50 μF.
It is connected to an AC source with f = 60.0 Hz and ε_m=150 V.



(A) Determine the impedance of the circuit.

Angular frequency: $\omega_D = 2\pi f = 2\pi\,(60.0)\,\mathrm{Hz} = 377\ \mathrm{s}^{-1}$

Resistance: $R = 425\ \Omega$

Inductive reactance: $X_L = \omega_D L = (377\ \mathrm{s}^{-1})(1.25\ \mathrm{H}) = 471\,\Omega$

Capacitive reactance: $X_C = 1/\omega_D C = 1/(377\ \mathrm{s}^{-1})(3.50\times 10^{-6}\ \mathrm{F}) = 758\,\Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(425\,\Omega)^2 + (471\,\Omega - 758\,\Omega)^2} = 513\,\Omega$$

(B) Find the peak current amplitude:

$$I_m = \frac{\varepsilon_m}{Z} = \frac{150\ \mathrm{V}}{513\,\Omega} = 0.293\ \mathrm{A}$$

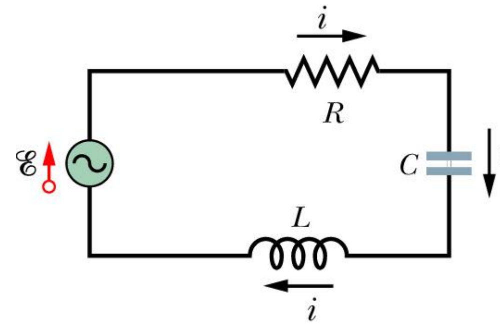
(C) Find the phase angle between the current and voltage.

$X_C > X_L$ (Capacitive) ➡ Current vector I_m leads the Voltage $\mathcal{E}_m$
Phase angle should be negative

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\,\Omega - 758\,\Omega}{425\,\Omega}\right) = -34.0^\circ = -0.593\ \mathrm{rad.}$$

Example 1: analyzing a series RLC circuit - continued

A series RLC circuit has R = 425 Ω, L = 1.25 H, C = 3.50 μF.
It is connected to an AC source with f = 60.0 Hz and ε_m=150 V.



(D) Find the instantaneous current across the RLC circuit.

$$I_m cos(\omega t + \varphi) = 0.292\ cos(377t - 0.593\ rad)$$

(E) Find the peak and instantaneous voltages across each circuit element.

$$V_{R,m} = I_m R = (0.292\ A)(425\ \Omega) = 124\ V$$

$$v_R(t) = V_R cos(\omega t + \varphi) = 124\ cos(377t - 0.593\ rad)$$

V_R in phase with I_m
V_R leads $\mathcal{E}_m$ by $|\Phi|$

$$V_{L,m} = I_m X_L = (0.292\ A)(471\ \Omega) = 138\ V$$

$$v_L(t) = V_L cos(\omega t + \varphi + \pi/2) = 124\ cos(377t - 0.593\ rad + \pi/2)$$

V_L leads V_R by 90°

$$V_{C,m} = I_m X_C = (0.293\ A)(758\ \Omega) = 222\ V$$

$$v_C(t) = V_C cos(\omega t + \varphi - \pi/2) = 222\ cos(377t - 0.593\ rad + \pi/2)$$

V_C lags V_R by 90°

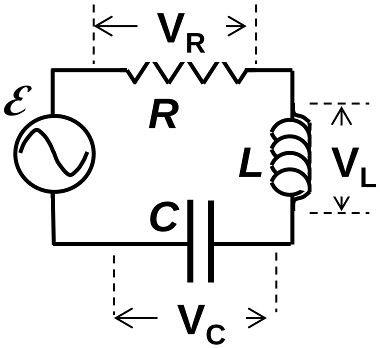
Note that: $V_R + V_L + V_C = 483V \neq 150V = \mathcal{E}_m$ Why not?

Voltages add with proper phases: $\mathcal{E}_m = \left(V_R^2 + [V_L - V_C]^2\right)^{1/2} = 150\ V$

Example 2: Resonance in a series LCR Circuit:

$R = 3000\ \Omega \qquad L = 0.33\ H \qquad C = 0.10\ mF \qquad \mathcal{E}_m = 100\ V.$

Find Z and Φ for $f_D = 200$ Hertz, $f_D = 876$ Hz, & $f_D = 2000$ Hz



Why should f_D make a difference?

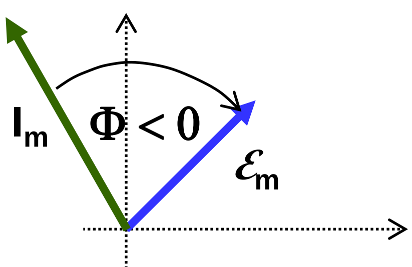
$$X_L = \omega_D L \qquad X_C = \frac{1}{\omega_D C} \qquad I_m \equiv \frac{\mathcal{E}_m}{Z}$$

$$Z \equiv [R^2 + (X_L - X_C)^2]^{1/2}$$

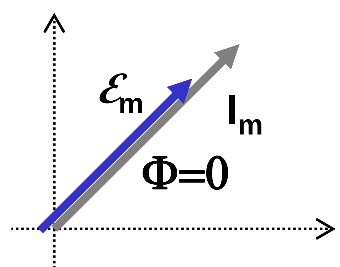
$$\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Frequency f	Resistance R	Reactance X_C	Reactance X_L	Impedance Z	Phase Angle Φ	Circuit Behavior
200 Hz	3000 Ω	7957 Ω	415 Ω	8118 Ω	- 68.3º	Capacitive $\mathcal{E}_m$ lags I_m
876 Hz	3000 Ω	1817 Ω	1817 Ω	3000 Ω Resonance	0º	Resistive Max current
2000 Hz	3000 Ω	796 Ω	4147 Ω	4498 Ω	+48.0º	Inductive $\mathcal{E}_m$ leads I_m

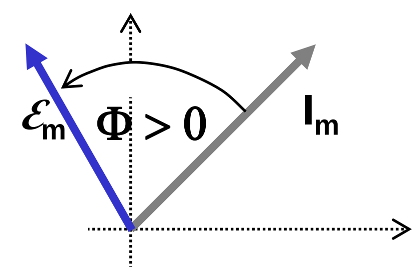
$X_C > X_L$



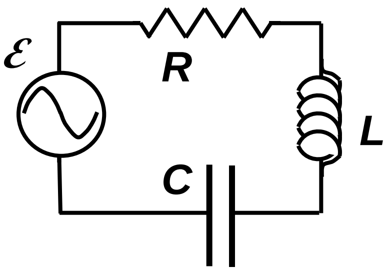
$X_C = X_L$



$X_L > X_C$



Resonance



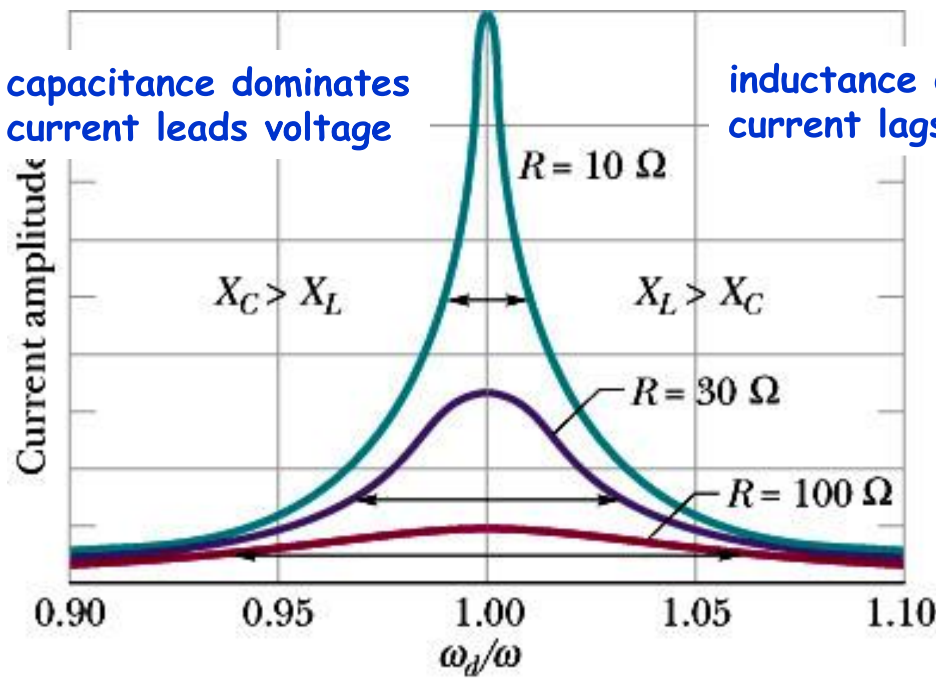
$$Z \equiv \left[R^2 + (X_L - X_C)^2\right]^{1/2} \qquad I_m = \frac{\mathcal{E}_m}{Z}$$

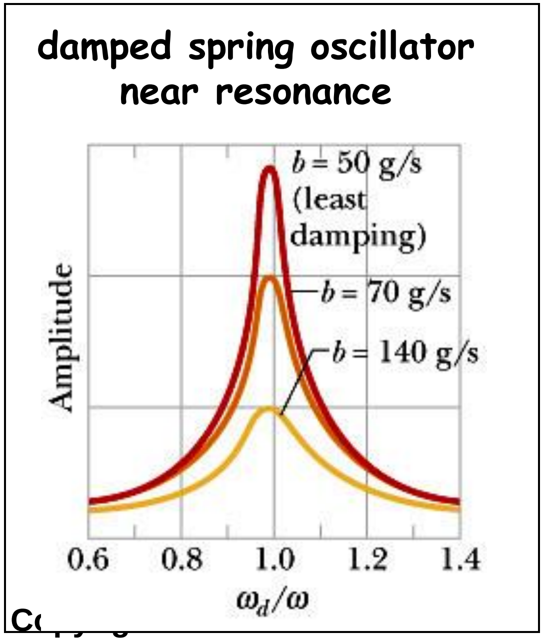
$$X_C \equiv 1/\omega_D C \qquad X_L \equiv \omega_D L \qquad R = \text{resistance}$$

Vary ω_D: At resonance maximum current flows & impedance is minimized

$$X_C = X_L \text{ when } \omega_D = 1/\sqrt{LC} \equiv \omega_{res} \quad \Rightarrow \quad Z = R, \quad I_m = \mathcal{E}_M / R, \quad \Phi = 0$$

width of resonance (selectivity, "Q") depends on R.
Large R → less selectivity, smaller current at peak



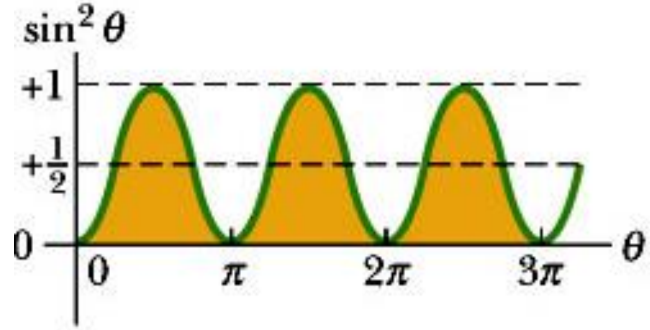


Power in AC Circuits

- Resistors always dissipate power, but the instantaneous rate varies as $i^2(t)R$
- No power is lost in pure capacitors and pure inductors in an AC circuit
  - In a capacitor, during one-half of a cycle energy is stored and during the other half the energy is returned to the circuit
  - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor. When the current in the circuit begins to decrease, the energy is returned to the circuit

Instantaneous and RMS (average) power

instantaneous power consumed by circuit $\equiv P_{inst} = i^2(t)R = I_m^2 R \sin^2(\omega t - \Phi)$



- Power is dissipated in R, not in L or C
- $\sin^2(x)$ is always positive, so P_{inst} is always positive. But, it is not constant.
- Pattern for power repeats every π radians (T/2)

The RMS power, current, voltage are useful, DC-like quantities

$$P_{av} \equiv \text{average of } P_{inst} \text{ over a whole cycle } (\omega \tau = 2\pi)$$

Integrate

$$P_{av} = I_m^2 R \frac{1}{T}\int_0^T \sin^2(\omega t - \Phi)dt = \frac{1}{2} I_m^2 R \quad \text{(the integral} = 1/2\text{)}$$

P_{av} is an "RMS" quantity:

- "Root Mean Square"
- Square a quantity (positive)
- Average over a whole cycle
- Compute square root.
- In practice, divide peak value by sqrt(2) for I_m or $\mathcal{E}_m$ since $\sin^2(x)$ appears

$$P_{rms} \equiv P_{av} = I_{rms}^2 R$$

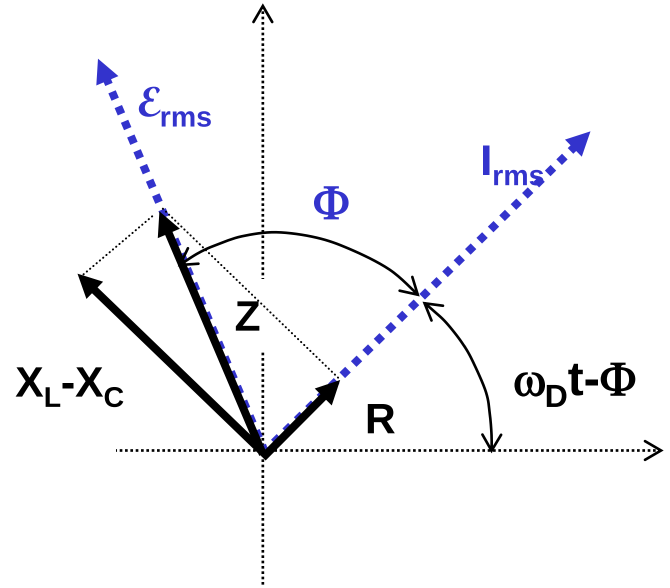
$$\mathcal{E}_{rms} \equiv \frac{\mathcal{E}_m}{\sqrt{2}} \qquad I_{rms} \equiv \frac{I_m}{\sqrt{2}} \qquad I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$$

$$V_{rms} \equiv \frac{V_m}{\sqrt{2}}$$ any component

Household power example: 120 volts RMS ←→ 170 volts peak

Power factor for AC Circuits

The *PHASE ANGLE* Φ determines the average RMS power actually absorbed by the RMS current and applied voltage in the circuit.



Result (proven below):

$$P_{av} \equiv P_{rms} = \vec{\mathcal{E}}_{rms} \circ \vec{I}_{rms} = \mathcal{E}_{rms} I_{rms} \cos(\Phi)$$

$$\cos(\Phi) \equiv R / Z \quad \text{is the "power factor"}$$

Proof: start with instantaneous power (not very useful, shift cos(x) by pi/2):

$$P_{inst}(t) = \mathcal{E}(t)\, I(t) = \mathcal{E}_m I_m \sin(\omega_D t) \sin(\omega_D t - \Phi)$$

Average it over one full cycle:

$$P_{av} \equiv \frac{1}{\tau}\int_0^{\tau} P_{inst}(t)\, dt = \mathcal{E}_m I_m \frac{1}{\tau}\int_0^{\tau} \sin(\omega_D t) \sin(\omega_D t - \Phi) dt$$

Note trig identities:

$$\sin(\omega_D t \pm \Phi) = \sin(\omega_D t)\cos(\Phi) \pm \cos(\omega_D t)\sin(\Phi)$$

$$\sin(\omega_D t)\cos(\omega_D t) = \frac{1}{2}\sin(2\omega_D t)$$

Power factor for AC Circuits - continued

Substitute trig identities:

$$P_{av} = \mathcal{E}_m I_m \cos(\Phi)\frac{1}{\tau}\int_0^\tau \sin^2(\omega_D t)dt \; - \; \mathcal{E}_m I_m \sin(\Phi)\frac{1}{2\tau}\int_0^\tau \sin(2\omega_D t)$$

Over a full period:

$$\omega_D\tau = 2\pi \qquad \frac{1}{\tau}\int_0^\tau \sin^2(\omega_D t)dt = \frac{1}{2} \qquad \frac{1}{2\tau}\int_0^\tau \sin(2\omega_D t) = 0$$

$$\therefore \; P_{av} = \mathcal{E}_m I_m \left\{\frac{\cos(\Phi)}{2} - 0\right\}$$

Recall: RMS values = Peak values divided by sqrt(2)

$$\therefore P_{av} \equiv P_{rms} = \mathcal{E}_{rms} I_{rms} \cos(\Phi)$$

Also note: $\therefore \mathcal{E}_{rms} = I_{rms} Z$ and $R = Z\cos(\Phi)$

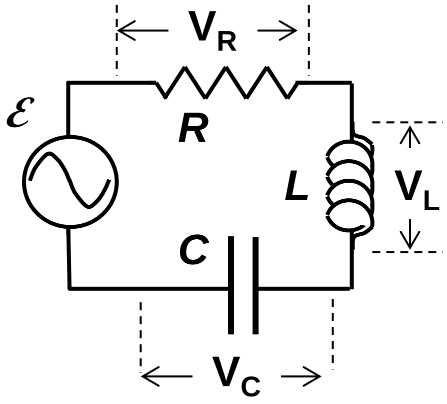
Alternate form: $P_{rms} = I^2_{rms} Z \cos(\Phi) = I^2_{rms} R$

If R=0 (pure LC circuit) $\Phi \rightarrow +/- \; \pi/2$ and $P_{av} = P_{rms} = 0$

Example 2 continued with RMS quantities:

$R = 3000\ \Omega$ $L = 0.33\ H$ $C = 0.10\ mF$ $\mathcal{E}_m = 100\ V.$

$f_D = 200\ Hz$



Find $\mathcal{E}_{rms}$: $\mathcal{E}_{rms} = \mathcal{E}_m / \sqrt{2} = 71\,V.$

Find I_{rms} at 200 Hz: $Z = 8118\,\Omega$ as before

$$I_{rms} = \mathcal{E}_{rms} / Z = 71\,V / 8118\,\Omega = 8.75\ mA.$$

Find the power factor:

$$\cos(\Phi) = R/Z = \frac{3000}{8118} = 0.369$$

Recall: do not use arc-cos to find Φ

Find the phase angle Φ:

$$\Phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = -68^0 \quad \text{as before}$$

Find the average power:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos(\Phi) = 71 \times 8.75 \times 10^{-3} \times 0.369 = 0.23\ \text{Watts}$$

or

$$P_{av} = I_{rms}^2 R = \left(8.75 \times 10^{-3}\right)^2 \times 3000 = 0.23\ \text{Watts}$$

Example 3 – LCR circuit analysis using RMS values

A 240 V (RMS), 60 Hz voltage source is applied to a series LCR circuit consisting of a 50-ohm resistor, a 0.5 H inductor and a 20 μF capacitor. $\omega_D = 2\pi f = 6.28 \times 60 = 377 \text{ rad/s}$

Find the capacitive reactance of the circuit: $X_C \equiv 1/\omega_D C = 1/(377 \times 2 \times 10^{-5}) = 133\,\Omega$

Find the inductive reactance of the circuit: $X_L \equiv \omega_D L = 377 \times .5 = 188.5\,\Omega$

The impedance of the circuit is: $Z \equiv [R^2 + (X_L - X_C)^2]^{1/2} = 74.7\,\Omega$

The phase angle for the circuit is: $\tan(\Phi) = \dfrac{X_L - X_C}{R} \Rightarrow \Phi = 48.0^0, \quad \cos(\Phi) = 0.669$

Φ is positive since $X_L > X_C$ (inductive)

The RMS current in the circuit is: $I_{rms} = \dfrac{\mathcal{E}_{rms}}{Z} = \dfrac{240}{74.7} = 3.2\text{ A.}$

The average power consumed in this circuit is:

$P_{rms} = I_{rms}^2 R = (3.2)^2 \times 50 = 516\text{ W.}$ or $P_{rms} = \mathcal{E}_{rms} I_{rms} \cos(\Phi)$ where $\cos(\Phi) = R/Z$

If the inductance could be changed to <u>maximize</u> the current through the circuit, what would the new inductance L' be? Current is a maximum at RESONANCE.

$$\omega_D = 377 = \frac{1}{\sqrt{LC}} \Rightarrow L' = \frac{1}{\omega_D^2 C} = \frac{1}{377^2 \times 2 \times 10^{-5}} = 0.352\text{ H.}$$

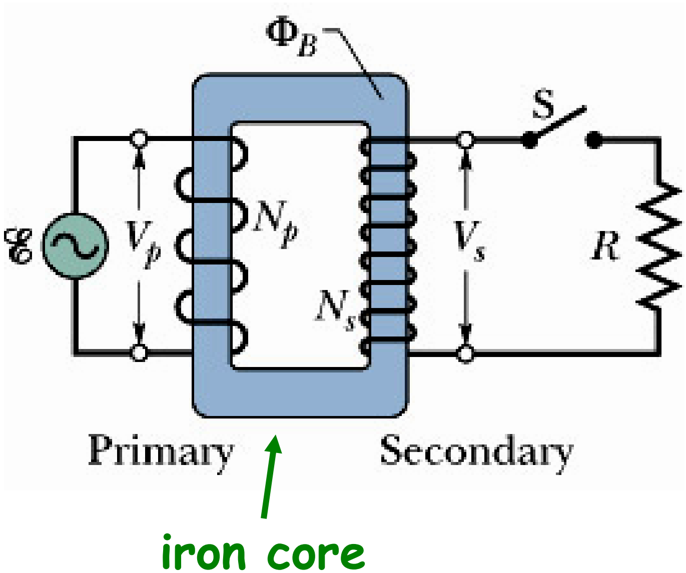
How much RMS current would flow in that case?

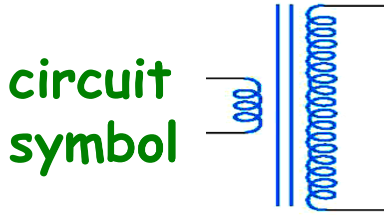
$$\text{At resonance } Z = R \Rightarrow I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{240\text{ V}}{50\,\Omega} = 4.8\text{ A.}$$

Transformers

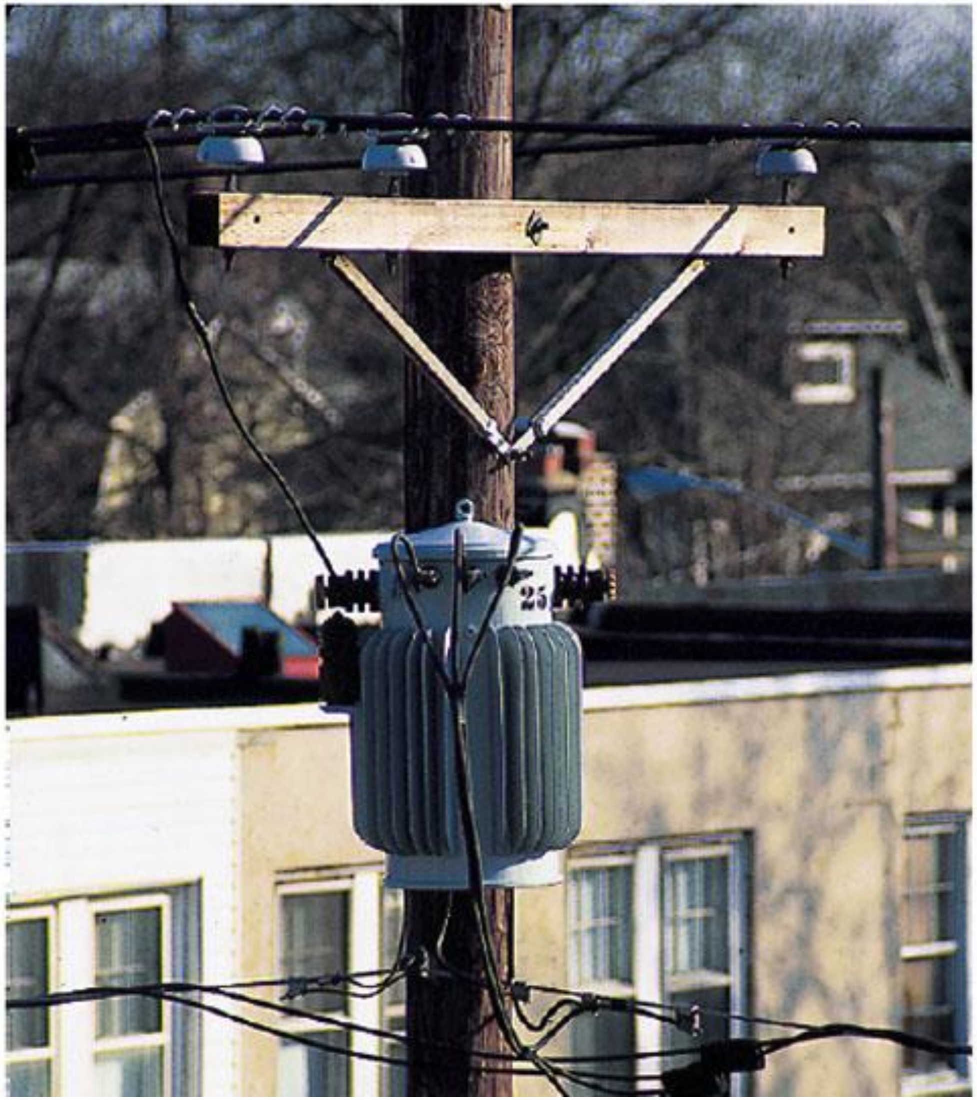
Devices used to change AC voltages. They have:

- Primary
- Secondary
- Power ratings





power transformer



Transformers

Assume zero internal resistances, EMFs $\mathcal{E}_p$, $\mathcal{E}_s$ = terminal voltages V_p, V_s

Faradays Law for primary and secondary:

$$V_p = -N_p \frac{d\Phi_B}{dt} \qquad V_s = -N_s \frac{d\Phi_B}{dt}$$

The same flux Φ_B cuts **each turn** in both primary and secondary windings of an ideal transformer (counting self- and mutual-induction)

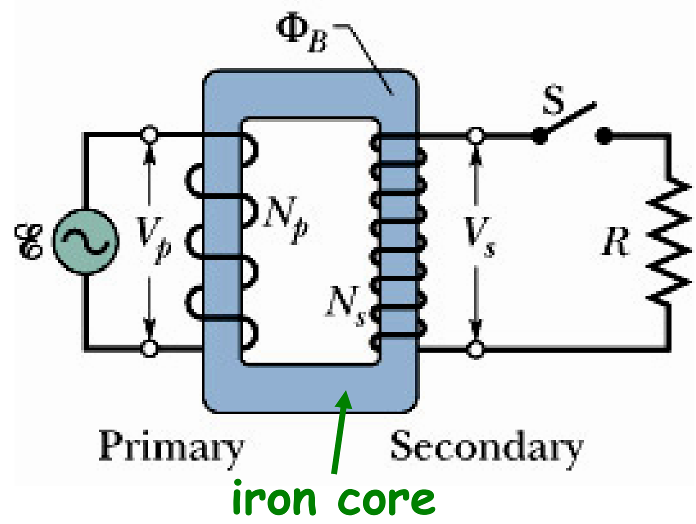
$$\frac{\text{induced voltage}}{\text{turn}} \equiv \frac{d\Phi_B}{dt} = \frac{V_p}{N_p} = \frac{V_s}{N_s}$$

$$\therefore\ V_s = \frac{N_s}{N_p} V_p$$

Turns ratio fixes the step up or step down voltage ratio

V_p, V_s are instantaneous (time varying) but can also be regarded as RMS averages, as can be the power and current.

Ideal Transformer



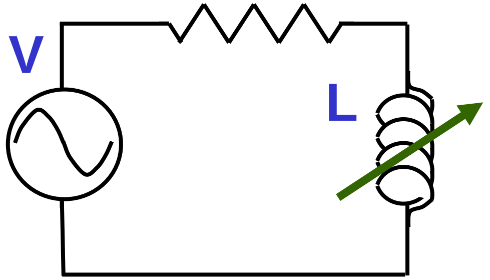
- zero resistance in coils
- no hysteresis losses in iron core
- all field lines are inside core

Assume no losses: energy and power are conserved

$$P_s = V_s I_s = \text{conserved} = P_p = V_p I_p$$

$$\therefore\ \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Example: A dimmer for lights using a variable inductance



$f = 60$ Hz $\qquad \omega = 377$ rad/sec

Without Inductor:

$$P_{rms} = \mathcal{E}^2_{rms}/R = 18 \text{ Watts}, \quad \Phi = 0$$

a) What value of the inductance would dim the lights to 5 Watts?

Recall: $P_{rms} = I^2_{rms} R \qquad I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{rms}}{R}\cos(\Phi)$

$$\therefore P_{rms} = \frac{\mathcal{E}^2_{rms}}{R}\cos^2(\Phi)$$

$$5 \text{ Watts} = 18 \text{ Watts} \times \cos^2(\Phi)$$

$$\cos(\Phi) = 0.527 = \cos(58.2^\circ)$$

$$\Phi = 58.2^o = \tan^{-1}[X_L / R]$$

$$X_L = R\tan(\Phi) = 50\tan(58.2^\circ) = 80.6\,\Omega = 2\pi f L$$

$$\therefore L = 80.6/377 = 214 \text{ mH}$$

b) What is the change in the RMS current?

Without inductor: $I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{30\text{ V}}{50\,\Omega} = 0.6\text{ A} \qquad P_{rms} = 18$ W.

With inductor: $I'_{rms} = \frac{\mathcal{E}_{rms}}{R}\cos(\Phi) = 0.6\text{ A} \times 0.527 = 0.316\text{ A} \qquad P_{rms} = 5$ W.

SUMMARY – AC CIRCUIT RELATIONS

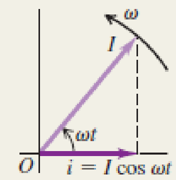
Phasors and alternating current: An alternator or ac source produces an emf that varies sinusoidally with time. A sinusoidal voltage or current can be represented by a phasor, a vector that rotates counterclockwise with constant angular velocity ω equal to the angular frequency of the sinusoidal quantity. Its projection on the horizontal axis at any instant represents the instantaneous value of the quantity.

For a sinusoidal current, the rectified average and rms (root-mean-square) currents are proportional to the current amplitude I. Similarly, the rms value of a sinusoidal voltage is proportional to the voltage amplitude V. (See Example 31.1.)

$$I_{rav} = \frac{2}{\pi} I = 0.637I \quad (31.3)$$

$$I_{rms} = \frac{I}{\sqrt{2}} \quad (31.4)$$

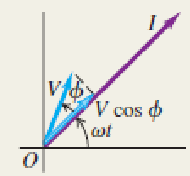
$$V_{rms} = \frac{V}{\sqrt{2}} \quad (31.5)$$



Voltage, current, and phase angle: In general, the instantaneous voltage between two points in an ac circuit is not in phase with the instantaneous current passing through those points. The quantity ϕ is called the phase angle of the voltage relative to the current.

$$i = I\cos\omega t \quad (31.2)$$

$$v = V\cos(\omega t + \phi)$$

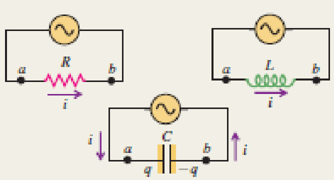


Resistance and reactance: The voltage across a resistor R is in phase with the current. The voltage across an inductor L leads the current by 90° ($\phi = +90°$), while the voltage across a capacitor C lags the current by 90° ($\phi = -90°$). The voltage amplitude across each type of device is proportional to the current amplitude I. An inductor has inductive reactance $X_L = \omega L$, and a capacitor has capacitive reactance $X_C = 1/\omega C$. (See Examples 31.2 and 31.3.)

$$V_R = IR \quad (31.7)$$

$$V_L = IX_L \quad (31.13)$$

$$V_C = IX_C \quad (31.19)$$

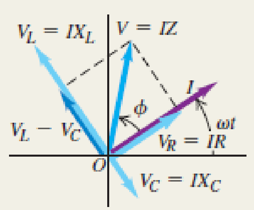


Impedance and the *L-R-C* series circuit: In a general ac circuit, the voltage and current amplitudes are related by the circuit impedance Z. In an L-R-C series circuit, the values of L, R, C, and the angular frequency ω determine the impedance and the phase angle ϕ of the voltage relative to the current. (See Examples 31.4 and 31.5.)

$$V = IZ \quad (31.22)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + [\omega L - (1/\omega C)]^2} \quad (31.23)$$

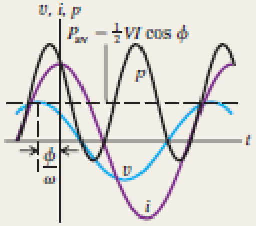
$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \quad (31.24)$$



Lecture 13/14 Chapter 31 - LCR and AC Circuits, Oscillations

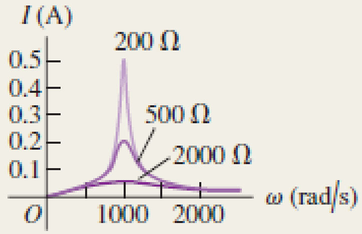
Power in ac circuits: The average power input P_{av} to an ac circuit depends on the voltage and current amplitudes (or, equivalently, their rms values) and the phase angle ϕ of the voltage relative to the current. The quantity $\cos\phi$ is called the power factor. (See Examples 31.6 and 31.7.)

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \qquad (31.31)$$



Resonance in ac circuits: In an *L-R-C* series circuit, the current becomes maximum and the impedance becomes minimum at an angular frequency called the resonance angular frequency. This phenomenon is called resonance. At resonance the voltage and current are in phase, and the impedance Z is equal to the resistance R. (See Example 31.8.)

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (31.32)$$



Transformers: A transformer is used to transform the voltage and current levels in an ac circuit. In an ideal transformer with no energy losses, if the primary winding has N_1 turns and the secondary winding has N_2 turns, the amplitudes (or rms values) of the two voltages are related by Eq. (31.35). The amplitudes (or rms values) of the primary and secondary voltages and currents are related by Eq. (31.36). (See Example 31.9.)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (31.35)$$

$$V_1 I_1 = V_2 I_2 \qquad (31.36)$$