

Chapter 21

Electric Charge and Electric Field

PowerPoint® Lectures for
University Physics, Thirteenth Edition
– *Hugh D. Young and Roger A. Freedman*

Lectures by Wayne Anderson

Goals for Chapter 21

- To study electric charge and charge conservation
- To see how objects become charged
- To calculate the electric force between objects using Coulomb's law
- To learn the distinction between electric force and electric field
- To calculate the electric field due to many charges
- To visualize and interpret electric fields
- To calculate the properties of electric dipoles

Introduction to Physics 121

Electricity and Magnetism

- New this Term:
 - Text: Young & Friedman, University Physics
 - Homework & Tutorial System: Mastering Physics
- Syllabus, rules, assignments, exams, etc. iClickers
 - Course content overview

First Lecture is Intro

- Review of vector operations
 - Dot product, cross product
- Scalar and vector fields in math and physics
- Gravitation as an example of a vector field
- Gravitational flux, shell theorems, flow fields
 - Methods for calculating fields

Vector Definitions

- Experiments tell us which physical quantities are scalars and vectors
- E&M uses vectors for fields, vector products for magnetic field and force

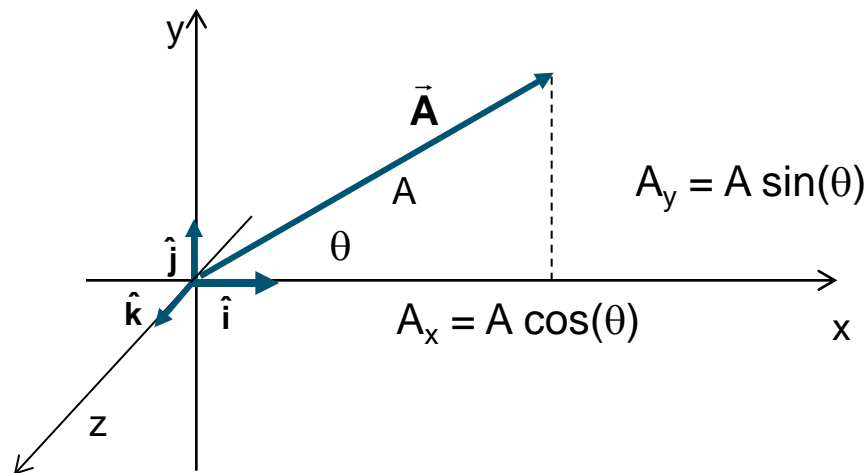
Representations in 2 Dimensions:

- Cartesian (x,y) coordinates

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$$

- Magnitude & direction

$$A = \sqrt{A_x^2 + A_y^2} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



- Addition and subtraction of vectors:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} \quad \text{means} \quad C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

$$\vec{\mathbf{C}} = -\vec{\mathbf{A}} \quad \text{means} \quad C_x = -A_x \quad \text{and} \quad C_y = -A_y$$

- Notation for vectors $\mathbf{F} = m\mathbf{a}$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

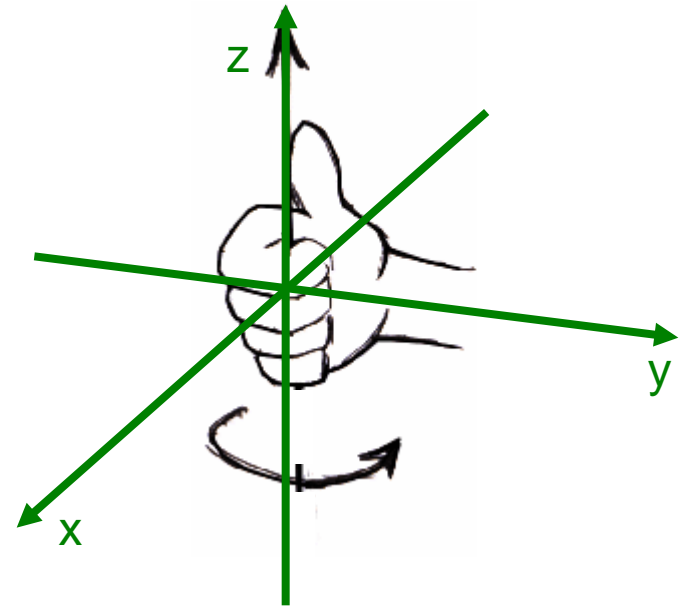
$$\underline{\mathbf{F}} = m\underline{\mathbf{a}}$$

Definition: Right-Handed Coordinate Systems

We **always** use right-handed coordinate systems.

In three-dimensions the right-hand rule determines which way the positive axes point.

Curl the fingers of your **RIGHT HAND** so they go from x to y . Your thumb will point in the positive z direction.



This course uses several right hand rules related to this one!

Vectors in 3 dimensions

Unit vector (Cartesian) notation:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Spherical polar coordinate representation:

1 magnitude and 2 directions

$$\vec{a} \equiv (a, \theta, \phi)$$

Conversion into x, y, z components

$$a_x = a \sin \theta \cos \phi$$

$$a_y = a \sin \theta \sin \phi$$

$$a_z = a \cos \theta$$

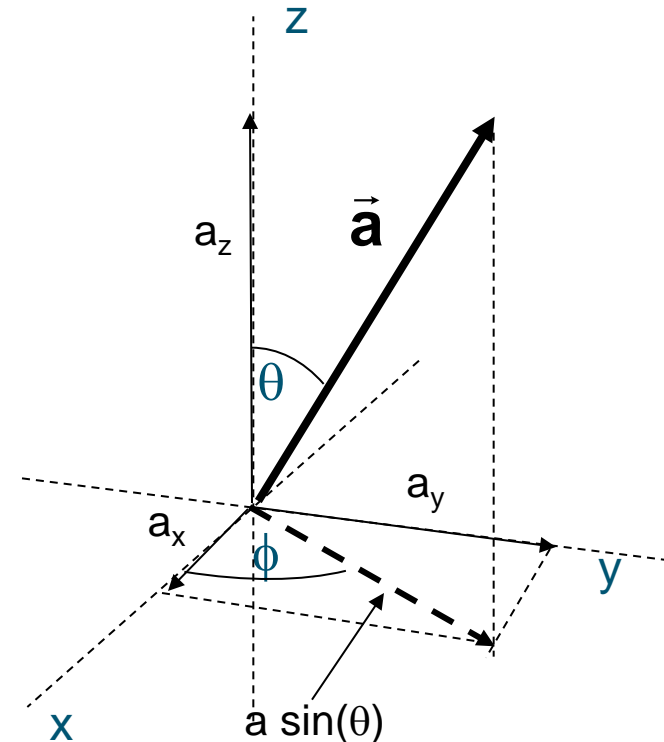
Conversion from x, y, z components

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\theta = \cos^{-1} a_z / a$$

$$\phi = \tan^{-1} a_y / a_x$$

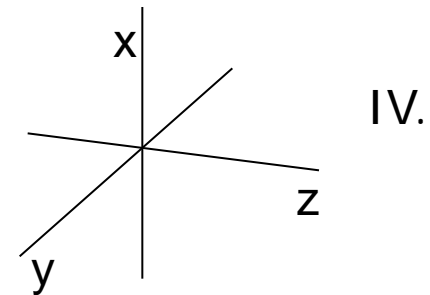
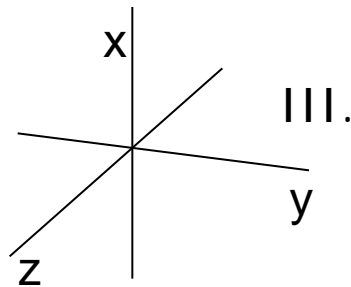
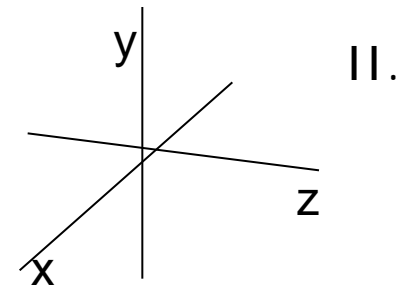
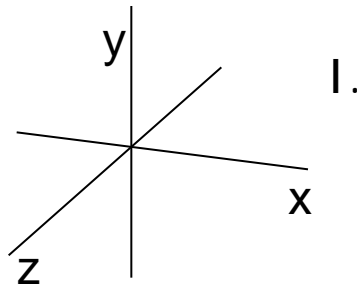
Rene Descartes
1596 - 1650



Right Handed Coordinate Systems

1-1: Which of these coordinate systems are right-handed?

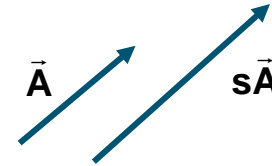
- A. I and II.
- B. II and III.
- C. I, II, and III.
- D. I and IV.
- E. IV only.



There are 3 Forms of Vector Multiplication

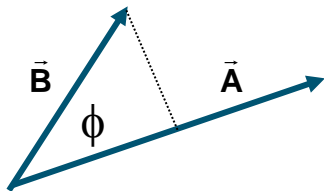
Multiplication of a vector by a scalar:

$$s\vec{A} = sA_x\hat{i} + sA_y\hat{j}$$



vector **times** scalar \rightarrow vector whose length is multiplied by the scalar

Dot product (or Scalar product or Inner product):



- vector **times** vector \rightarrow scalar
- projection of A on B or B on A
- commutative

$$\vec{A} \circ \vec{B} \equiv AB\cos(\varphi) = \vec{B} \circ \vec{A} = A_xB_x + A_yB_y + A_zB_z$$

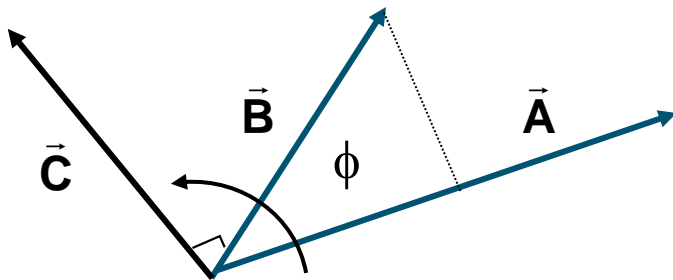
unit vectors measure
perpendicularity:

$$\begin{aligned} \hat{i} \circ \hat{j} &= 0, & \hat{j} \circ \hat{k} &= 0, & \hat{i} \circ \hat{k} &= 0 \\ \hat{i} \circ \hat{i} &= 1, & \hat{j} \circ \hat{j} &= 1, & \hat{k} \circ \hat{k} &= 1 \end{aligned}$$

Vector multiplication, continued

Cross product (or Vector product or Outer product):

- Vector **times** vector \rightarrow another vector perpendicular to the plane of \underline{A} and \underline{B}
- Draw \underline{A} & \underline{B} tail to tail: right hand rule shows direction of \underline{C}



$$\vec{C} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad (\text{not commutative})$$

$$\text{magnitude: } |\vec{C}| = AB \sin(\phi)$$

where ϕ is the smaller angle from \vec{A} to \vec{B}

- If \underline{A} and \underline{B} are parallel or the same, $\underline{A} \times \underline{B} = 0$
- If \underline{A} and \underline{B} are perpendicular, $\underline{A} \times \underline{B} = AB$ (max)

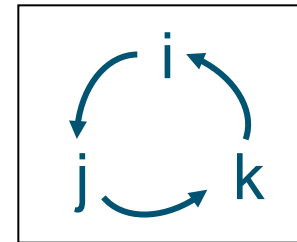
Algebra:

$$\begin{aligned} \text{distributive rule: } & \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \\ \text{associative rules: } & s\vec{A} \times \vec{B} = (s\vec{A}) \times \vec{B} = \vec{A} \times (s\vec{B}) \\ & (\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C}) \end{aligned}$$

$$\begin{aligned} \text{Unit vector representation: } & \hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{i} \times \hat{k} = -\hat{j} \\ & \hat{i} \times \hat{i} = 0, \quad \hat{j} \times \hat{j} = 0, \quad \hat{k} \times \hat{k} = 0 \end{aligned}$$

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$



Applications:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Example:

A force $F = -8\hat{i} + 6\hat{j}$ Newtons acts on a particle with position vector $r = 3\hat{i} + 4\hat{j}$ meters relative to the coordinate origin. What are a) the torque on the particle about the origin and b) the angle between the directions of r and F .

a) Use: $\vec{\tau} = \vec{r} \times \vec{F}$

$$\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} = (3\hat{i} + 4\hat{j}) \times (-8\hat{i} + 6\hat{j}) \\ &= -(3 \times 8)\hat{i} \times \hat{i} + (3 \times 6)\hat{i} \times \hat{j} - (4 \times 8)\hat{j} \times \hat{i} + (4 \times 6)\hat{j} \times \hat{j} \\ &= 18\hat{k} + 32\hat{k}\end{aligned}$$

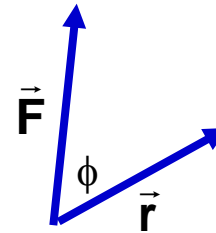
$$\therefore \hat{\tau} = 50\hat{k} \text{ N.m} \quad |\hat{\tau}| = 50 \quad \text{along z axis}$$

b) Can Use: $|\vec{\tau}| = r F \sin(\phi)$

$$r = [3^2 + 4^2]^{1/2} = 5 \quad F = [8^2 + 6^2]^{1/2} = 10$$

$$r F \sin(\phi) = 50 \sin(\phi) \quad \therefore \sin(\phi) = 1$$

$$\therefore \phi = 90^\circ \text{ that is } \vec{F} \perp \vec{r}$$



Better to Use: $\vec{r} \cdot \vec{F} = r F \cos(\phi) = 50 \cos(\phi)$

$$\begin{aligned}\vec{r} \cdot \vec{F} &= -(3 \times 8)\hat{i} \cdot \hat{i} + (3 \times 6)\hat{i} \cdot \hat{j} - (4 \times 8)\hat{j} \cdot \hat{i} + (4 \times 6)\hat{j} \cdot \hat{j} \\ &= -24 + 24 = 0\end{aligned}$$

$$\therefore 50 \cos(\phi) = 0 \text{ so } \phi = 90^\circ \text{ that is } \vec{F} \perp \vec{r}$$

Why?

Introduction

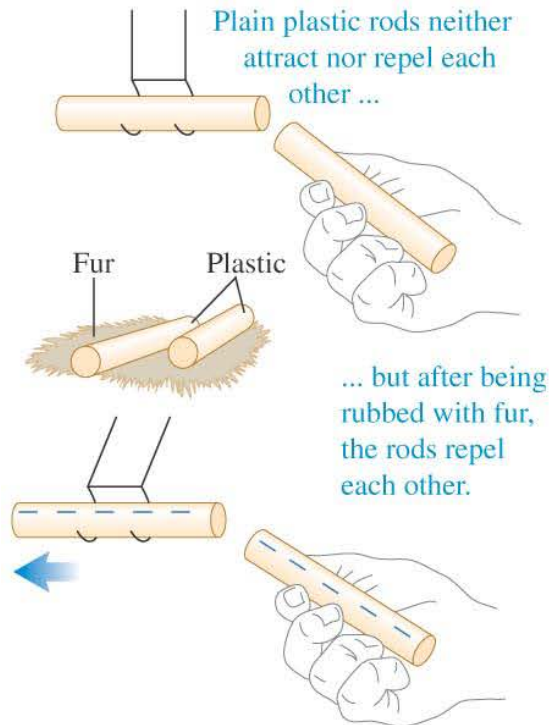
- Water makes life possible as a solvent for biological molecules. What electrical properties allow it to do this?
- We now begin our study of *electromagnetism*, one of the four fundamental forces.
- We start with electric charge and look at electric fields.



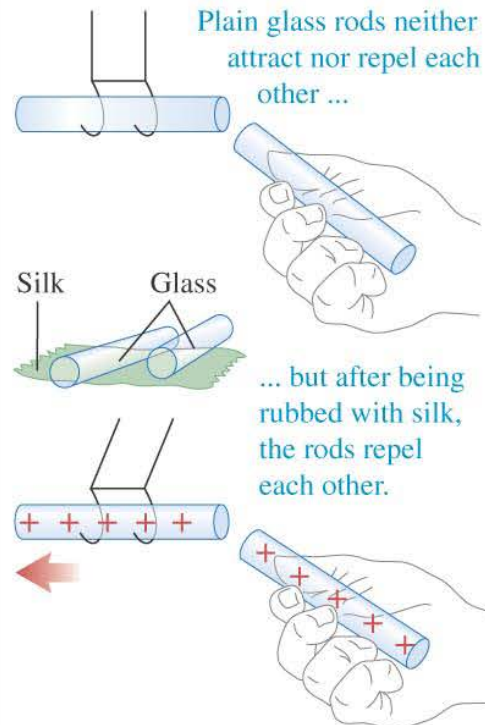
Electric charge

- Two positive or two negative charges repel each other. A positive charge and a negative charge attract each other.
- Figure 21.1 below shows some experiments in *electrostatics*.

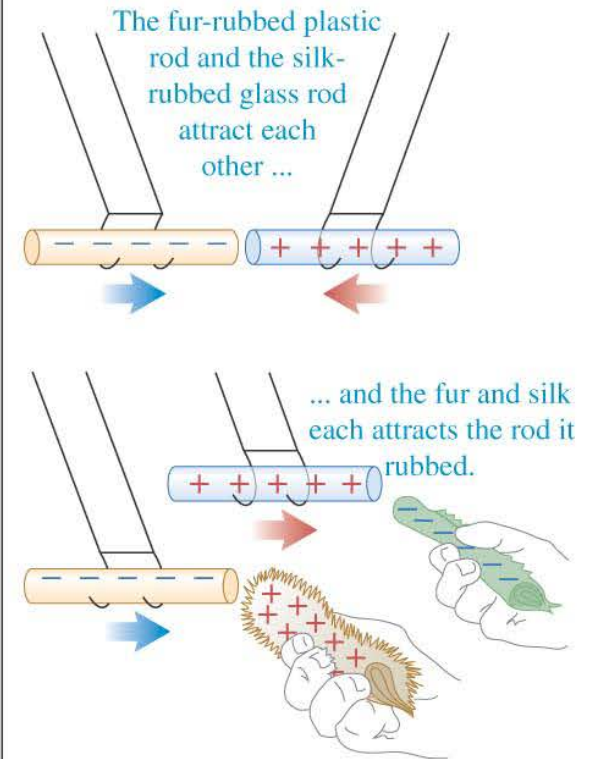
(a) Interaction between plastic rods rubbed on fur



(b) Interaction between glass rods rubbed on silk

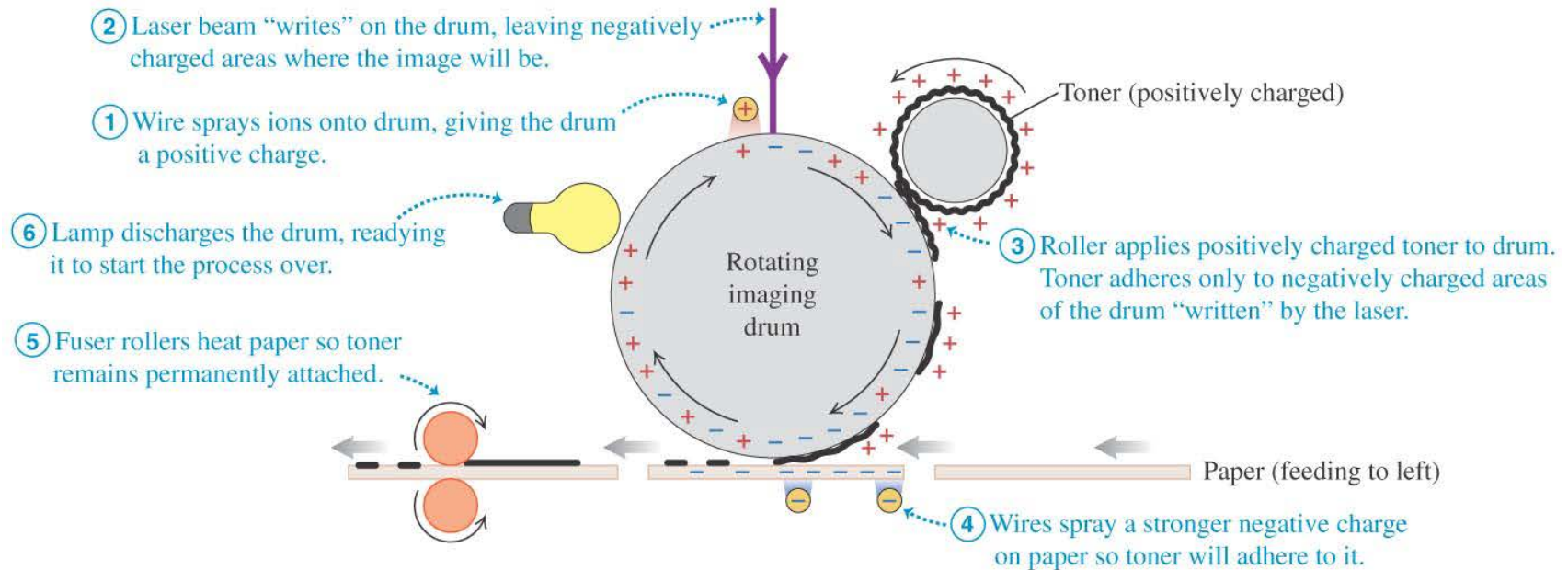


(c) Interaction between objects with opposite charges



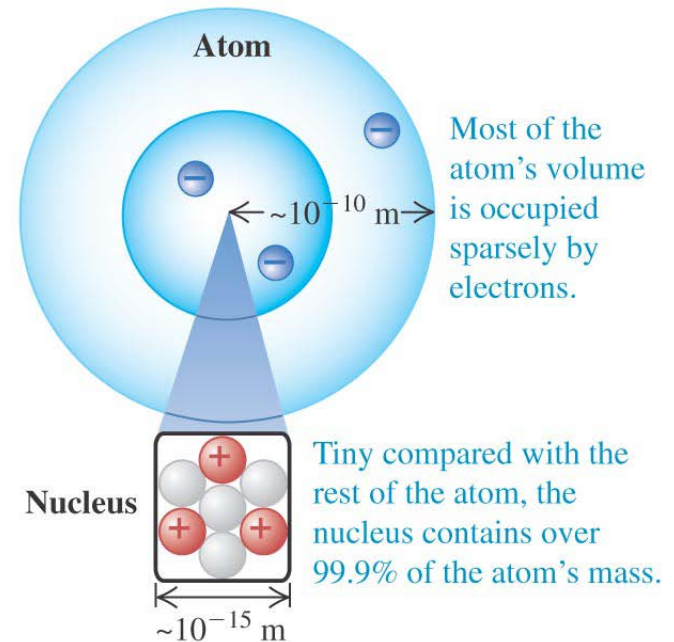
Laser printer

- A laser printer makes use of forces between charged bodies.



Electric charge and the structure of matter

- The particles of the atom are the negative *electron*, the positive *proton*, and the uncharged *neutron*.
- Protons and neutrons make up the tiny dense nucleus which is surrounded by electrons (see Figure 21.3 at the right).
- The electric attraction between protons and electrons holds the atom together.



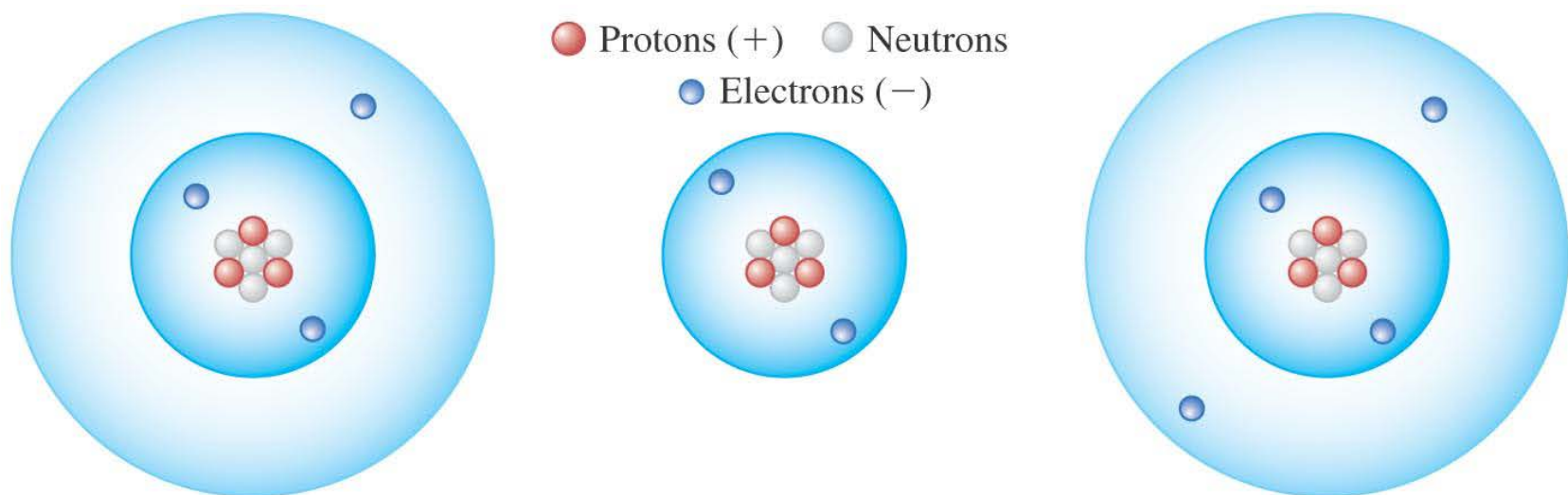
- **Proton:** Positive charge
Mass = 1.673×10^{-27} kg
- **Neutron:** No charge
Mass = 1.675×10^{-27} kg
- **Electron:** Negative charge
Mass = 9.109×10^{-31} kg

The charges of the electron and proton are equal in magnitude.

Name	Symbol	Value
Speed of light in vacuum	c	2.99792458×10^8 m/s
Magnitude of charge of electron	e	$1.602176487(40) \times 10^{-19}$ C
Gravitational constant	G	$6.67428(67) \times 10^{-11}$ N·m ² /kg ²
Planck's constant	h	$6.62606896(33) \times 10^{-34}$ J·s
Boltzmann constant	k	$1.3806504(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214179(30) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol·K
Mass of electron	m_e	$9.10938215(45) \times 10^{-31}$ kg
Mass of proton	m_p	$1.672621637(83) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.674927211(84) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A·m
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817 \dots \times 10^{-12}$ C ² /N·m ²
	$1/4\pi\epsilon_0$	$8.987551787 \dots \times 10^9$ N·m ² /C ²

Atoms and ions

- A neutral atom has the same number of protons as electrons.
- A *positive ion* is an atom with one or more electrons removed. A *negative ion* has gained one or more electrons.



(a) Neutral lithium atom (Li):

3 protons (3+)

4 neutrons

3 electrons (3-)

Electrons equal protons:

Zero net charge

(b) Positive lithium ion (Li^+):

3 protons (3+)

4 neutrons

2 electrons (2-)

Fewer electrons than protons:

Positive net charge

(c) Negative lithium ion (Li^-):

3 protons (3+)

4 neutrons

4 electrons (4-)

More electrons than protons:

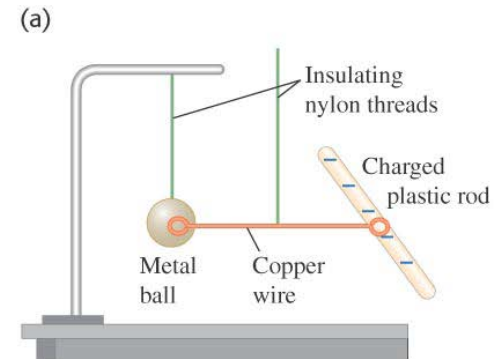
Negative net charge

Conservation of charge

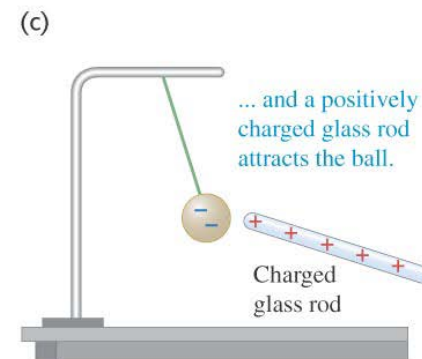
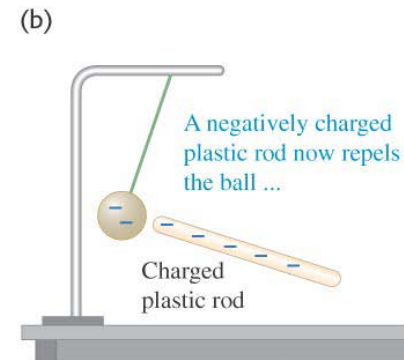
- The proton and electron have the same magnitude charge.
- The magnitude of charge of the electron or proton is a natural unit of charge. All observable charge is *quantized* in this unit.
- The universal *principle of charge conservation* states that the algebraic sum of all the electric charges in any closed system is constant.

Conductors and insulators

- A *conductor* permits the easy movement of charge through it. An *insulator* does not.
- Most metals are good conductors, while most nonmetals are insulators. (See Figure 21.6 at the right.)
- *Semiconductors* are intermediate in their properties between good conductors and good insulators.

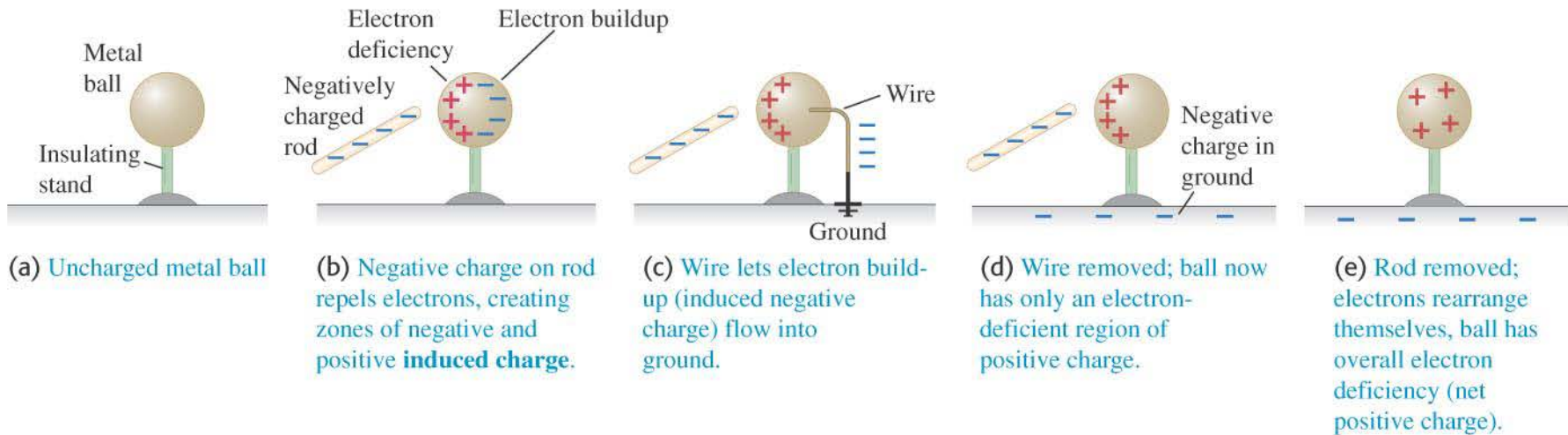


The wire conducts charge from the negatively charged plastic rod to the metal ball.



Charging by induction

- In Figure 21.7 below, the negative rod is able to charge the metal ball without losing any of its own charge. This process is called charging by *induction*.



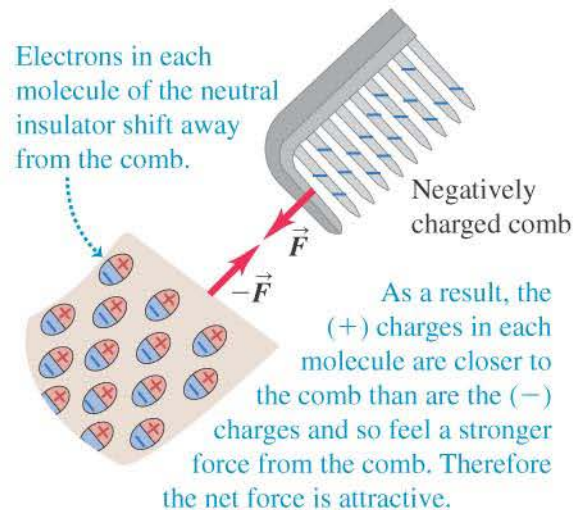
Electric forces on uncharged objects

- The charge within an insulator can shift slightly. As a result, two neutral objects can exert electric forces on each other, as shown in Figure 21.8 below.

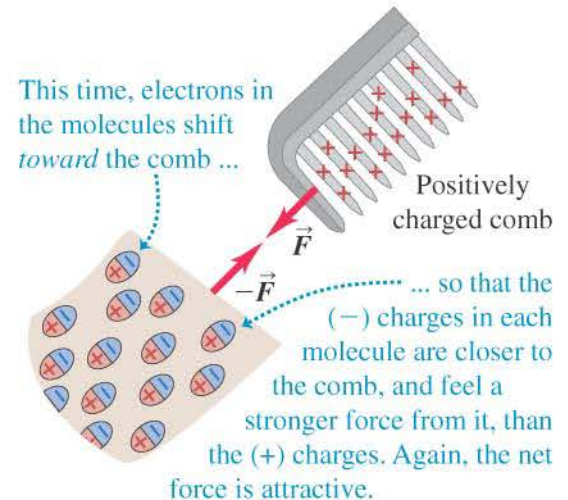
(a) A charged comb picking up uncharged pieces of plastic



(b) How a negatively charged comb attracts an insulator

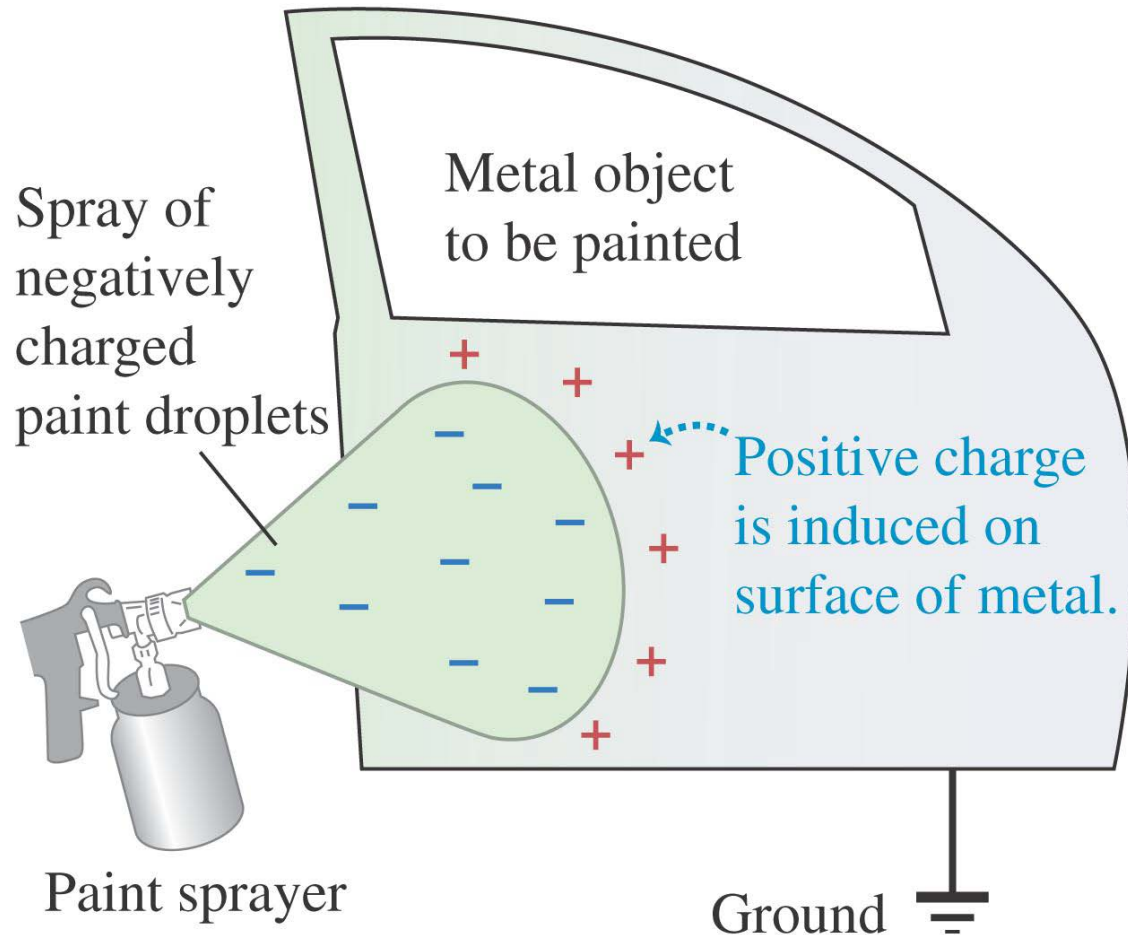


(c) How a positively charged comb attracts an insulator



Electrostatic painting

- Induced positive charge on the metal object attracts the negatively charged paint droplets.

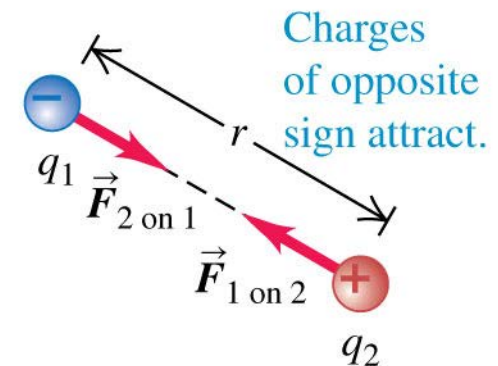
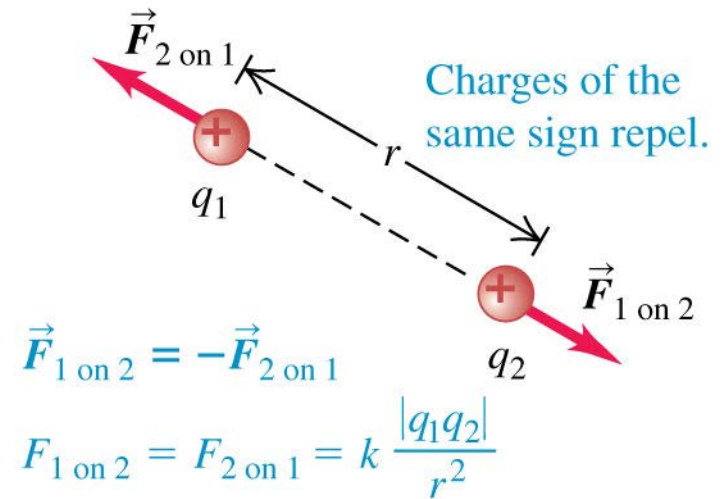


Coulomb's law

- *Coulomb's Law*: The magnitude of the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. (See the figure at the right.)

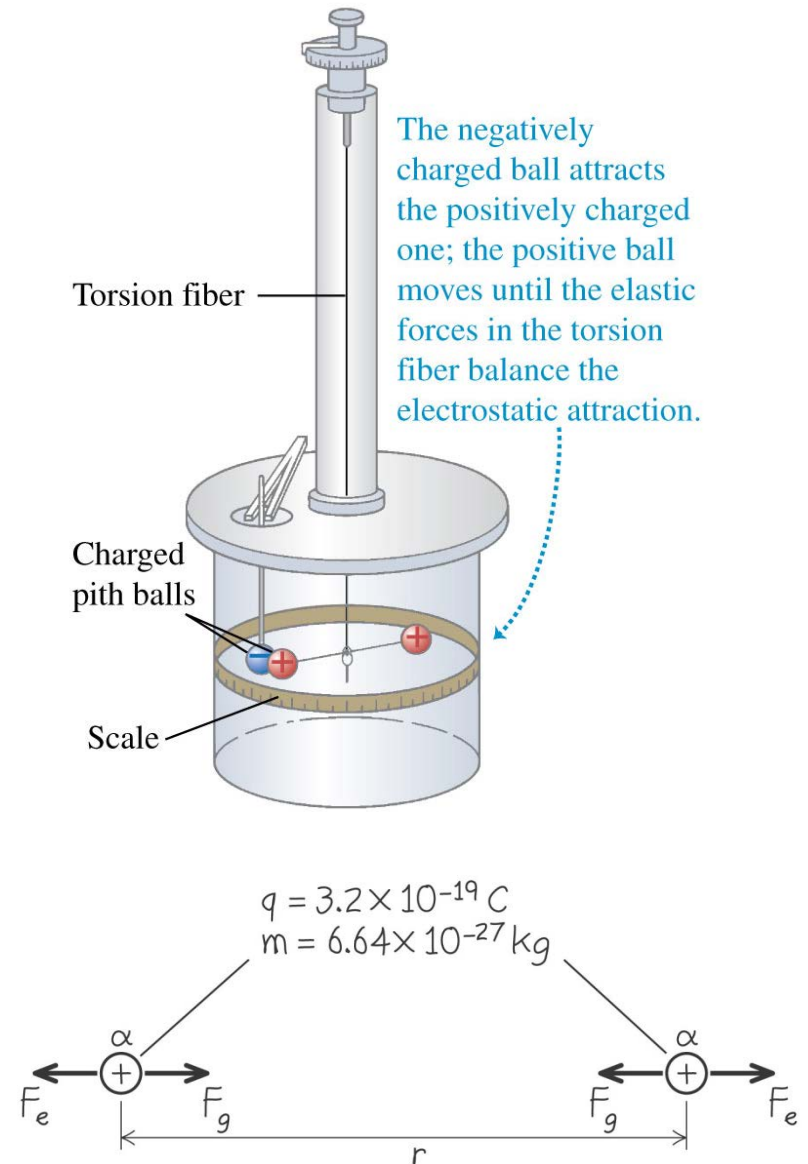
- Mathematically:

$$F = k/|q_1q_2|/r^2 = (1/4\pi\epsilon_0)/|q_1q_2|/r^2$$



Measuring the electric force between point charges

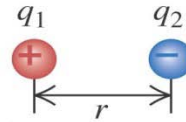
- The figure at the upper right illustrates how Coulomb used a torsion balance to measure the electric force between point charges.
- Example 21.1 compares the electric and gravitational forces. Follow it using Figure 21.11 at the lower right.



Force between charges along a line

- Read Problem-Solving Strategy 21.1.
- Follow Example 21.2 for two charges, using Figure 21.12 at the right.
- Follow Example 21.3 for three charges, using Figure 21.13 below.

(a) The two charges



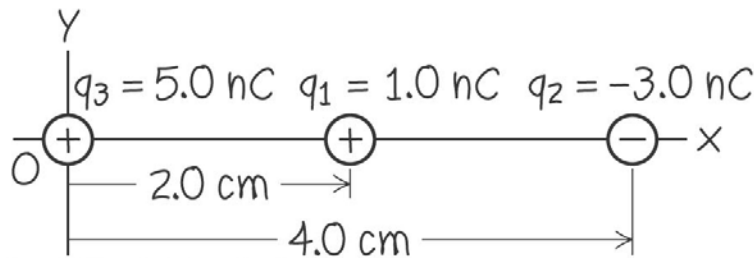
(b) Free-body diagram for charge q_2



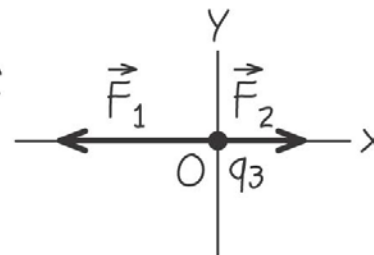
(c) Free-body diagram for charge q_1



(a) Our diagram of the situation

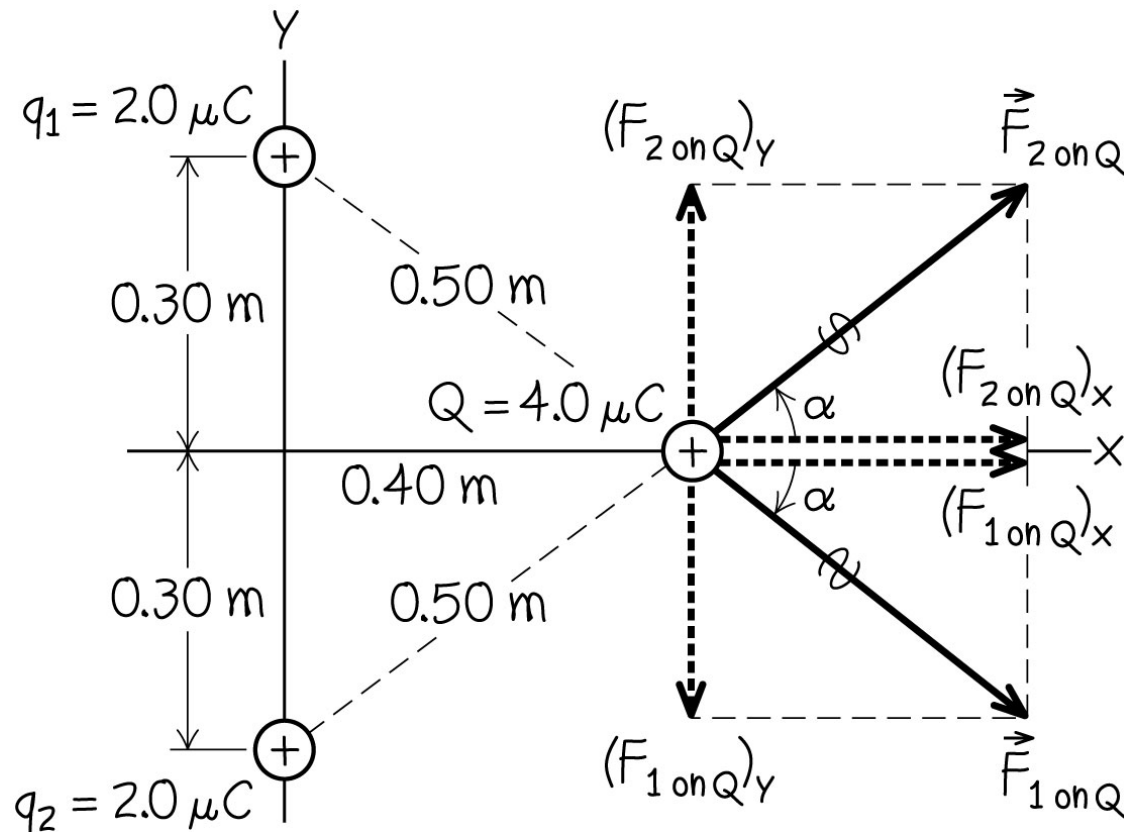


(b) Free-body diagram for q_3



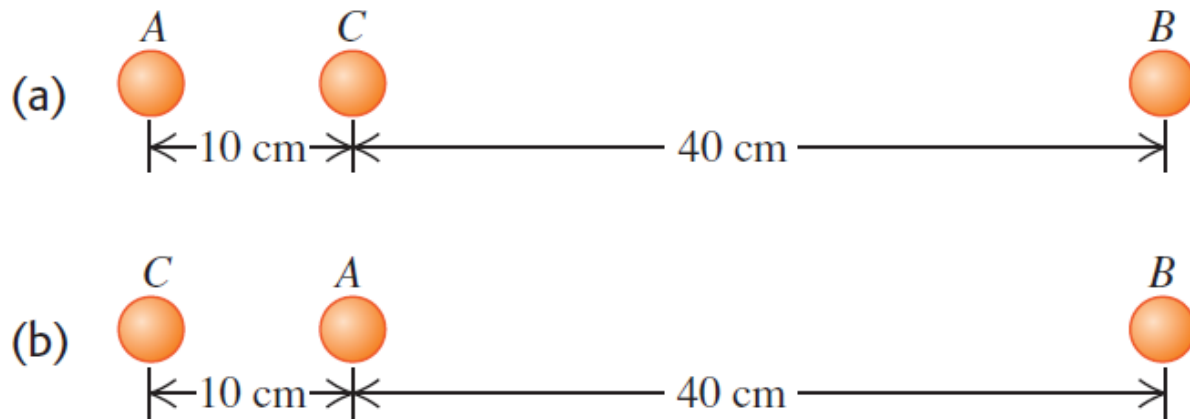
Vector addition of electric forces

- Example 21.4 shows that we must use vector addition when adding electric forces. Follow this example using Figure 21.14 below.



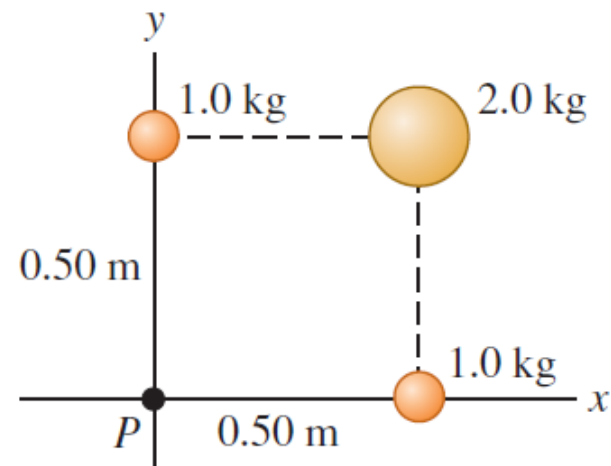
13.6 •• Find the magnitude and direction of the net gravitational force on mass A due to masses B and C in Fig. E13.6. Each mass is 2.00 kg.

Figure **E13.6**



13.43 • Three uniform spheres are fixed at the positions shown in Fig. P13.43. (a) What are the magnitude and direction of the force on a 0.0150-kg particle placed at P ? (b) If the spheres are in deep outer space and a 0.0150-kg particle is released from rest 300 m from the origin along a line 45° below the $-x$ -axis, what will the particle's speed be when it reaches the origin?

Figure **P13.43**



21.7 •• An average human weighs about 650 N. If two such generic humans each carried 1.0 coulomb of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their 650-N weight?

21.21 •• Two point charges are located on the y -axis as follows: charge $q_1 = -1.50$ nC at $y = -0.600$ m, and charge $q_2 = +3.20$ nC at the origin ($y = 0$). What is the total force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00$ nC located at $y = -0.400$ m?

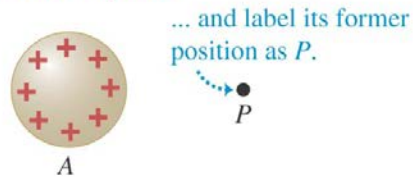
Electric field

- A charged body produces an *electric field* in the space around it (see Figure 21.15 at the lower left).
- We use a small *test charge* q_0 to find out if an electric field is present (see Figure 21.16 at the lower right).

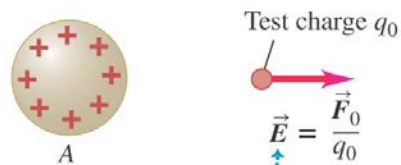
(a) A and B exert electric forces on each other.



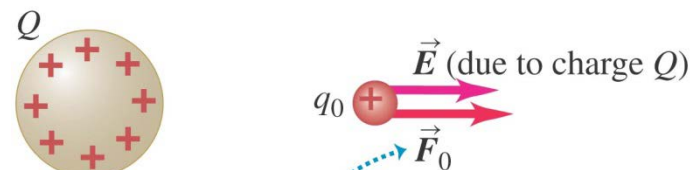
(b) Remove body B ...



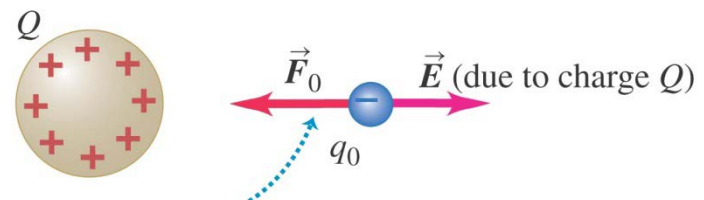
(c) Body A sets up an electric field \vec{E} at point P .



\vec{E} is the force per unit charge exerted by A on a test charge at P .



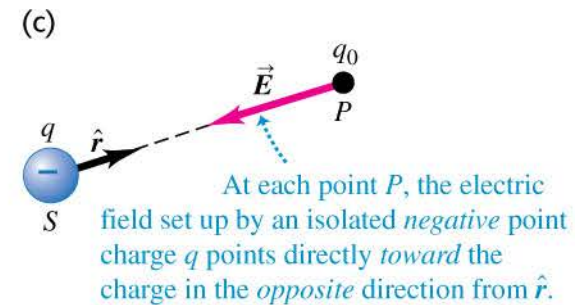
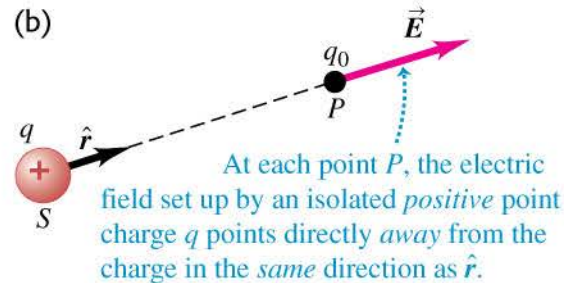
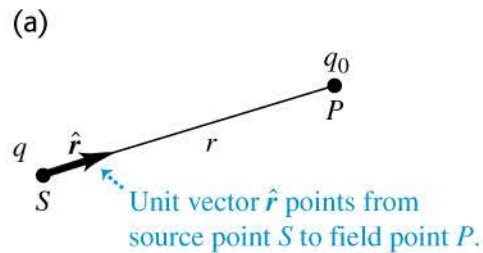
The force on a positive test charge q_0 points in the direction of the electric field.



The force on a negative test charge q_0 points opposite to the electric field.

Definition of the electric field

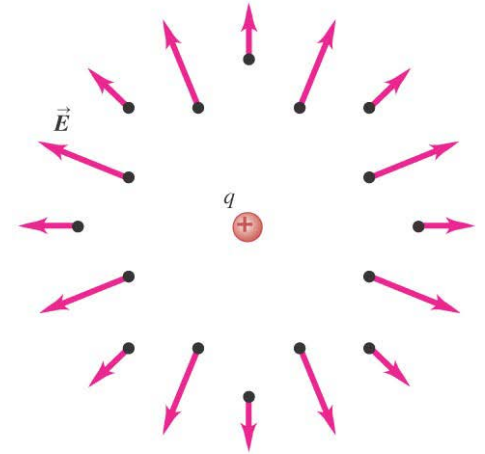
- Follow the definition in the text of the electric field using Figure 21.17 below.



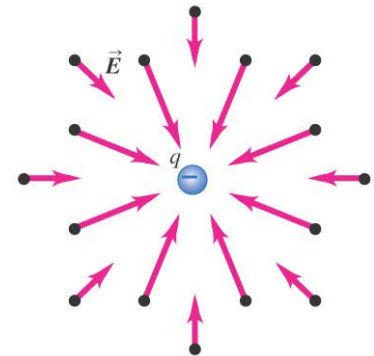
Electric field of a point charge

- Follow the discussion in the text of the electric field of a point charge, using Figure 21.18 at the right.
- Follow Example 21.5 to calculate the magnitude of the electric field of a single point charge.

(a) The field produced by a positive point charge points *away from* the charge.

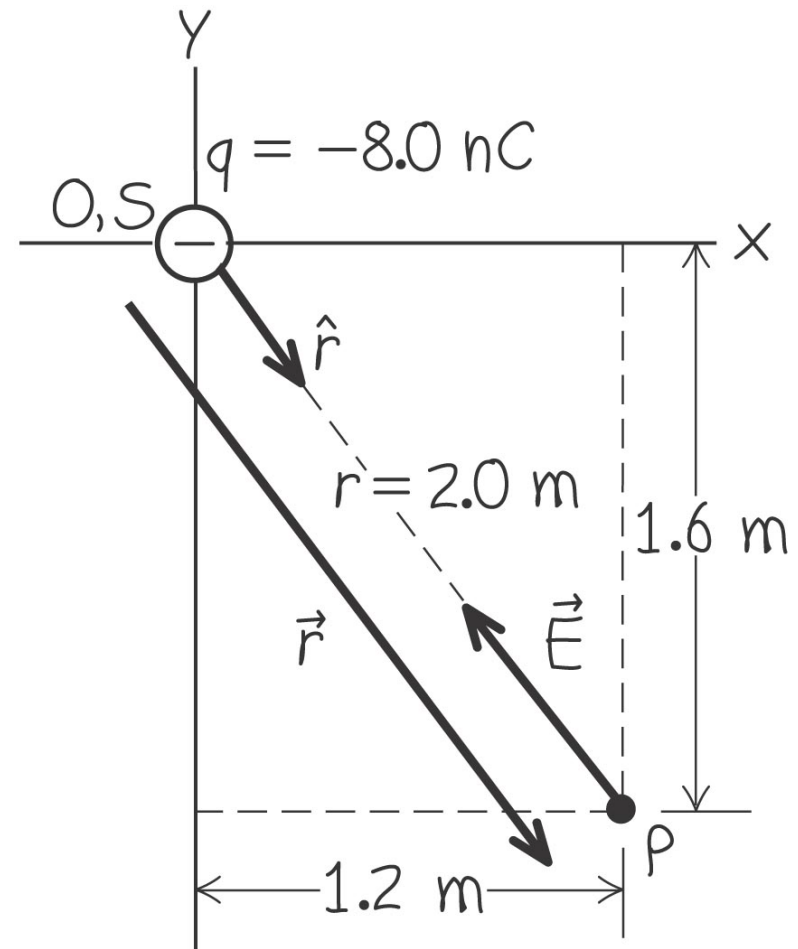


(b) The field produced by a negative point charge points *toward* the charge.



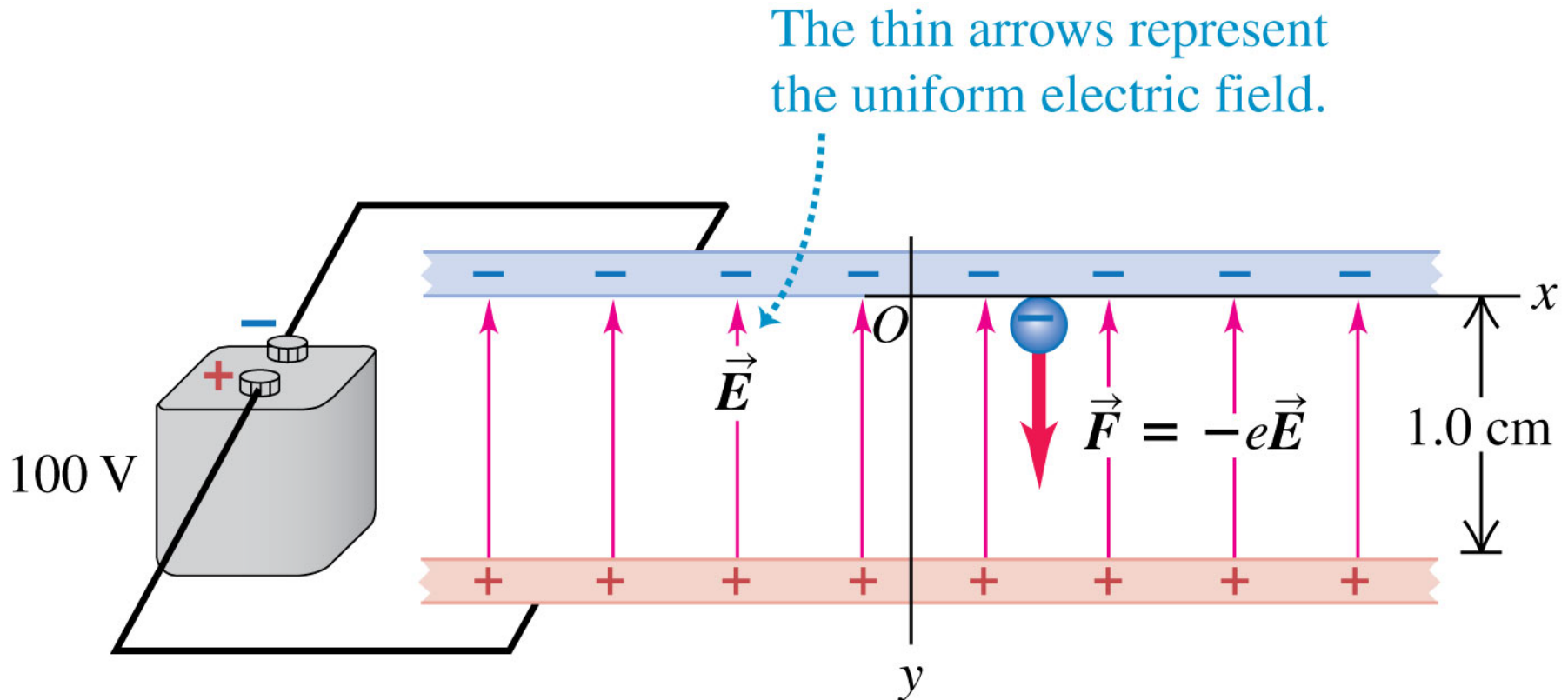
Electric-field vector of a point charge

- Follow Example 21.6 to see the vector nature of the electric field. Use Figure 21.19 at the right.



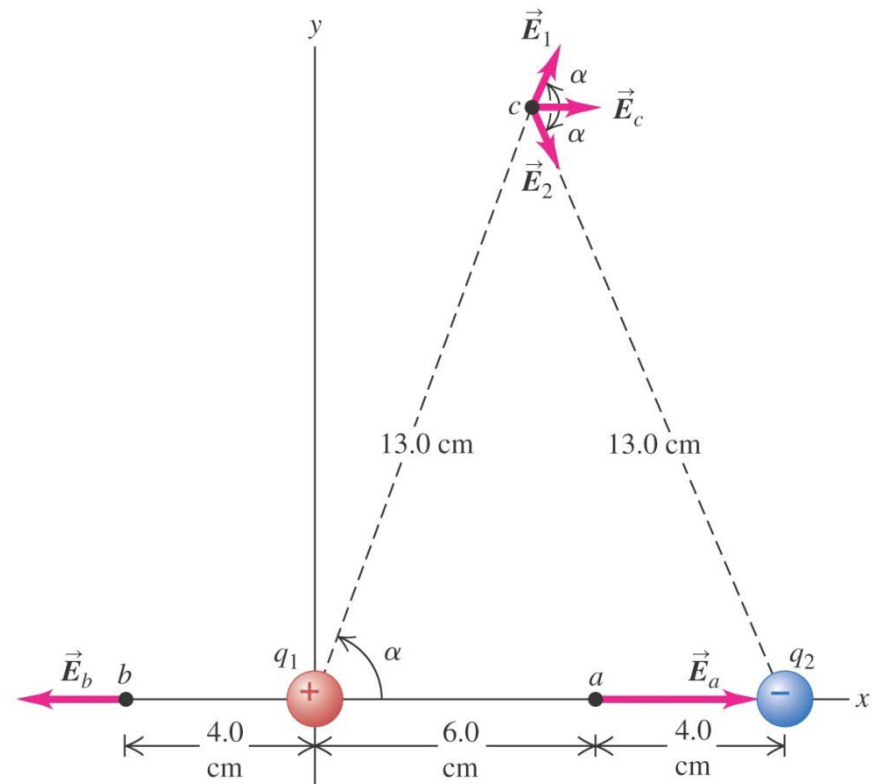
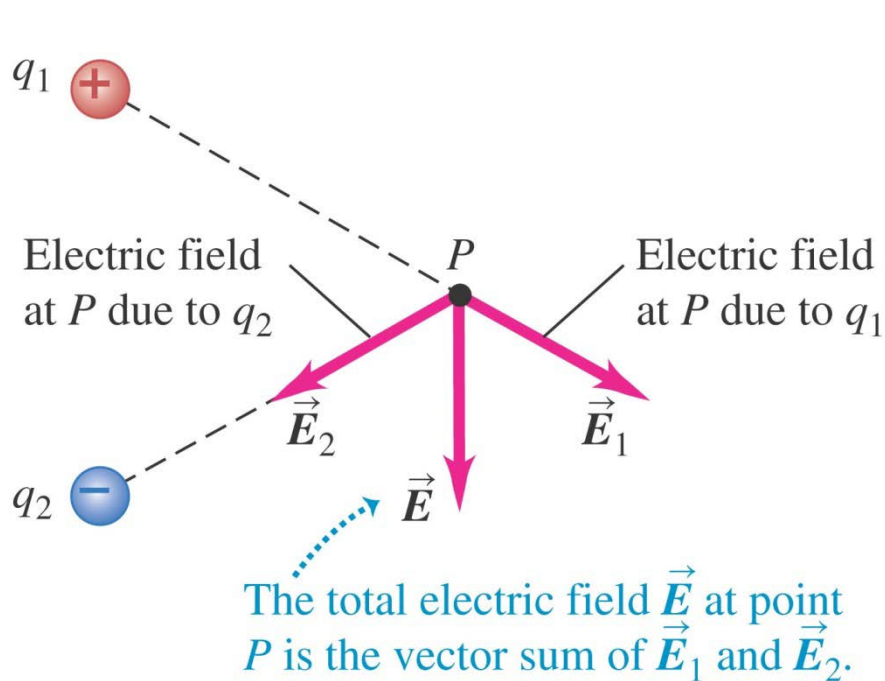
Electron in a uniform field

- Example 21.7 requires us to find the force on a charge that is in a known electric field. Follow this example using Figure 21.20 below.



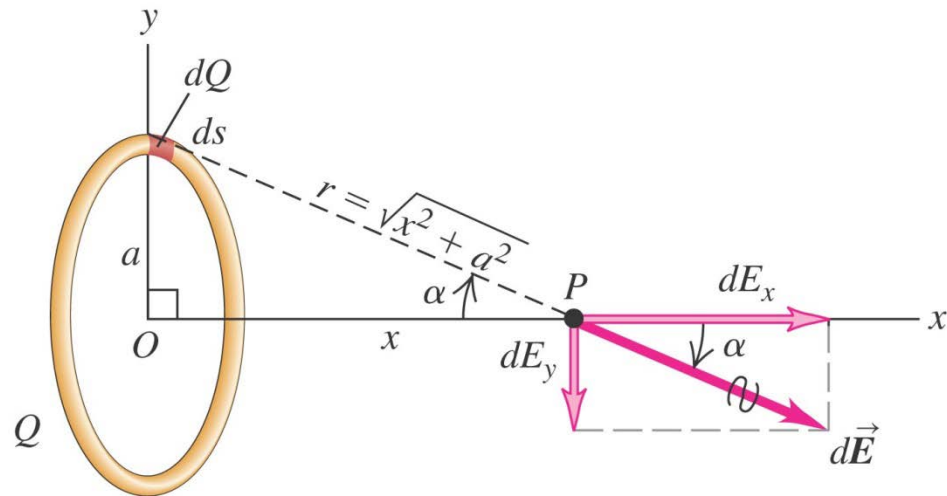
Superposition of electric fields

- The total electric field at a point is the vector sum of the fields due to all the charges present. (See Figure 21.21 below right.)
- Review Problem-Solving Strategy 21.2.
- Follow Example 21.8 for an electric dipole. Use Figure 21.22 below.



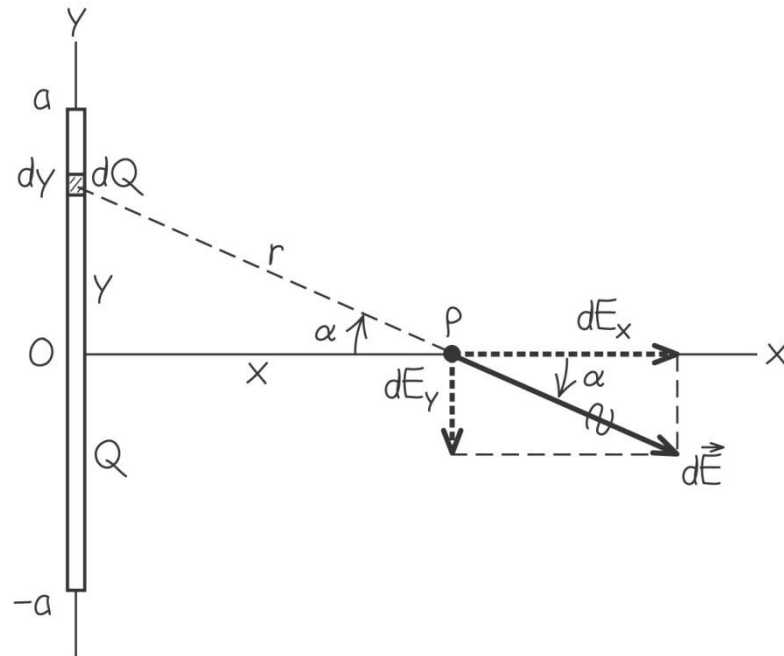
Field of a ring of charge

- Follow Example 21.9 using Figure 21.23 below.



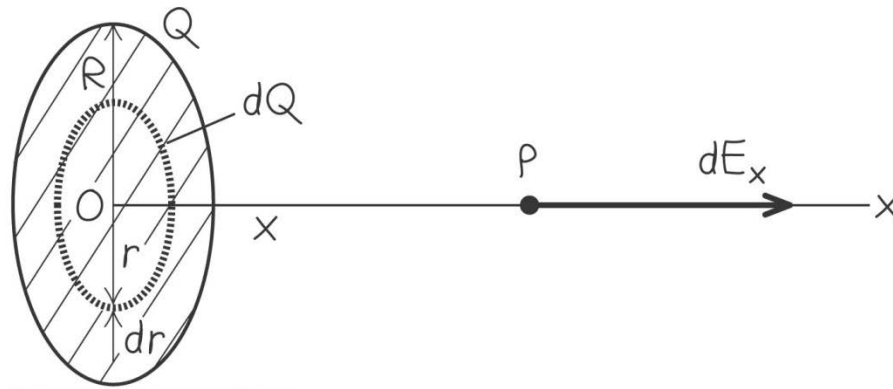
Field of a charged line segment

- Follow Example 21.10 and Figure 21.24 below.



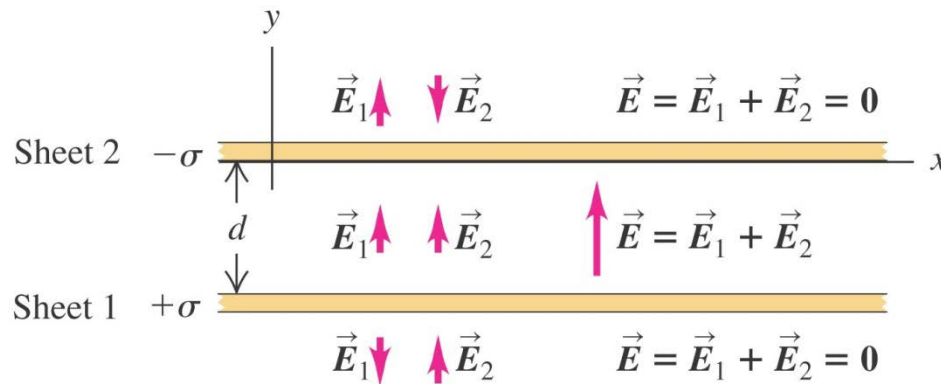
Field of a uniformly charged disk

- Follow Example 21.11 using Figure 21.25 below.



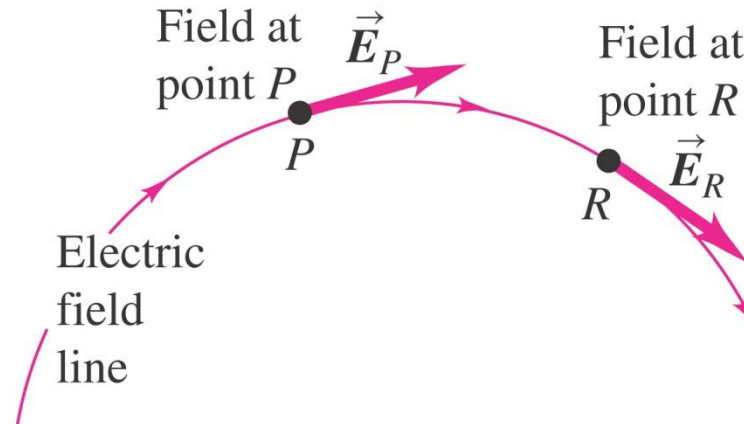
Field of two oppositely charged infinite sheets

- Follow Example 21.12 using Figure 21.26 below.



Electric field lines

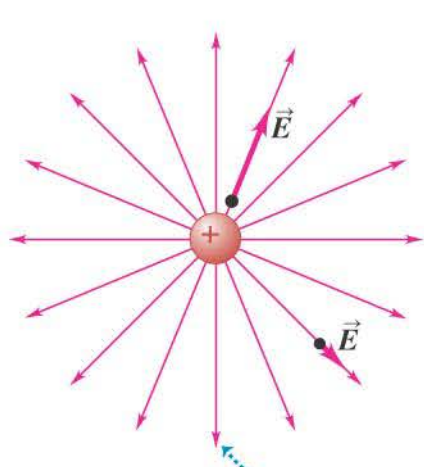
- An *electric field line* is an imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point. (See Figure 21.27 below.)



Electric field lines of point charges

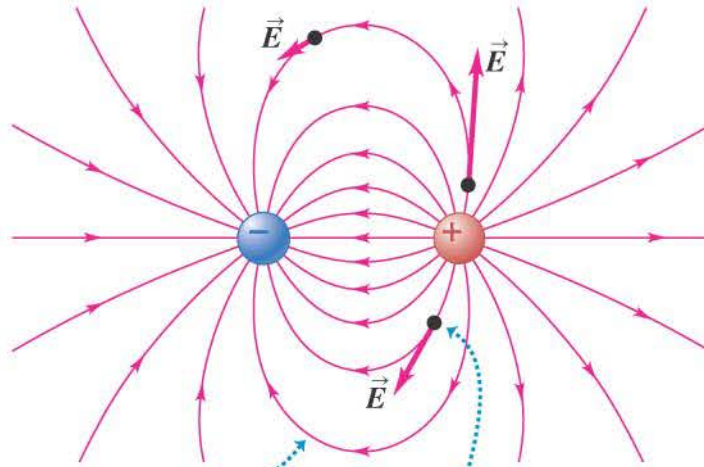
- Figure 21.28 below shows the electric field lines of a single point charge and for two charges of opposite sign and of equal sign.

(a) A single positive charge



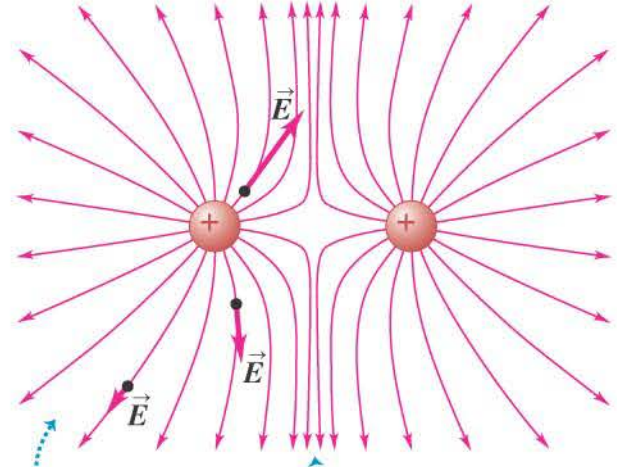
Field lines always point away from (+) charges and toward (-) charges.

(b) Two equal and opposite charges (a dipole)



At each point in space, the electric field vector is *tangent* to the field line passing through that point.

(c) Two equal positive charges

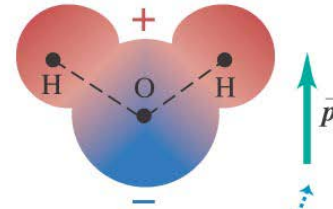


Field lines are close together where the field is strong, farther apart where it is weaker.

Electric dipoles

- An *electric dipole* is a pair of point charges having equal but opposite sign and separated by a distance.
- Figure 21.30 at the right illustrates the water molecule, which forms an electric dipole.

(a) A water molecule, showing positive charge as red and negative charge as blue



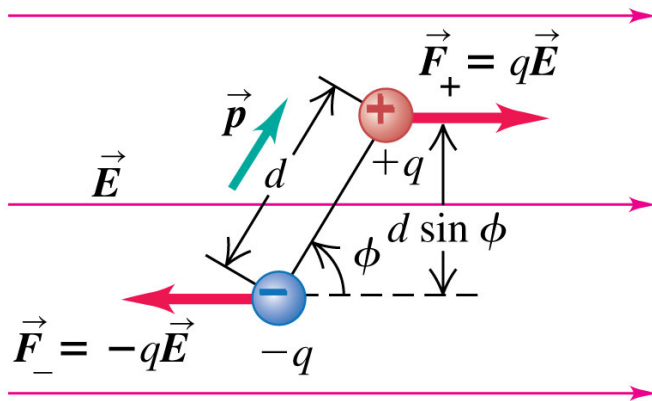
The electric dipole moment \vec{p} is directed from the negative end to the positive end of the molecule.

(b) Various substances dissolved in water

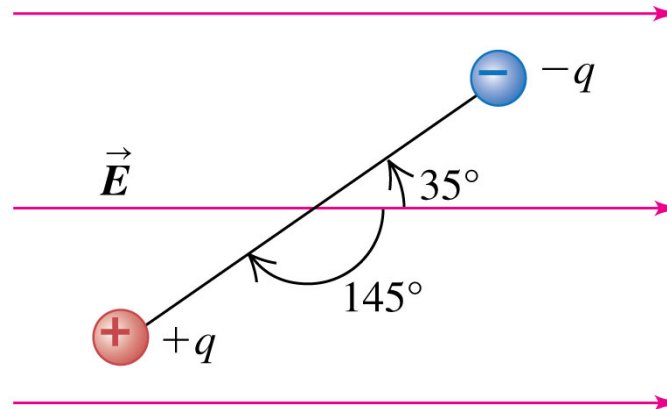


Force and torque on a dipole

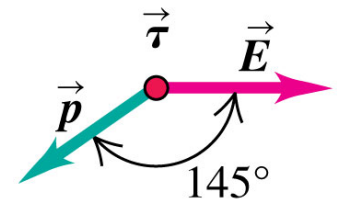
- Figure 21.31 below left shows the force on a dipole in an electric field.
- Follow the discussion of force, torque, and potential energy in the text.
- Follow Example 21.13 using Figure 21.32 below right.



(a)



(b)



Electric field of a dipole

- Follow Example 21.14 using Figure 21.33.

