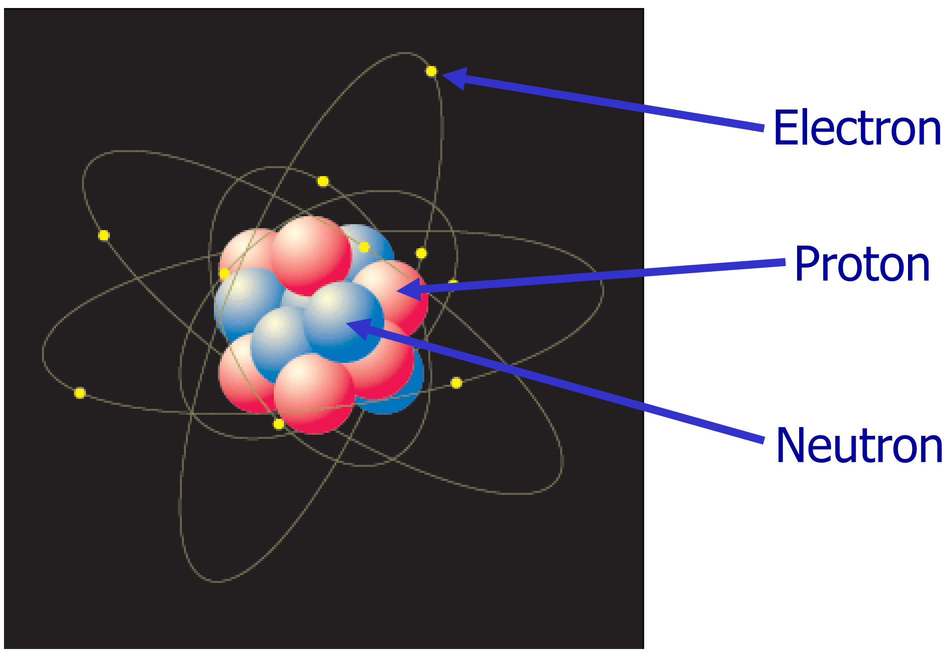
Name	Symbol	Value
Speed of light in vacuum	c	2.99792458×10^{8} m/s
Magnitude of charge of electron	e	$1.602176487(40) \times 10^{-19}$ C
Gravitational constant	G	$6.67428(67) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Planck's constant	h	$6.62606896(33) \times 10^{-34}\ \mathrm{J \cdot s}$
Boltzmann constant	k	$1.3806504(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214179(30) \times 10^{23}$ molecules/mol
Gas constant	R	$8.314472(15)\ \mathrm{J/mol \cdot K}$
Mass of electron	m_{e}	$9.10938215(45) \times 10^{-31}$ kg
Mass of proton	m_{p}	$1.672621637(83) \times 10^{-27}$ kg
Mass of neutron	m_{n}	$1.674927211(84) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{Wb/A \cdot m}$
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817\ldots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$
	$1/4\pi\epsilon_0$	$8.987551787\ldots \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$

The physical world we see is electrical

MODERN ATOMIC PHYSICS (1900 – 1932): Rutherford, Bohr

- Charge is *quantized*: $e = 1.6 \times 10^{-19}$ coulombs (small).
  - Every charge is an exact multiple of +/- e. Milliken ~1900.
- Protons (+e) are in a small nucleus, $r \sim 10^{-15}$ m , $m_p = 1.67 \times 10^{-27}$ kg,
  - Ze = nuclear charge, Z = atomic number. There are neutrons too.
- Electrons (-e) are in stable orbital clouds with radius ~ 10^{-10} m.,
  - mass $m_e = 9.1 \times 10^{-31}$ kg. (small). Atoms are mostly space.
- A neutral atom has equal # of electrons and protons.
- An ion is charged + or – and will attract opposite charges to become neutral



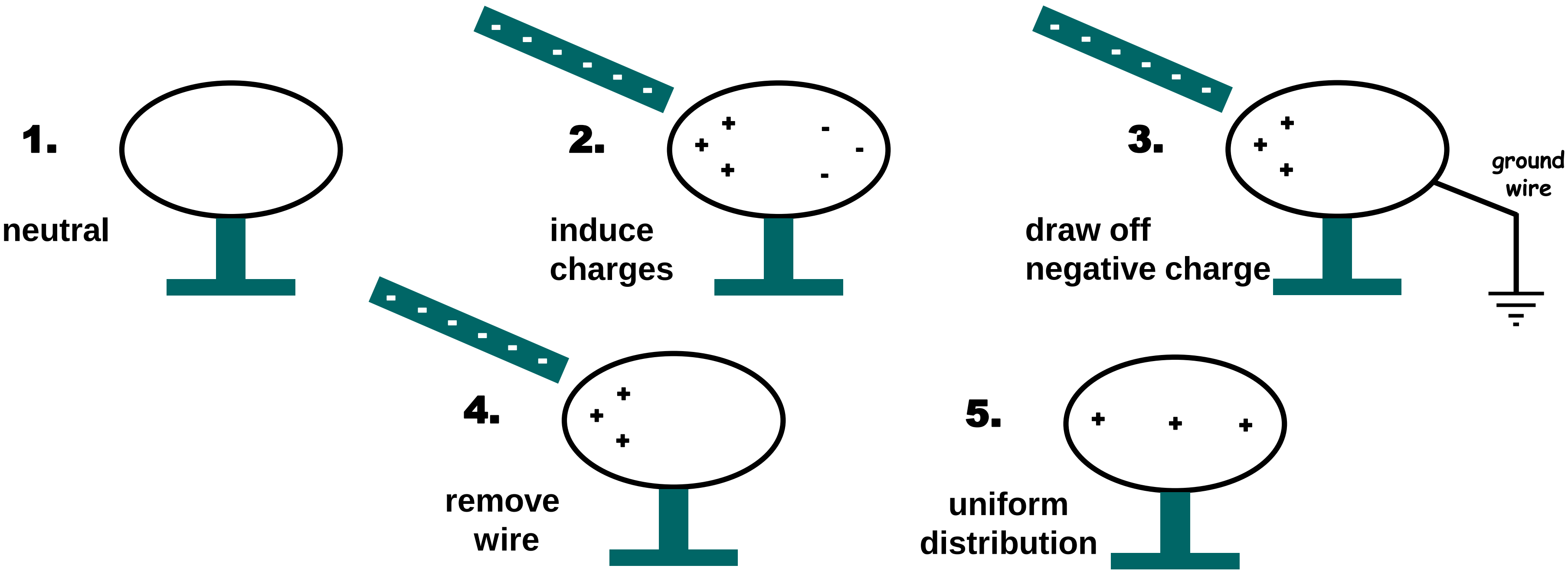
2-1: Which type of charge is easiest to pull out of an atom?

A. Proton

B. Electron

The protons all repel each other
Why don't nuclei all fly apart?

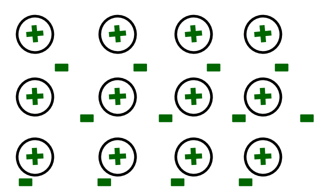
Charging a conducting sphere by induction



Atomic view of Conductors: charges free to move

SOLID CONDUCTORS

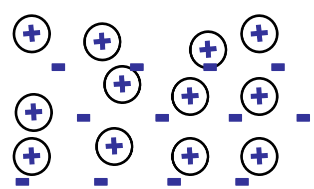
metals for example



- Regular lattice of fixed + ions
- conduction band free electrons wander when E field is applied

LIQUID & GASEOUS CONDUCTORS

sea water, humans, hot plasmas

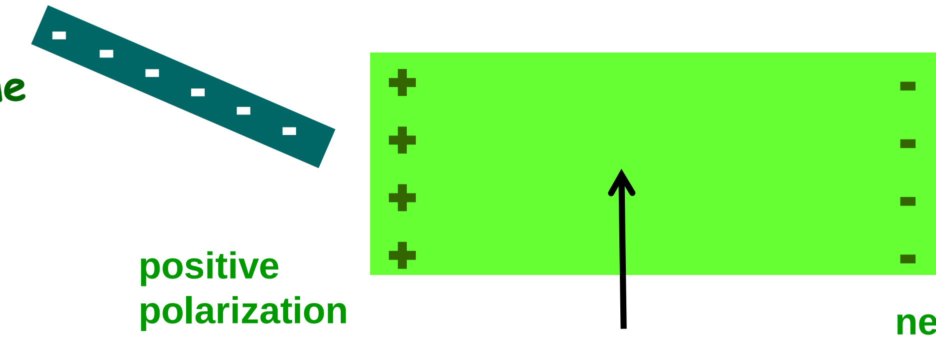


- Electrons and ions both free to move independently
- Normally random motion
- Electrons & ions move in opposite directions in an E field.

Insulators: Charges not free to move

Electrons are tightly bound to ions...
...BUT insulators can be induced to polarize by nearby charges

Polarization means charge separates but does not leave home

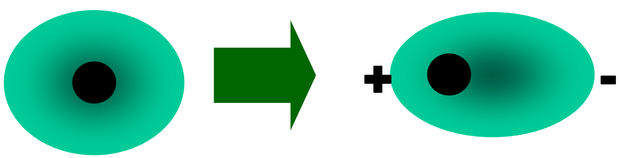


positive polarization charge

field inside material is smaller than it would be in vacuum

negative polarization charge

MOLECULES CAN HAVE PERMANENT POLARIZATION... ...OR MAY BE INDUCED TO ROTATE OR DISTORT WHEN CHARGE IS NEAR

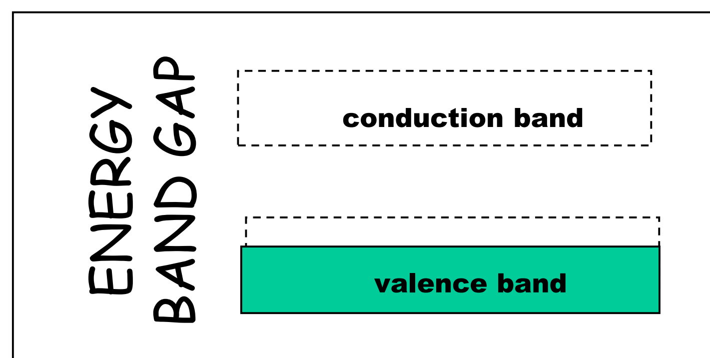


Insulators can be solids, liquids, or gases

THE *DIELECTRIC CONSTANT* MEASURES MATERIALS' ABILITY TO *POLARIZE*

Semiconductors:

- Normally insulators, but can be weak conductors when voltage or doping is applied
- Bands are formed by overlapping atomic energy levels

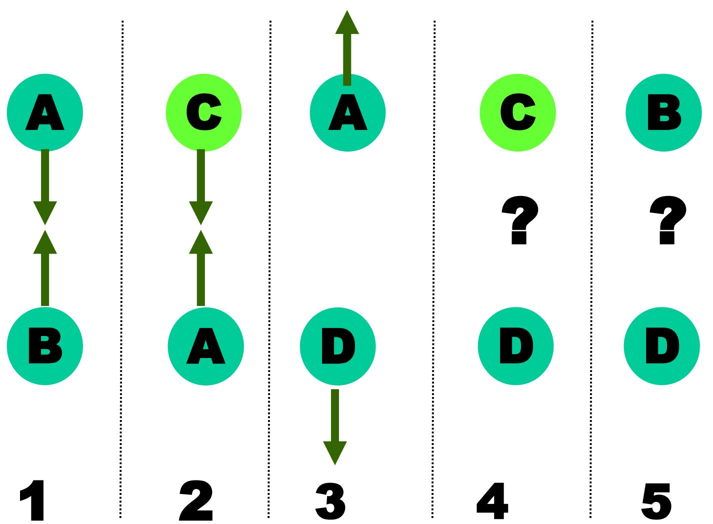


Insulators and Conductors

2-2: Which of the following are good conductors of electricity?

A. A plastic rod.
B. A glass rod.
C. A rock.
D. A wooden stick.
E. A metal rod.

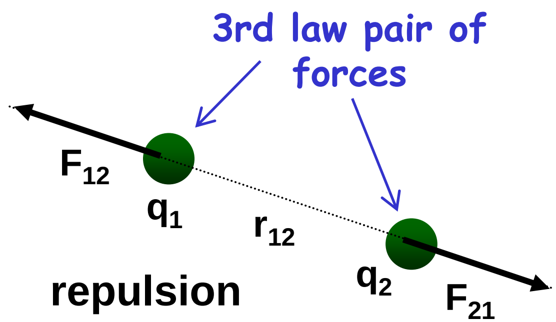
2-3: Balls A, B, and D are charged plastic. Ball C is made of metal and has zero net charge on it. The forces between pairs are as shown in sketches 1, 2, 3. In sketches 4 and 5 are the forces between balls attractive or repulsive?



Answer choices

A. 4 is attractive, 5 is repulsive
B. 4 is attractive, 5 is attractive
C. 4 is repulsive, 5 is repulsive
D. 4 is repulsive, 5 is attractive
E. not enough information

ELECTROSTATIC FORCE LAW (coulomb, 1785)



repulsion shown

Constant
$k=8.89\times10^9\ Nm^2/coul^2$

Force on q_1 due to q_2 (magnitude)

$$F_{12} = \frac{k\ q_1 q_2}{r_{12}^2}$$

- Force is between pairs of point charges
- An electric field transmits the force (no contact, action at a distance)
- Symmetric in q_1 & q_2 so $F_{12} = - F_{21}$
- Inverse square law
- Electrical forces are strong compared to gravitation
- $\varepsilon_0 = 8.85 \times 10^{-12}$ Why this value? Units

$$k \equiv \frac{1}{4\pi\varepsilon_0} \approx 9\text{x}10^9$$

Gravitation is weak

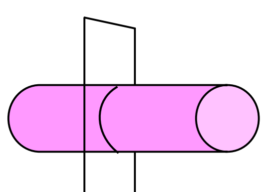
$$F_{12} = \frac{G\ m_1 m_2}{r_{12}^2}$$

$$G \approx 6.67\text{x}10^{-11} N.m^2/kg^2$$

Unit of charge = Coulomb

1 Coulomb = charge passing through a cross section of a wire carrying 1 Ampere of current in 1 second

1 Coulomb = $6.24\text{x}10^{18}$ electrons [= 1/e]



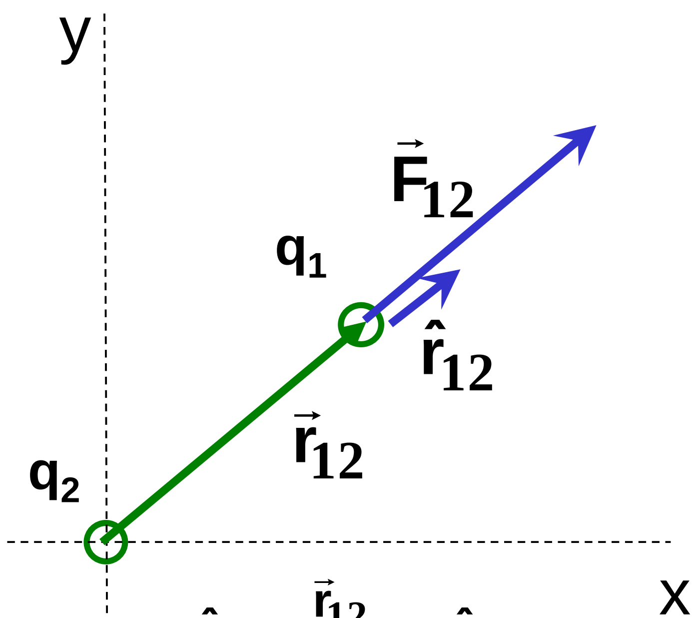
chargeflow $dq = i\,dt$ ...or... current $\equiv i = \frac{dq}{dt}$

Vector Form of Coulomb's Law

$\vec{F}_{12} \equiv$ Force **on** q_1 due to q_2

$\vec{r}_{12} \equiv$ Displacement **to** q_1 **from** q_2

$\hat{r}_{12} \equiv$ Unit vector pointing radially away from q_2 at location of q_1



$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}} = -\hat{r}_{21}$$

Sketch shows repulsion:

$\vec{F}_{12}$ is parallel to $\hat{r}_{12}$

For attraction (opposite charges):

$\vec{F}_{12}$ in opposite direction to $\hat{r}_{12}$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r_{12}|^3}\vec{r}_{12} \quad \text{or} \quad \vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r_{12}|^2}\hat{r}_{12}$$

cubed

squared

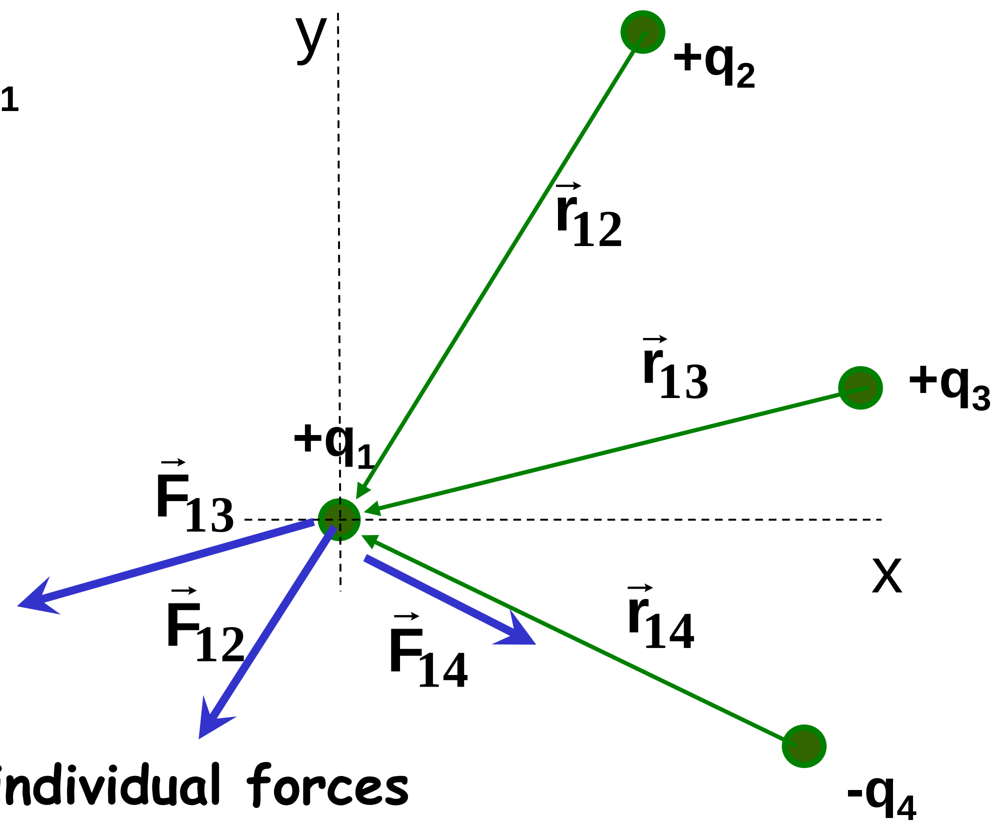
Superposition of forces & fields

The electrostatic force between *a specific pair* of point charges does not depend on interaction with *other* charges that may be nearby – there are no 3-body forces (same as gravitation)

Example: Find NET force on q_1

Method: find forces for all pairs of charges involving q_1, then add the forces vectorially

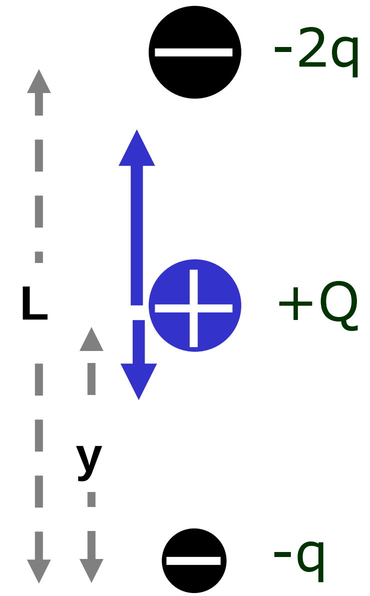
$$\vec{F}_{net\,on\,1} = \sum_{i=2}^{n} \vec{F}_{1,i} = \vec{F}_{1,2} + \vec{F}_{1,3} + \vec{F}_{1,4+.....}$$



- F_{11} is meaningless
- Use coulombs law to calculate individual forces
- Keep track of direction, usually using unit vectors
- Find the vector sum of individual forces at the point
- For continuous charge distributions, integrate instead of summing

Calculate the Exact Location for the forces to balance

- Force on test charge Q is attractive toward both negative charges, hence they could cancel.
- By symmetry, position for +Q in equilibrium is along y-axis
- Coordinate system: call the total distance L and call y the position of charge +Q from charge $-q$.
- Net force is sum of the two force vectors, and has to be zero, so



$$F = F_{from2q} + F_{fromq} = k\frac{2qQ}{(L-y)^2} - k\frac{qQ}{y^2} = 0$$

- k, q, and Q all cancel due to zero on the right, so our answer does not depend on knowing the + charge value. We end up with

$$\frac{2}{(L-y)^2} = \frac{1}{y^2}$$

$$\frac{(L-y)^2}{y^2} = 2 \quad \Rightarrow \quad \frac{L-y}{y} = +/-\sqrt{2}$$

- Solving for y, $y = \frac{L}{1+\sqrt{2}} = 0.412L$

y is less than half-way to the top negative charge

How do we know that electrostatic force - not gravitational force - holds atoms & molecules together?

Example: Find the ratio of those forces in Hydrogen

- 1 proton in nucleus, neutral Hydrogen atom has 1 electron
- Both have charge $e = 1.6 \times 10^{-19}$ C.
- m_e = electron mass = 9.1×10^{-31} kg.
- m_p = proton mass = 1833 m_e
- a_0 = radius of electron orbit = 10^{-8} cm = 10^{-10} m.

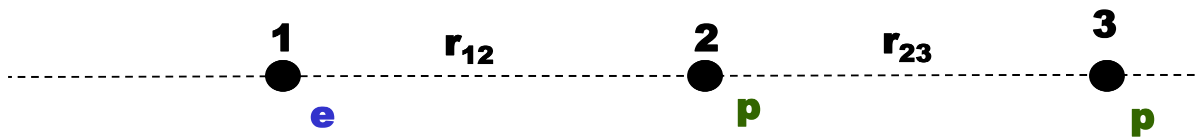
$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{a_0^2} \qquad F_{grav} = G\frac{m_e m_p}{a_0^2}$$

$$\frac{F_{elec}}{F_{grav}} = \frac{1}{4\pi\varepsilon_0 G}\frac{a_0^2}{a_0^2}\frac{e^2}{1833\, m_e^2} = \frac{9 \times 10^9}{6.67 \times 10^{-11}}\frac{(1.6 \times 10^{-19})^2}{1833\,(9.11 \times 10^{-31})^2}$$

$$\therefore \quad \frac{F_{elec}}{F_{grav}} = 3.1 \times 10^{39}$$

- Electrostatics dominates atomic structure by a factor of ~ 10^{39}
- So why is gravitation important at all? Matter is normally Neutral!
- Why don’t atomic nuclei with several protons break apart?

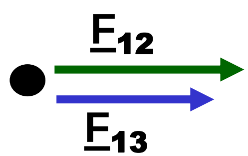
Example: Electron and two protons along a line (1 D)



By symmetry, all forces lie along the horizontal axis

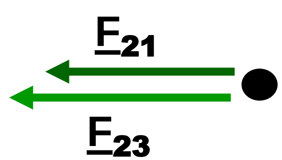
Show forces on each of the objects in free body diagrams for # 1, 2, & 3

FBD for 1



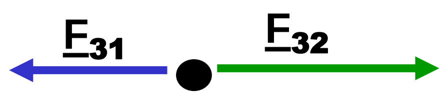
$$|\vec{F}_{12}| = 9\times 10^9 \frac{e^2}{r_{12}^2} \qquad |\vec{F}_{13}| = 9\times 10^9 \frac{e^2}{r_{13}^2}$$

FBD for 2



$$\vec{F}_{21} = -\vec{F}_{12} \qquad |\vec{F}_{23}| = 9\times 10^9 \frac{e^2}{r_{23}^2}$$

FBD for 3



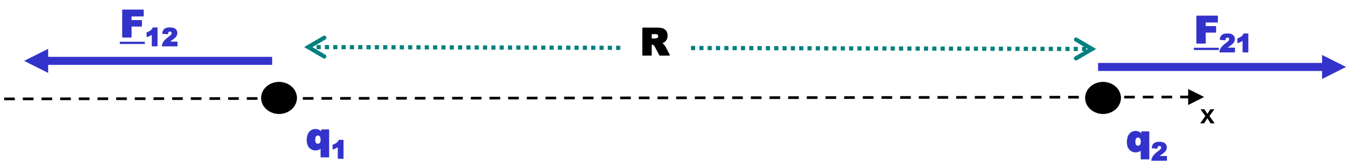
$$\vec{F}_{31} = -\vec{F}_{13} \qquad \vec{F}_{32} = -\vec{F}_{23}$$

- There are 3 action-reaction pairs of forces
- Which forces are attractive/repulsive?
- Could net force on any individual charge be zero above?
- What if $r_{12} = r_{23}$?

EXAMPLE: Vector-based solution…
… Start with two point charges on x-axis

Find $\underline{F}_{12}$ - the Force on q_1 due to q_2

$\underline{F}_{21}$ has the same magnitude, opposite direction



Let: $q_1 = +e = +1.6 \times 10^{-19}$ C. $q_2 = +2e = +3.2 \times 10^{-19}$ C.

$R = 0.02$ m

Solution:

$$\left|\vec{F}_{12}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1| \times |q_2|}{R^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 3.2 \times 10^{-19}}{0.02^2}$$

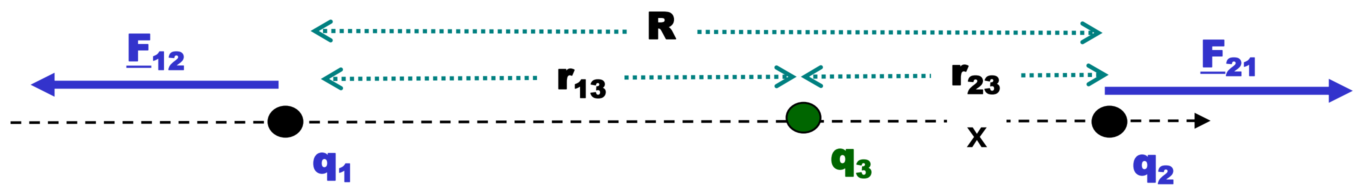
$$\boxed{\left|\vec{F}_{12}\right| = 1.15 \times 10^{-24}\ \text{N}}$$

small!

In vector notation:

$$\vec{F}_{12} = -\,1.15 \times 10^{-24}\ \hat{i}\ \text{N} \qquad \vec{F}_{21} = +\,1.15 \times 10^{-24}\ \hat{i}\ \text{N}$$

EXAMPLE continued: add a 3rd charge on x-axis

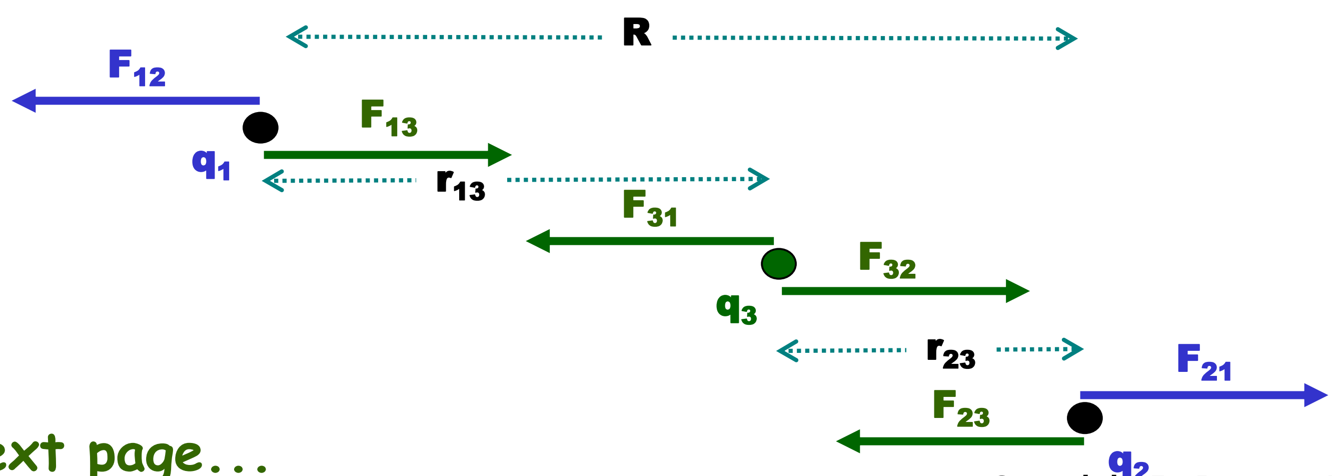


Let:

q_3 = - 2e = -3.2×10^{-19} C. $r_{13} = \frac{3}{4}R$, $r_{23} = \frac{1}{4}R$

What changes?

- Superposition implies: F_{12} & F_{21} not affected...but...
- Four new forces appear - with only two distinct new magnitudes
  F_{13} and F_{23} .... FBDs below.



...see next page...

EXAMPLE continued: calculate magnitudes

$$|F_{13}| = K\frac{|q_1| \times |q_3|}{r_{13}^{\ 2}} = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 3.2 \times 10^{-19}}{(0.75R)^2}$$

$$|F_{13}| = 2.05 \times 10^{-24}\ \text{N}$$

$$\vec{F}_{13} = +2.05 \times 10^{-24}\ \hat{i}\ \text{N} \qquad \vec{F}_{31} = -\vec{F}_{13} = -\ 2.05 \times 10^{-24}\ \hat{i}\ \text{N}$$

$$|F_{23}| = K\frac{|q_2| \times |q_3|}{r_{23}^{\ 2}} = 9 \times 10^9 \times \frac{3.2 \times 10^{-19} \times 3.2 \times 10^{-19}}{(0.25R)^2}$$

$$|F_{13}| = 3.69 \times 10^{-23}\ \text{N}$$

$$\vec{F}_{23} = -3.69 \times 10^{-23}\ \hat{i}\ \text{N} \qquad \vec{F}_{32} = -\vec{F}_{23} = +\ 3.69 \times 10^{-23}\ \hat{i}\ \text{N}$$

Find net forces on each of the three charges by applying superposition

On 1: $\vec{F}_{net,1} = \vec{F}_{12} + \vec{F}_{13} = -1.15 \times 10^{-24}\ \hat{i} + 2.05 \times 10^{-24}\ \hat{i}$ $\qquad \vec{F}_{net,1} = 9.0 \times 10^{-25}\ \hat{i}\ \text{N.}$

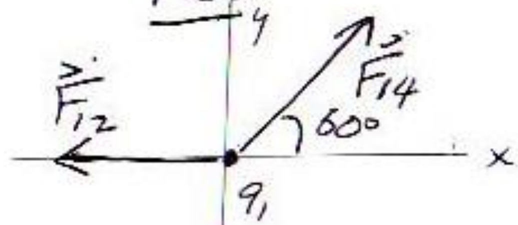
On 2: $\vec{F}_{net,2} = \vec{F}_{23} + \vec{F}_{21} = -3.69 \times 10^{-23}\ \hat{i} + 1.15 \times 10^{-24}\ \hat{i}$ $\qquad \vec{F}_{net,2} = -3.58 \times 10^{-23}\ \hat{i}\ \text{N.}$

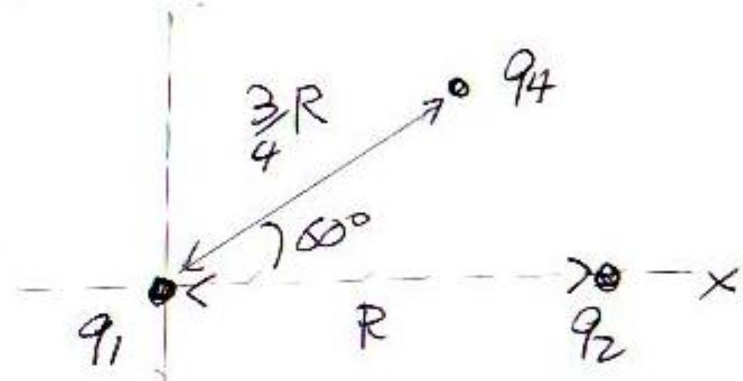
On 3: $\vec{F}_{net,3} = \vec{F}_{31} + \vec{F}_{32} = -2.05 \times 10^{-24}\ \hat{i} + 3.69 \times 10^{-23}\ \hat{i}$ $\qquad \vec{F}_{net,3} = -3.49 \times 10^{-23}\ \hat{i}\ \text{N.}$

Now move charge off-axis – Now 2 dimensional

Ex 22-1-C

Find $\vec{F}_{NET,1}$





$|F_{12}| = K\frac{q_1 q_2}{R^2} = 1.15 \times 10^{-24}\ N$

$\vec{F}_{12} = -1.15 \times 10^{-24}\ N.$ unchanged

$|\vec{F}_{14}| = 9\times10^9 \cdot \frac{q_1 q_4}{(\frac{3}{4}R)^2} = \frac{9\times10^9 \times 1.6\times10^{-19} (+) 3.2\times10^{-19}}{(.75)^2 (.02)^2}$

$|F_{14}| = 2.05 \times 10^{-24}$

$\vec{F}_{14} = 2.05\times10^{-24} \times \hat{r}$

$\hat{r} = \cos 60^\circ\, \hat{i} + \sin 60^\circ\, \hat{j}$

$\vec{F}_{net,1} = \vec{F}_{12} + \vec{F}_{14}$ (SUPERPOSITION)

$= \hat{i}\,(-1.15\times10^{-24} + 2.05\times10^{-24} \times \frac{1}{2})$

$+\hat{j}\,(2.05\times10^{-24} \times .866$

$\vec{F}_{NET,1} = 1.25\times10^{-25}\,\hat{i} + 1.78\times10^{-24}\,\hat{j}$

As before:

q_1 = +e = +1.6 x 10^{-19} C.

q_2 = +2e = +3.2 x 10^{-19} C.

R = 0.02 m

Now: q_3 gone

q_4 = -2e = -3.2 x 10^{-19} C.

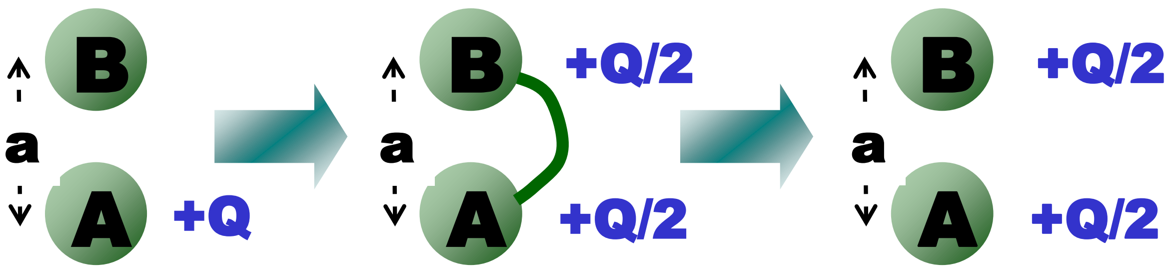
cos(60°) = .5

sin(60°) = .866

Example:

- Two **identical conducting** spheres. Radii small compared to a.
- Initially: Sphere A has charge +Q, sphere B is neutral
- Connect them by a wire. What happens and why?

Charge redistributes until ......???



Are the final charges equal if the spheres are not identical?

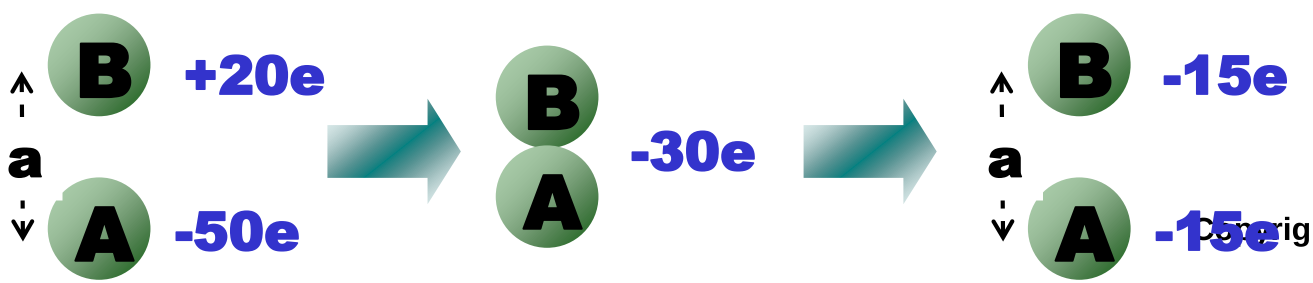
- Spheres repel each other. How big is the force?

We applied shell theorem outside

- **Do the spheres approximate point charges? What about polarization?**

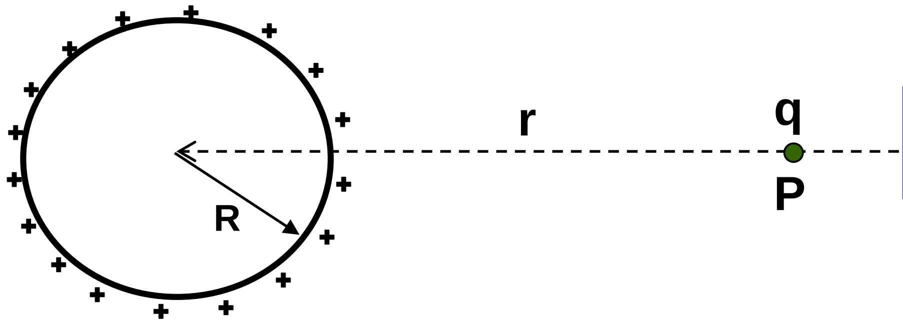
$$F_{AB} = K\frac{(Q/2)^2}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2$$

Another Example: Find the final charges after the spheres below touch?



What determines how much charge ends up on each?

What force does a spherical charge distribution (fixed) exert on a point charge nearby?



Proof by lengthy integration or symmetry + Gauss Law

Shell Theorem (Similar to gravitational Shell Theorem)

- Holds for spherical symmetry only: shell or solid sphere
- Shell has radius R, total charge Q
- Uniform surface charge density σ - not free to move

$$\sigma \equiv Q/A = Q/4\pi R^2$$ dimensions are Coulombs/ m^2

Outside a shell, r > R: A uniform shell of charge attracts or repels a charged particle q outside it as if all the shell's charge is concentrated as a point charge at the center of the sphere.

Inside a shell, r < R: A charged particle q located inside a uniform hollow shell of charge feels zero net electrostatic force from the shell

Why does the shell theorem work? spherical symmetry → cancellations

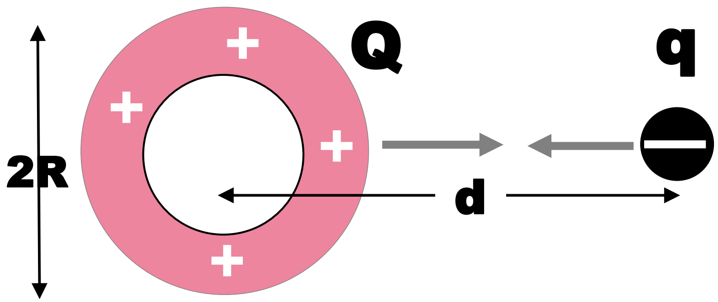
Charge on a spherical conductor – charge free to move

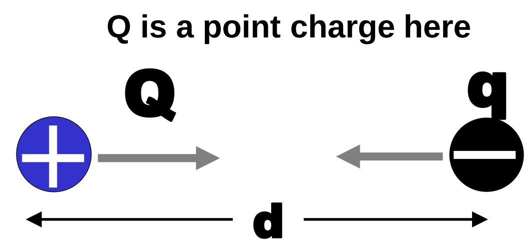
- Net charge Q on a conducting (metal) spherical shell will spread out everywhere over the surface.
- Tiny free charges inside the conductor push on the others and move as far apart as they can go. When do they stop moving?

If no other charges are nearby ("isolated" system)

- Charges spread themselves out uniformly because of symmetry
- Sphere acts as if all of it's charge is concentrated at the center (for points outside the sphere) as in the Shell Theorem.

Bring test charge q<<Q near the sphere





The test charge induces polarization charge on the sphere

- Charge distribution will NOT be uniform any more
- Extra surface charge is induced on the near and far surfaces of the sphere
- The net force will be more attractive than for point charges

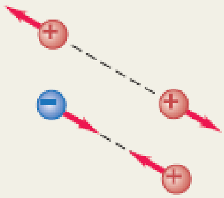
The Shell Theorem works approximately for conductors if the test charge induces very little polarization; e.g., if q << Q and/or d >> R.

CHAPTER 21 SUMMARY

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

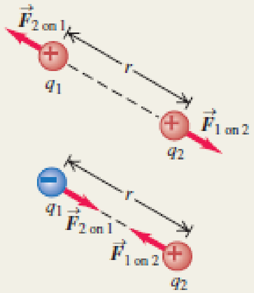


Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product $q_1 q_2$ and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \qquad (21.2)$$

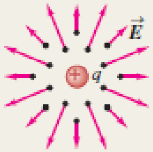
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$



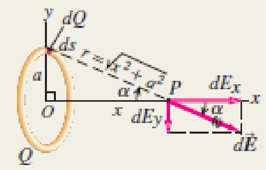
Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \qquad (21.3)$$

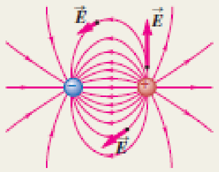
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad (21.7)$$



Superposition of electric fields: The electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ, surface charge density σ, and volume charge density ρ. (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.



Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ has magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.13 and 21.14.)

$$\tau = pE\sin\phi \qquad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (21.16)$$

$$U = -\vec{p} \cdot \vec{E} \qquad (21.18)$$