Physics 121 - Electricity and Magnetism
Lecture 06 - Capacitance
Y&F Chapter 24 Sec. 1 - 6

- **Overview**
- **Definition of Capacitance**
- **Calculating the Capacitance**
- **Parallel Plate Capacitor**
- **Spherical and Cylindrical Capacitors**
- **Capacitors in Parallel and Series**
- **Energy Stored in an Electric Field**
- **Atomic Physics View of Dielectrics**
- **Electric Dipole in an Electric Field**
- **Capacitors with a Dielectric**
- **Dielectrics and Gauss Law**
- **Summary**

What Capacitance Measures

How much charge does an arrangement of conductors hold when a given voltage is applied?

- The charge needed depends on a geometrical factor called capacitance.

$$Q = C\Delta V$$

Example:

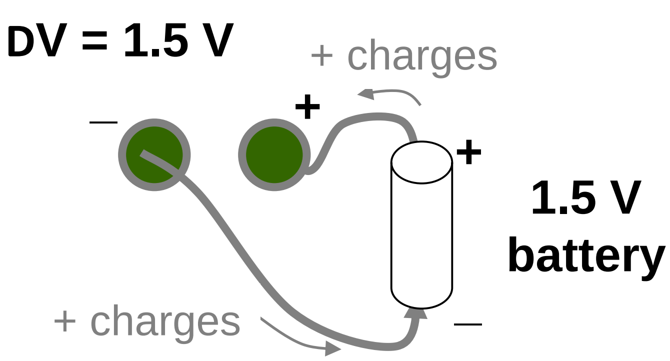
- Two conducting spheres: Radii R_1 and $R_2 = 2R_1$. Different charges Q_1 and Q_2.
- Spheres touch and come to the same potential ΔV,
- Apply point charge potential formula, V(infinity) = 0

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1} \equiv \frac{Q_1}{C_1} \quad \text{and also} \quad \Delta V = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2} \equiv \frac{Q_2}{C_2} \Rightarrow \frac{Q_1}{Q_2} \equiv \frac{C_1}{C_2} = \frac{R_1}{R_2} = \frac{1}{2}$$

Capacitance of a single isolated sphere: $C = 4\pi\varepsilon_0 R$

Example: A primitive capacitor

- The right ball's potential is the same as the + side of the battery. Similarly for the – ball.
- How much charge flows onto each ball to produce a potential difference of 1.5 V ?
- The answer depends on the capacitance.



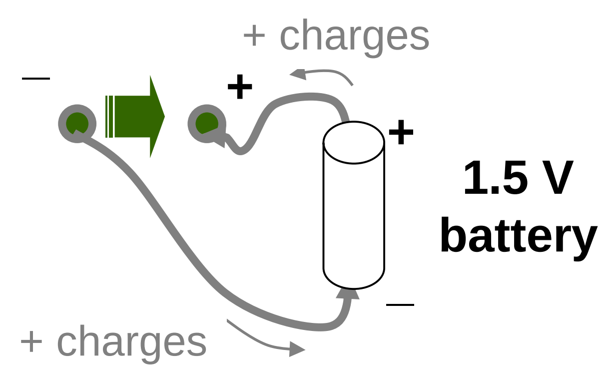
Definition of *CAPACITANCE* :

$$C \equiv \frac{Q}{\Delta V} \quad \text{or} \quad Q \equiv C\,\Delta V \qquad \left[\frac{\text{Coulombs}}{\text{Volt}}\right]$$

- Measures the charge needed per volt of potential difference
- Does not depend on applied DV or charge Q.   Always positive.
- Depends on geometry (and on dielectric materials) **only**
- Units: 1 *FARAD* = 1 Coulomb / Volt. - Farads are very large
  1 mF = 10^{-6} F.   1 pF = 1 pico-Farad = 10^{-12} F = 10^{-6} μF = 1 $\mu\mu$F

Example - Capacitance depends on geometry

- Move the balls at the ends of the wires closer together while still connected to the battery
- The potential difference ΔV cannot change.
- But: $\Delta V = -\int \vec{E} \cdot d\vec{s} \approx \vec{E}_{av} \circ \Delta\vec{s}$
- The distance Ds between the balls decreased so the E field had to increase as did the stored energy.
- Charge flowed from the battery to the balls to increase E.
- The two balls now hold more charge for the same potential difference: i.e. the capacitance increased.



constant

$Q = C\Delta V$

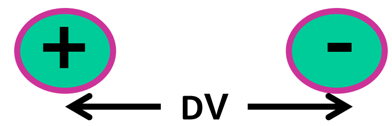
increases   increases

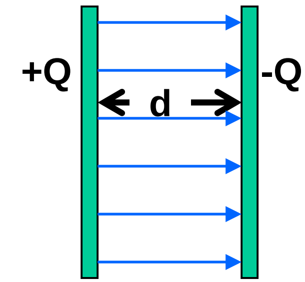
Capacitors are charge storage devices

- Two conductors not in electrical contact
- Electrically neutral before & after being charged

$$Q_{enc} = Q_{net} = 0$$

- Current can flow from + plate to – plate if there is a conducting path (complete circuit)
- Capacitors store charge and potential energy
  - memory bits - radio circuits - power supplies
- Common type: "parallel plate", sometimes tubular





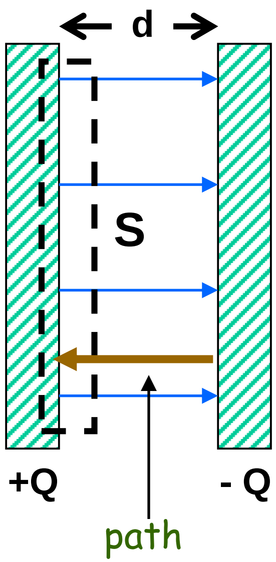
Method for calculating capacitance from geometry:

- Assume two conducting plates (equipotentials) with equal and opposite charges +Q and –Q
- Possibly use Gauss' Law to find E between the plates
- Calculate ΔV between plates using a convenient path
- Capacitance $C = Q/\Delta V$
- Certain materials ("dielectrics") can reduce the E field between plates by "polarizing" - capacitance increases

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0} = \oint_S \vec{E} \circ d\vec{A}$$

$$\Delta V_{fi} = -\int_i^f \vec{E} \circ d\vec{s}$$

EXAMPLE: CALCULATE C for a PARALLEL PLATE CAPACITOR

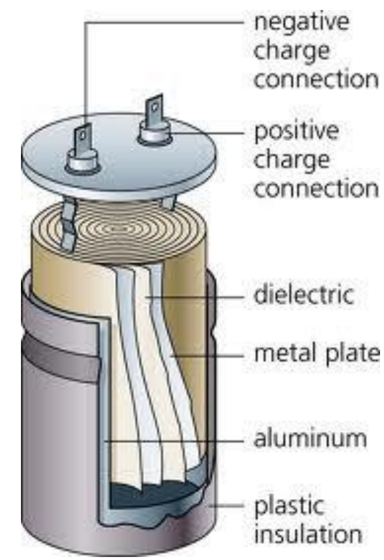


Find E between plates

- A = plate area. Treat plates as infinite sheets
- $|s^+| = |- s^-| = s = Q / A$ = uniform surface charge density
- E is uniform between the plates (d << plate size)
- Use Gaussian surface S (one plate). Flux through ends and attached conductors is zero. Total flux is EA
- $Q_{enc} = \sigma A = e_0 f = \varepsilon_0 EA$

$$\therefore\ E = \sigma/\varepsilon_0 \quad \text{i.e.,} \quad E = Q/\varepsilon_0 A$$

(infinite conducting sheet)



Find potential difference DV:

- Choose V = 0 on negative plate (grounded)
- Choose path from – plate to + plate, opposite to E field

$$\Delta V_{fi} = -\int_{path} \vec{E} \circ d\vec{s} = (-)(-)Ed = +\frac{\sigma}{\varepsilon_0} d = \frac{Qd}{\varepsilon_0 A} \equiv \frac{Q}{C}$$

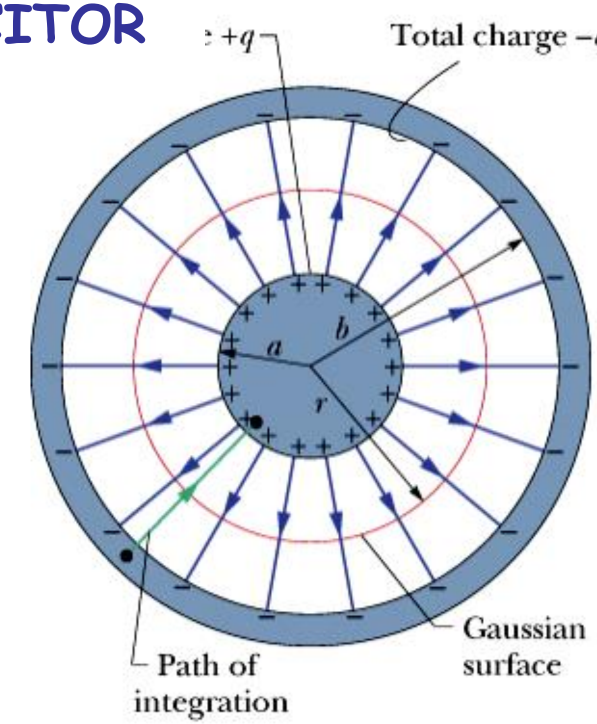
$$\therefore\ C \equiv \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- C DEPENDS ONLY ON GEOMETRY
- C → infinity as plate separation d → 0
- C directly proportional to plate area A

- Other formulas for other geometries

EX 24.03: FIND C for a SPHERICAL CAPACITOR

- 2 concentric spherical, conducting shells, radii a & b
- Charges are +q (inner sphere), -q (outer sphere)
- All charge on the outer sphere is on its inner surface (by Gauss's Law)
- Choose Gaussian surface S as shown and find field using Gauss's Law:



$$\varepsilon_0 \int_S \vec{E} \cdot d\vec{A} = q \qquad q = \varepsilon_0 E A = \varepsilon_0 E(4\pi r^2)$$

- As before: $E = q/(4\pi\varepsilon_0 r^2)$

- To find potential difference use outward radial integration path from r = a to r = b.

$$\Delta V = V_b - V_a = -\int_{r=a}^{r=b} \vec{E} \cdot d\vec{s} = \frac{-q}{4\pi\varepsilon_0} \int_{r=a}^{r=b} \frac{dr}{r^2} = \frac{-q}{4\pi\varepsilon_0} \frac{(-)1}{r}\Big|_a^b$$

$$\Delta V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{a-b}{ba}\right)$$

Negative For b > a

$V_b < V_a$

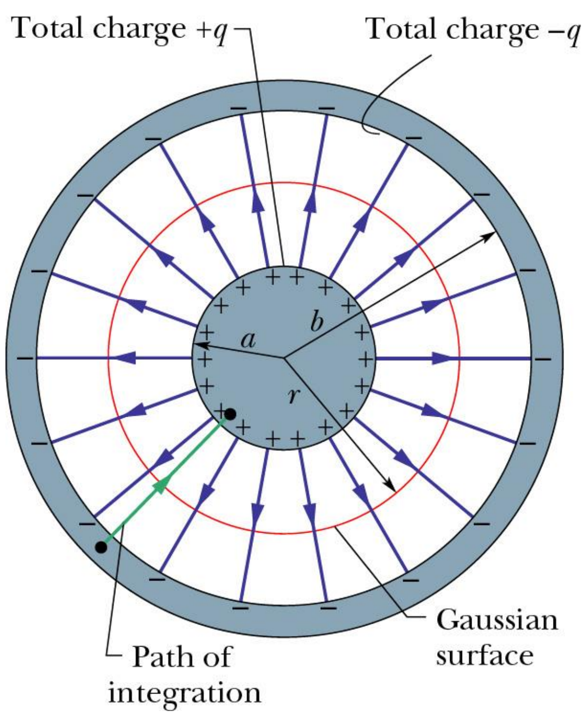
$$\therefore \; C \equiv \frac{q}{|\Delta V|} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

Let b → infinity. Then a/b → 0 and result becomes the earlier formula for the isolated sphere:

$$C \rightarrow \frac{4\pi\varepsilon_0 ab}{b} = 4\pi\varepsilon_0 a$$

EX 24.04: Find C for a CYLINDRICAL CAPACITOR

- 2 concentric, long cylindrical conductors
- Radii a & b and length L >> b => neglect end effects
- Charges are +q (inner) and -q (outer), λ is uniform
- All charge on the outer conductor is on its inner surface (by Gauss's Law)
- Choose Gaussian surface S between plates and find field at radius r.
- E is perpendicular to endcaps => zero flux contribution



$$\varepsilon_0\Phi_{cyl} = \varepsilon_0\int_S \vec{E}\cdot d\vec{A} = q \qquad q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

- So: $E = q/(2\pi\varepsilon_0 rL) = \lambda/(2\pi\varepsilon_0 r)$

- To find potential difference use outward radial integration path from r = b to r = a.

$$\Delta V = V_b - V_a = -\int_{r=a}^{r=b}\vec{E}\cdot d\vec{s} = \frac{-q}{2\pi\varepsilon_0 L}\int_{r=a}^{r=b}\frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 L}\ln(r)\Big|_a^b = \frac{-q}{2\pi\varepsilon_0 L}\ln(b/a)$$

$$C = q/\Delta V = 2\pi\varepsilon_0\frac{L}{\ln(b/a)}$$

$V_b < V_a$ For b > a

$C \rightarrow 0$ as $b/a \rightarrow$ inf

$C \rightarrow$ inf as $b/a \rightarrow 1$

C depends only on geometrical parameters

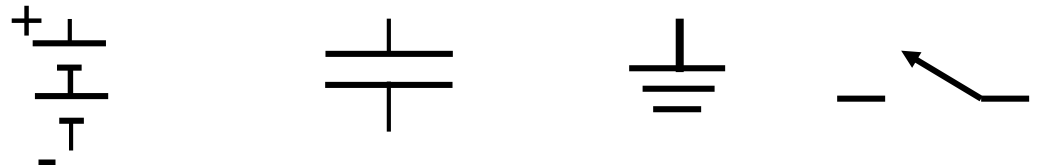
Examples of Capacitance Formulas

- **Capacitance for isolated Sphere** $$C = 4\pi\varepsilon_0 R$$
- **Parallel Plate Capacitor** $$C = \frac{\varepsilon_0 A}{d}$$
- **Concentric Cylinders Capacitor** $$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$
- **Concentric Spheres Capacitor** $$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$
- **Units: F (Farad) = C^2/Nm = C/ Volt = $\varepsilon_0 \times$length**
  - **named after Michael Faraday. [note: ε_0 = 8.85 pF/m]**

All of these formulas depend only on geometrical factors

Capacitors in circuits

CIRCUIT SYMBOLS:



CIRCUIT DEFINITIONS:

Current ≡ i ≡ dq/dt

= rate of + charge flow past a point in the circuit

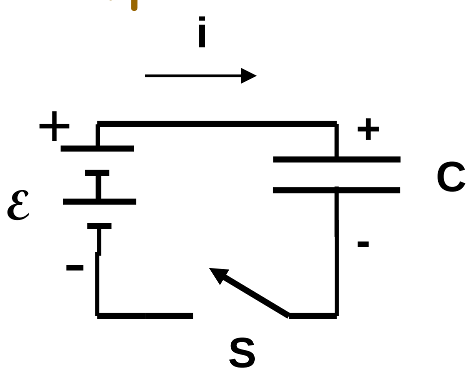
Open Circuit: NO closed path. No current. Conductors are equi-potentials

Closed Circuit: There is/are completed paths through which current can flow.

Loop Rule: Potential is a conservative field

→ Potential CHANGE around ANY closed path = 0

Example: CHARGING A CAPACITOR



- Current flows when switch is CLOSED, completing circuit
- Battery (EMF) maintains DV (= EMF $\mathcal{E}$), and supplies energy by moving free + charges from – to + terminal, internal to battery

Convention: i flows from + to – outside of battery

When switch closes, current (charge) flows until DV across capacitor equals battery voltage E.

Then current stops as E field in wire → 0

DEFINITION: EQUIVALENT CAPACITANCE

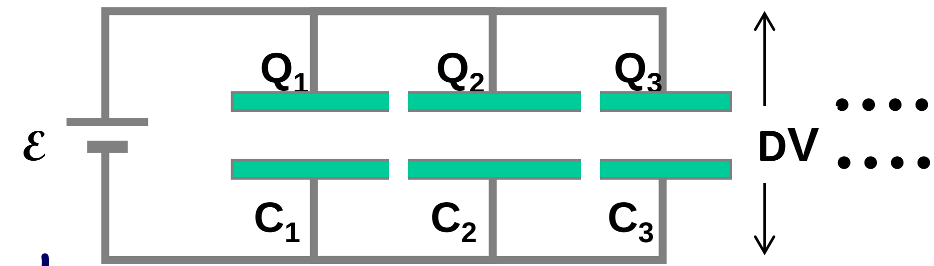
- Capacitors can be connected in series, parallel, or more complex combinations
- The "equivalent capacitance" is the capacitance of a SINGLE capacitor that would have the same capacitance as the combination.
- The equivalent capacitance can replace the original combination in analysis.

Parallel capacitors - Equivalent capacitance

The actual parallel circuit...

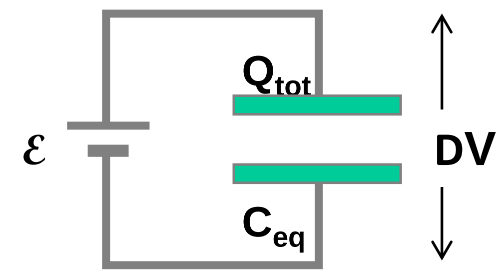
$$Q_i = C_i \,\Delta V$$

ΔV is the same for each branch



...and the equivalent circuit:

$$Q_{tot} \equiv C_{eq} \Delta V$$



The parallel capacitors are just like a single capacitor with larger plates so....

$$Q_{tot} = \sum Q_i \quad \text{(parallel)}$$

Charges on parallel capacitors add

$$\therefore \quad Q_{tot} = C_1 \Delta V + C_2 \Delta V + C_3 \Delta V + \ldots \;=\; (C_1 + C_2 + C3 + \ldots)\Delta V$$

Parallel capacitances add directly

$$C_{eq} = \sum C_i \quad \text{(parallel)}$$

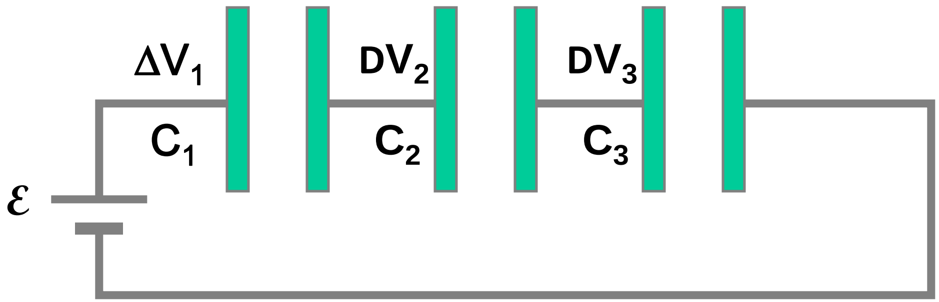
Question: Why is DV the same for all elements in parallel?

Answer: Potential is conservative field, for ANY closed loop around circuit:

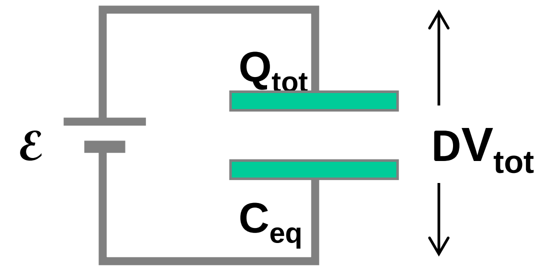
$$\sum \Delta V_i = 0 \quad \text{(KirchoffLoopRule)}$$

Series capacitors - equivalent capacitance

The actual series circuit...



The equivalent circuit...

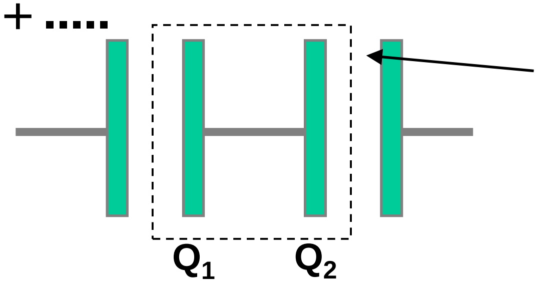


DV_i are NOT necessarily the same for each capacitor in series

$$Q_{tot} \equiv C_{eq} \Delta V_{tot}$$

$$\Delta V_{tot} = \sum \Delta V_i = \Delta V_1 + \Delta V_2 + \Delta V_3 + ...$$

But... charges on series capacitors are all equal - here's why.....



Gaussian surface
$Q_{enc} = 0$
neutral..so
$Q_1 = - Q_2$

so $\Delta V_i = Q / C_i$ same Q

$$\therefore \quad \Delta V_{tot} = Q/C_1 + Q/C_2 + Q/C_3 + ... = Q \sum 1/C_i \equiv Q/C_{eq}$$

Reciprocals of series capacitances add

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i} \quad \text{(series)}$$

For two capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2} \Rightarrow C_{eq} = \frac{C_2 C_1}{C_1 + C_2}$$

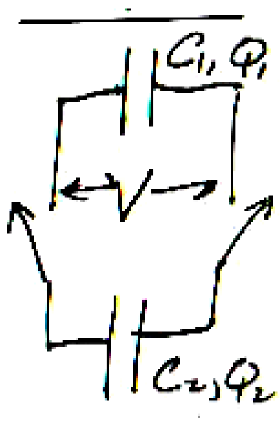
Example 1: A 33μF and a 47 μF capacitor are connected in parallel
Find the equivalent capacitance

Solution: $C_{para} = C_1 + C_2 = 80\ \mu F$

Example 2: Same two capacitors as above, but now in series connection

Solution: $$C_{ser} = \frac{C_1 C_2}{C_1 + C_2} = \frac{33 \times 47}{33 + 47} = 19.4\ \mu F$$

Example 3: A pair of capacitors is connected as shown



- $C_1 =$ 10 μF, charged initially to 100V = V_i
- $C_2 =$ 20 μF, uncharged initially

Close switches. Find final potentials across C_1 & C_2.

Solution:

- C's are in parallel → Same potential V_f for each
- Total initial charge: $Q_{tot} = Q_{1i} = C_1 V_i = 10^{-3}$ C.
- Charge is conserved – it redistributes on both C_1 & C_2

$$C_{eq} = Q_{tot} / V_f = C_1 + C_2 \Rightarrow V_f = \frac{10^{-3}}{30 \times 10^{-6}} = 33\ V.$$

- Final charge on each:

$$Q_{1f} = C_1 V_f = 3.3 \times 10^{-4}\ C. \quad Q_{2f} = C_2 V_f = 6.7 \times 10^{-4}\ C.$$

Three Capacitors in Series

6-2: The equivalent capacitance for two capacitors in series is:

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

Which of the following is the equivalent capacitance formula for three capacitors in series?

A. $$C_{eq} = \frac{C_1 C_2 C_3}{C_1 + C_2 + C_3}$$

B. $$C_{eq} = \frac{C_1 C_2 + C_2 C_3 + C_1 C_3}{C_1 + C_2 + C_3}$$

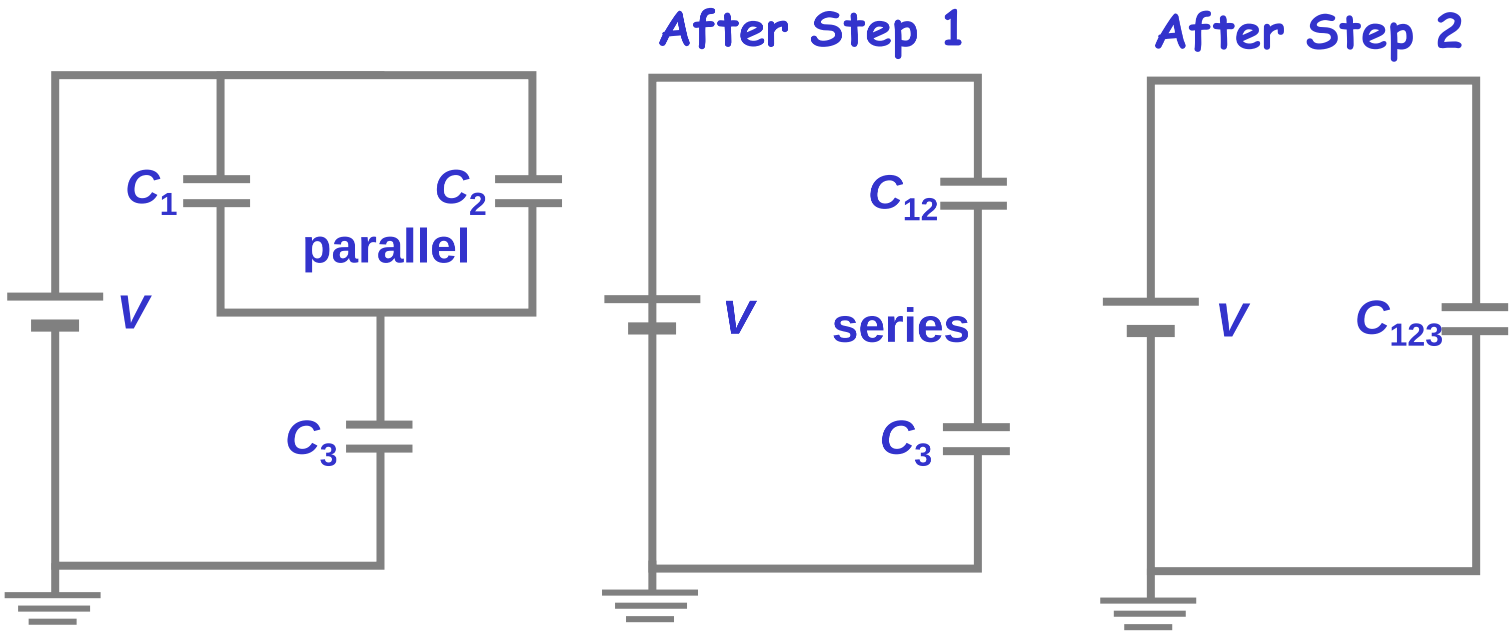
C. $$C_{eq} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_1 C_2 C_3}$$

D. $$C_{eq} = \frac{C_1 + C_2 + C_3}{C_1 C_2 C_3}$$

E. $$C_{eq} = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

Apply formula for C_{eq} twice

Example: Reduce circuit to find $C_{eq}=C_{123}$ for mixed series-parallel capacitors



$$C_{12} = C_1 + C_2$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$C_{123} = \frac{C_{12}C_3}{C_{12} + C_3}$$

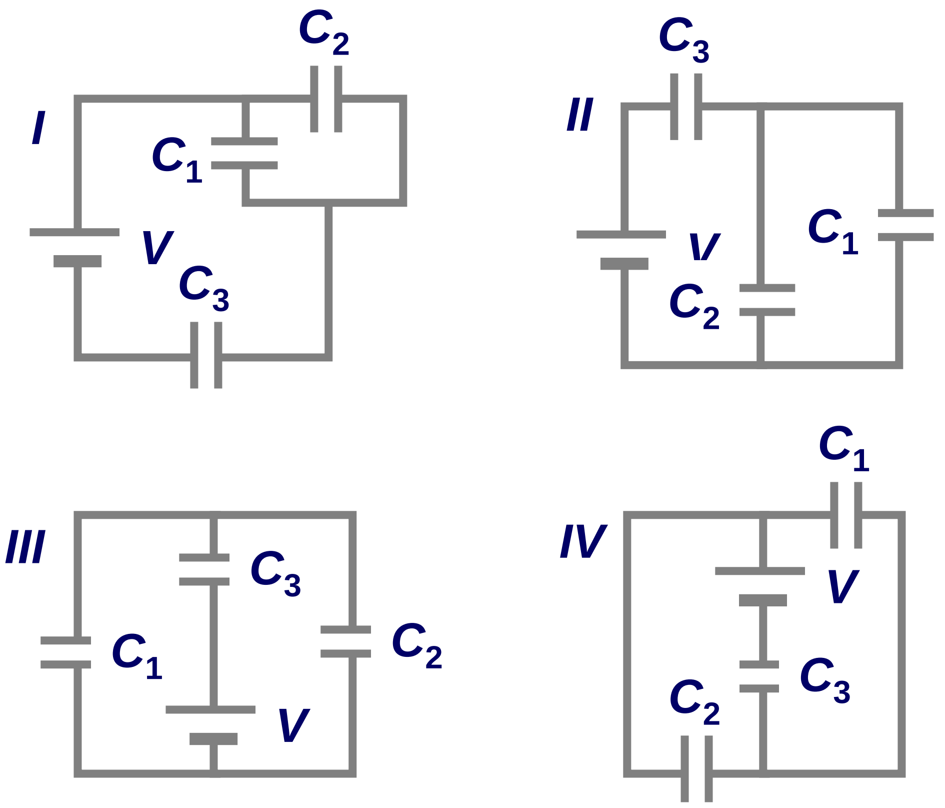
Values: $C_1 = 12.0\ \mu F,\ \ C_2 = 5.3\ \mu F,\ \ C_3 = 4.5\ \mu F$

$$C_{123} = (12 + 5.3)\ 4.5\ /\ (12 + 5.3 + 4.5)\ \mu F = 3.57\ \mu F$$

Series or Parallel?

6-3: In the circuits below, which ones show capacitors 1 and 2 connected in series?

A. I, II, III
B. I, III
C. II, IV
D. III, IV
E. None



Energy Stored in a Capacitor

When charge flows in the sketch, energy stored in the battery is depleted. Where does it go?

- **Charge distributions have potential energy. Charges that are separated in a neutral body store energy.**
- **The electric potential is defined to be**

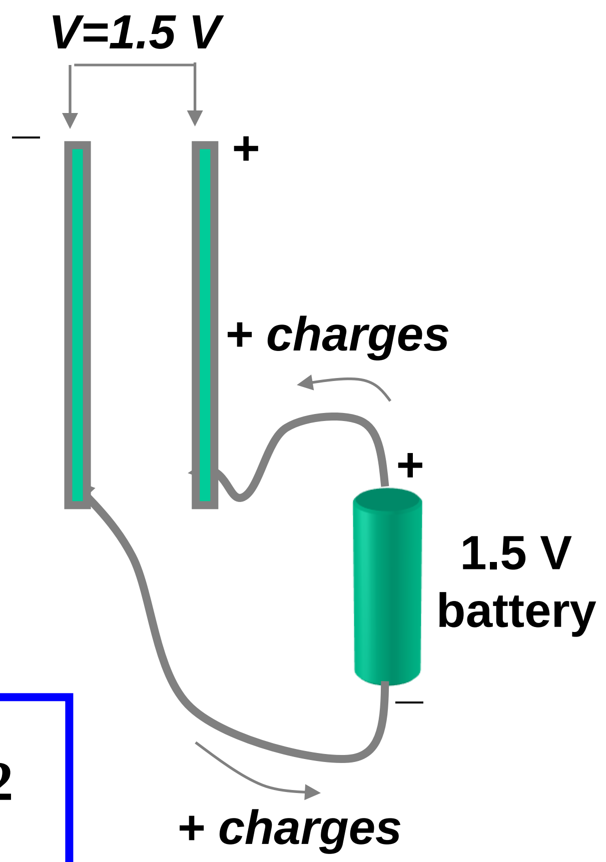
  $$V = U/q, \quad U = qV$$

- **A small element of charge dq on each plate of a capacitor stores potential energy:**

  $$dU = V\,dq$$

- **The energy stored by charging a capacitor from charge 0 to Q is the integral:**

$$V = \frac{q}{C}$$



$$U = \int_0^Q dU = \frac{1}{C}\int_0^Q q'\,dq' = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Capacitors Store Energy in the Electrostatic Field

- The total energy in a parallel plate capacitor is

$$U = \tfrac{1}{2}CV^2 = \frac{\varepsilon_0 A}{2d}V^2$$

- The volume of space filled by the electric field in the capacitor is = Ad, so the ***energy density u*** is

$$u \equiv \frac{U}{vol} = \frac{\varepsilon_0 A}{2dAd}V^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

- But for a parallel plate capacitor,

$$V = -\int \vec{E}\cdot d\vec{s} = Ed$$

- so

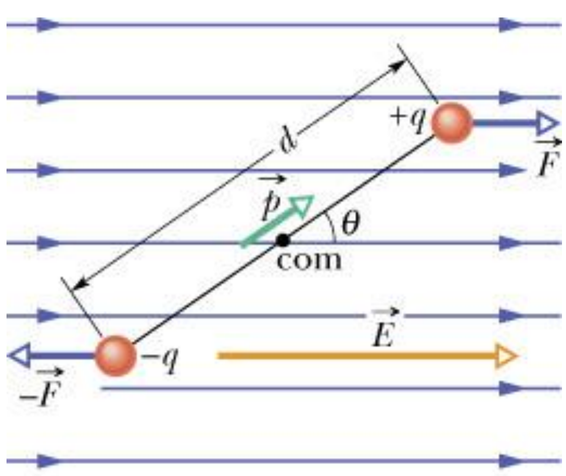
$$u = \tfrac{1}{2}\varepsilon_0 E^2$$

Energy is stored in the electric field

Model for a Molecule that can Polarize

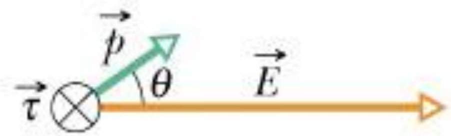
A dipole in a uniform external field..

..feels torque, stores electrostatic potential energy



See Lecture 3

$$\vec{p} \equiv q\vec{d}$$

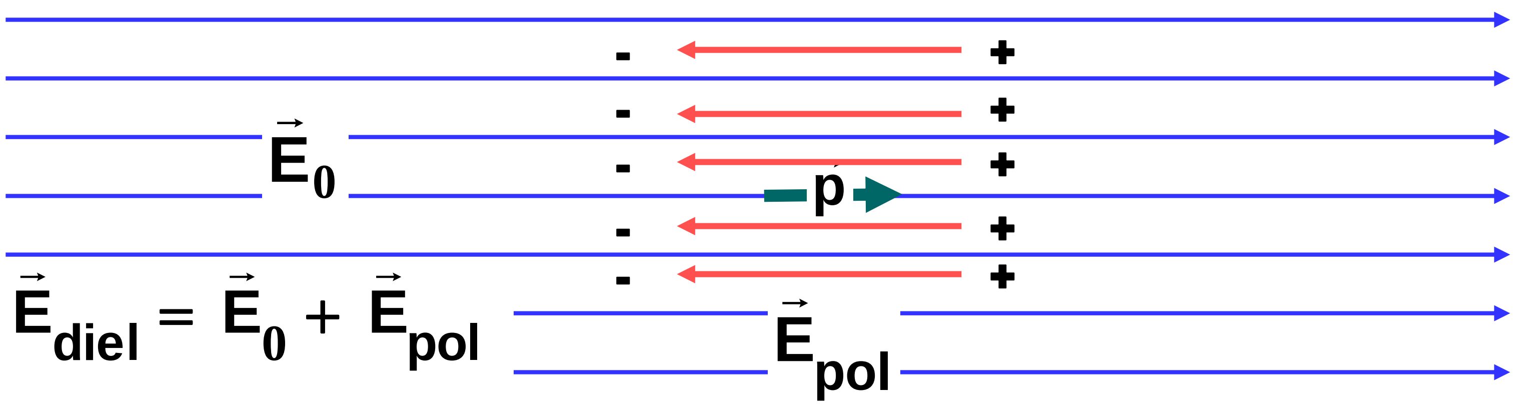


$$\vec{\tau} = \vec{p} x \vec{E}$$

- |torque| = 0 at q = 0 or q = π
- |torque| = pE at q = +/- π/2
- RESTORING TORQUE: t(-q) = t(+q)

$$U_E = -\vec{p} \cdot \vec{E}$$

Polarization: An external field aligns dipoles in a material, causing polarization that reduces the field

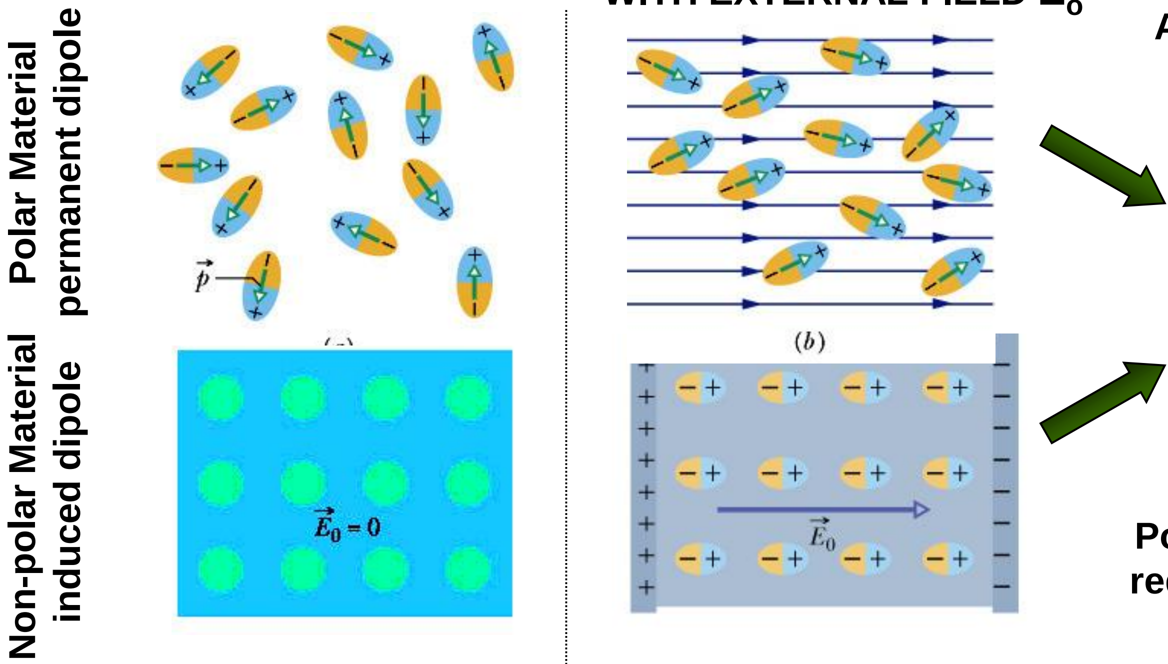


Dielectric materials in capacitors

- Insulators *POLARIZE* when an external electric field is applied
- The NET field inside the material is reduced.

MOLECULAR VIEW

NO EXTERNAL FIELD | WITH EXTERNAL FIELD E_o

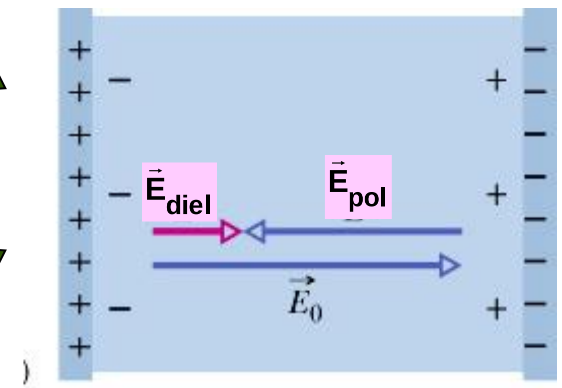


$E_0 = E_{vac}$ is field due to free charge

Response to E_{vac} is the polarization field E_{pol}

Actual weakened net field inside is

$E_{diel} = E_0 - E_{pol} = E_{net}$



Polarization surface charge density reduces free surface charge density

$\sigma_{net} = s_{free} - s_{pol}$

$(s_{free} = s_{ext} = s_{vac})$

Dielectrics increase capacitance

For a given DV, more movable charge s_{free} is needed

$$\text{Dielectric constant} \equiv K \equiv \frac{C_{dielectric}}{C_{vacuum}} \geq 1$$

Inside conductors, polarization reduces E_{net} to zero

Representing Dielectrics

- ε_0 is the free space permittivity.
- All materials (water, paper, plastic, air) polarize to some extent and have different permittivities $\varepsilon = \kappa\varepsilon_0$
- κ is the *dielectric constant* - a dimensionless number.
- Wherever you see ε_0 for a vacuum, you can substitute $\kappa\varepsilon_0$ when considering dielectric materials.

- For example, the capacitance of a parallel plate capacitor increases when the space is filled with a dielectric:

$$C_{diel} = \frac{\kappa\varepsilon_0 A}{d} = \kappa C_{vac}$$

- A dielectric weakens the field, compared to what it would be for a vacuum

$$\vec{E}_{diel} = \vec{E}_{vac}/\kappa \equiv \vec{D}/\varepsilon_0\kappa$$

TABLE 26.1

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

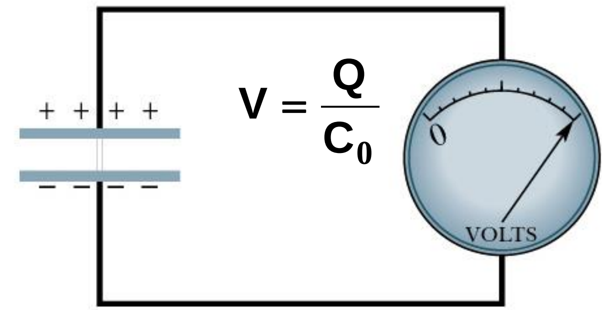
What happens as you insert a dielectric?

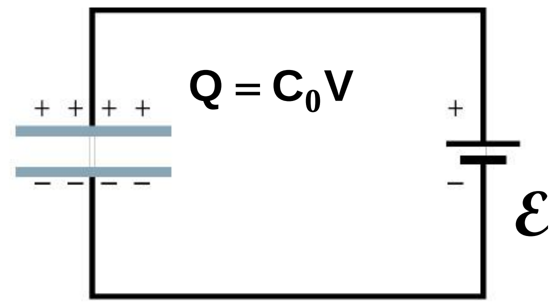
Initially, charge capacitor C_0 to voltage V, charge Q, field E_{net}.

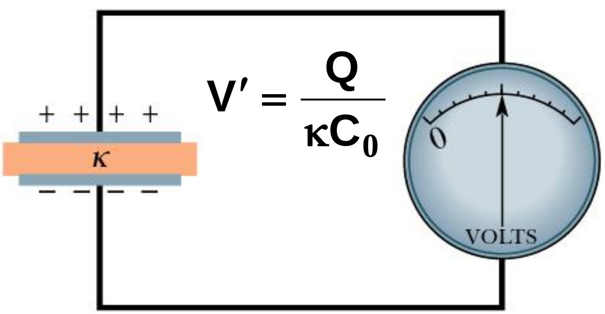
- With battery detached insert dielectric
- Q remains constant, E_{net} is reduced
- Voltage (fixed Q) drops to V'.
- Dielectric reduced E_{net} and V.

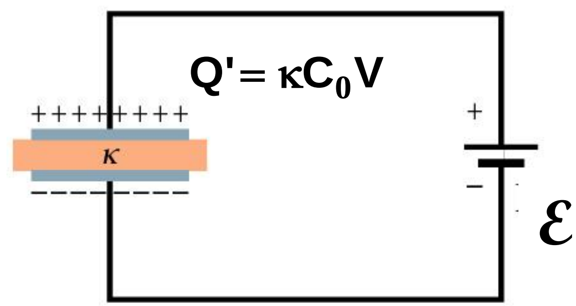
OR

- With battery attached, insert dielectric.
- E_{net} and V are momentarily reduced but battery maintains voltage $\mathcal{E}$
- Charge flows to the capacitor as dielectric is inserted until V and E_{net} are back to original values.









Q = a constant

$$V = -\int \vec{E} \cdot d\vec{s} = Ed$$

V = a constant

$Q' > Q$

Gauss' Law with a dielectric

OPTIONAL TOPIC

$$\varepsilon_0 \int_S \vec{E}_{diel} \circ d\vec{A} = q_{free} - q_{pol} = q_{net}$$

Alternatively:

$$\int_S \varepsilon_0 \vec{E}_{vac} \circ d\vec{A} \;=\; q_{free} = \int_S \varepsilon_0 K \vec{E}_{diel} \circ d\vec{A} \equiv \int_S \vec{D} \circ d\vec{A}$$

free charge on plates

field not counting polarization = $\varepsilon_0 E_{vac}$

The "Electric Displacement" D measures field that would be present due to the "free" charge only, i.e. without polarization field from dielectric

$$E_{vac} = K E_{diel} \qquad D \equiv \varepsilon_0 E_{vac} = \varepsilon_0 K E_{diel} \equiv \varepsilon E_{diel}$$

- κ could vary over Gaussian surface S. Usually it is constant and factors
- Flux is still measured using field **without** dielectric: $E_{vac} = K E_{diel} = D/e_0$)

$$d\Phi = \vec{E}_{vac} \circ d\vec{A} = K \vec{E}_{diel} \circ d\vec{A}$$

- Only the free charges q_{free} (excluding polarization) are counted as q_{enc} in the above. Using κ on the left compensates for the polarization.
- When applying the above include only q_{free}.
  Ignore polarization charges inside the Gaussian surface

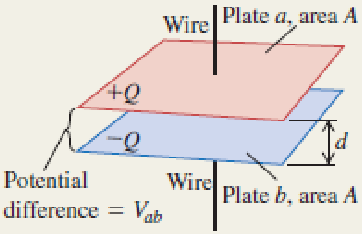
CHAPTER 24 SUMMARY

Capacitors and capacitance: A capacitor is any pair of conductors separated by an insulating material. When the capacitor is charged, there are charges of equal magnitude Q and opposite sign on the two conductors, and the potential V_{ab} of the positively charged conductor with respect to the negatively charged conductor is proportional to Q. The capacitance C is defined as the ratio of Q to V_{ab}. The SI unit of capacitance is the farad (F): $1\ \text{F} = 1\ \text{C/V}$.

A parallel-plate capacitor consists of two parallel conducting plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance depends only on A and d. For other geometries, the capacitance can be found by using the definition $C = Q/V_{ab}$. (See Examples 24.1–24.4.)

$$C = \frac{Q}{V_{ab}} \quad (24.1)$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \quad (24.2)$$



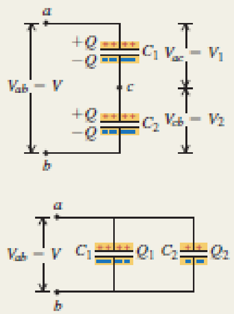
Capacitors in series and parallel: When capacitors with capacitances $C_1, C_2, C_3, \ldots$ are connected in series, the reciprocal of the equivalent capacitance C_{eq} equals the sum of the reciprocals of the individual capacitances. When capacitors are connected in parallel, the equivalent capacitance C_{eq} equals the sum of the individual capacitances. (See Examples 24.5 and 24.6.)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad (24.5)$$

(capacitors in series)

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad (24.7)$$

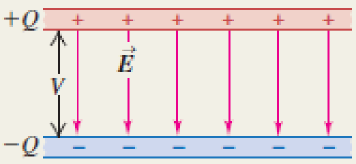
(capacitors in parallel)



Energy in a capacitor: The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor. This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is proportional to the square of the electric-field magnitude. (See Examples 24.7–24.9.)

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \tag{24.9}$$

$$u = \frac{1}{2}\epsilon_0 E^2 \tag{24.11}$$



Dielectrics: When the space between the conductors is filled with a dielectric material, the capacitance increases by a factor K, called the dielectric constant of the material. The quantity $\epsilon = K\epsilon_0$ is called the permittivity of the dielectric. For a fixed amount of charge on the capacitor plates, induced charges on the surface of the dielectric decrease the electric field and potential difference between the plates by the same factor K. The surface charge results from polarization, a microscopic rearrangement of charge in the dielectric. (See Example 24.10.)

Under sufficiently strong fields, dielectrics become conductors, a situation called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.

In a dielectric, the expression for the energy density is the same as in vacuum but with ϵ_0 replaced by $\epsilon = K\epsilon$. (See Example 24.11.)

Gauss's law in a dielectric has almost the same form as in vacuum, with two key differences: $\vec{E}$ is replaced by $K\vec{E}$ and Q_{encl} is replaced by $Q_{encl\text{-}free}$, which includes only the free charge (not bound charge) enclosed by the Gaussian surface. (See Example 24.12.)

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d}$$

(parallel-plate capacitor filled with dielectric) (24.19)

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2 \tag{24.20}$$

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{encl\text{-}free}}{\epsilon_0} \tag{24.23}$$

Dielectric between plates