Physics 121 - Electricity and Magnetism
Lecture 09 - Charges & Currents in Magnetic Fields
Y&F Chapter 27, Sec. 1 - 8

- **What Produces Magnetic Field?**
- **Properties of Magnetic versus Electric Fields**
- **Force on a Charge Moving through Magnetic Field**
- **Magnetic Field Lines**
- **A Charged Particle Circulating in a Magnetic Field – Cyclotron Frequency**
- **The Cyclotron, the Mass Spectrometer, the Earth's Field**
- **Crossed Electric and Magnetic Fields**
- **The e/m Ratio for Electrons**
- **Magnetic Force on a Current-Carrying Wire**
- **Torque on a Current Loop: the Motor Effect**
- **The Magnetic Dipole Moment**
- **Summary**

Magnetic Field

Electrostatic & gravitational forces act through vector fields $\vec{E}$ and $\vec{g}$

Now: Magnetic force (quite different): $\vec{B} \equiv$ magnetic field

Force law first: How are charges & currents affected by a given B field?

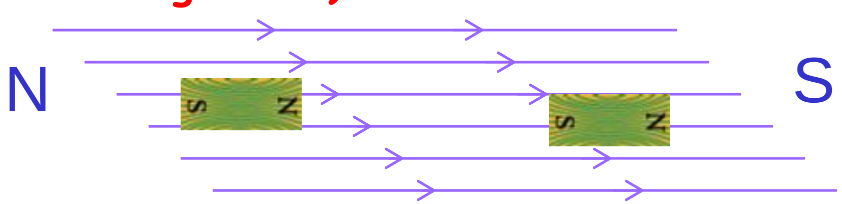
Then: How to create B field (next lecture)

- Currents in loops of wire
- Intrinsic spins of $e^-, p^+ \rightarrow$ currents $\rightarrow$ magnetic dipole moment
- Spins can align permanently to form natural magnets

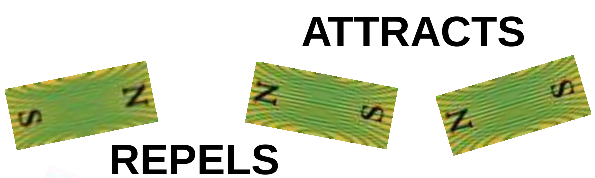
Unification: Maxwell's equations, electromagnetic waves

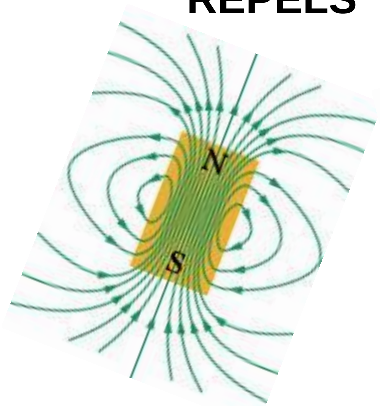
"Permanent Magnets": North- and South- *seeking* poles

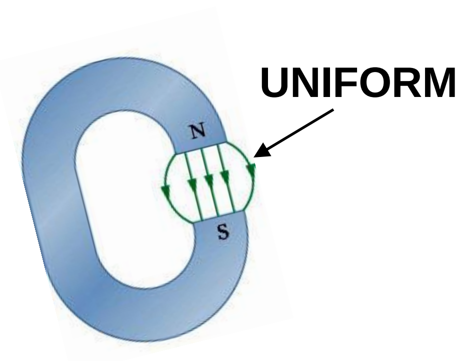
- Natural magnets known since antiquity
- Earth's magnetic field, compass
- Ferromagnetic materials make a magnet when cooled in B field (Fe, Ni, Co) - - Spins align into "domains" (magnets)
- Other materials (plastic, copper, wood,…) slightly or not affected (para- and dia- magnetism)



DIPOLES ALIGN IN B FIELD



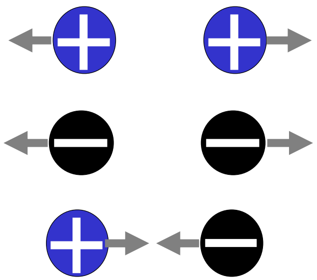


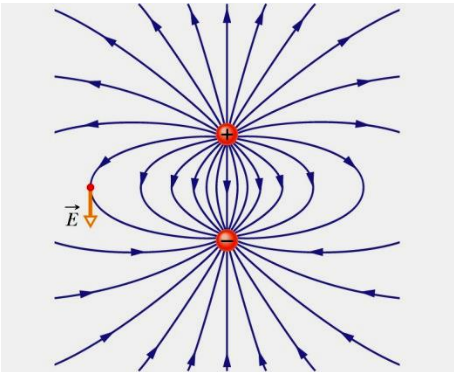


Electric Field versus Magnetic Field

- Electric force acts at a distance through electric field.
- Vector field, E.
- Source: electric charge.

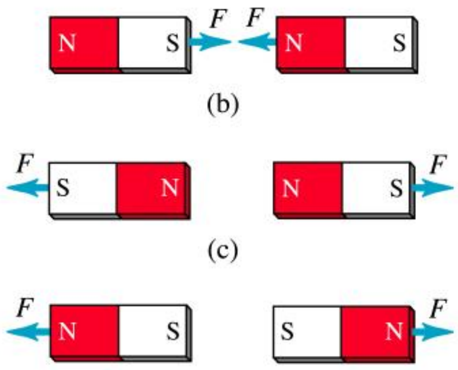
- Positive charge (+) and negative charge (-).
- Opposite charges attract
- Like charges repel.
- Electric field lines visualizing the direction and magnitude of E.

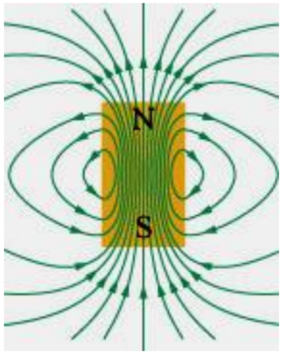




- Magnetic force acts at a distance through magnetic field.
- Vector field, B
- Source: *moving* electric charge (current, even in substances such as permanent magnets).
- North pole (N) and south pole (S)

- Opposite poles attract
- Like poles repel.
- Magnetic field lines visualizing the direction and magnitude of B.





Differences between magnetic & electrostatic field

Test charge and electric field

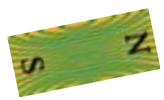
$$\vec{E} = \frac{\vec{F}_E}{q}$$

Single electric poles exist

Test monopole and magnetic field ?

$$\vec{B} = \frac{\vec{F}_B}{p}$$

Magnetic poles are always found in pairs. A single magnetic pole has never been found.

Cut up a bar magnet  ➡ small, complete magnets  

There is no magnetic monopole...

...dipoles are the basic units

Electrostatic Gauss Law $\int_S \vec{E} \circ d\vec{A} = q_{enc} / \varepsilon_0$

Magnetic Gauss Law $\int_S \vec{B} \circ d\vec{A} = 0$

Magnetic flux through each and every Gaussian surface = 0

Magnetic field exerts force on moving charges (current) only

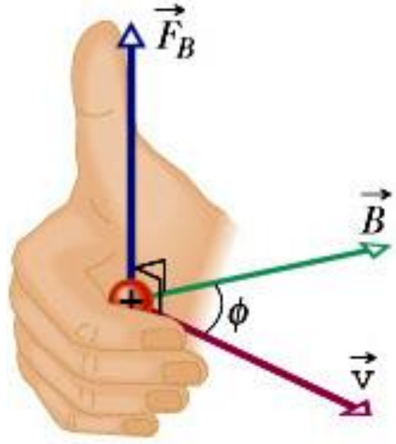
Magnetic Force on a Charged Particle

Define B by the magnetic force F_B it exerts on a charged particle moving with a velocity v

$$F_B = qvB\sin(\phi)$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

"LORENTZ FORCE"



Typical Magnitudes:
- Earth's field: 10^{-4} T.
- Bar Magnet: 10^{-2} T.
- Electromagnet: 10^{-1} T.

Units: $[1\,\text{Tesla}] \equiv \dfrac{[\text{Newtons}]}{[\text{Coulomb}][\text{m/s}]}$

1 "GAUSS" = 10^{-4} Tesla

See Lecture 01 for cross product definitions and examples

- F_B is proportional to speed v as well as charge q and field B.
- F_B is geometrically complex – depends on cross product
  - F = 0 if v is parallel to B.
  - F is otherwise normal to plane of both v and B.
  - F_B reverses sign for opposite sign of charge
  - Source is also qv [current x length].
- Electric force can do work on a charged particle…
- BUT magnetic force cannot do work on moving particles since $F_B \cdot v = 0$.

The Vector Product $\vec{c} = \vec{a} \times \vec{b}$ in terms of Vector Components

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \ , \ \vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k} \ , \ \vec{c} = c_x\hat{i} + c_y\hat{j} + c_z\hat{k}$$

The vector components of vector $\vec{c}$ are given by the equations:

$$c_x = a_y b_z - a_z b_y \ , \ c_y = a_z b_x - a_x b_z \ , \ c_z = a_x b_y - a_y b_x$$

Note : Those familiar with the use of determinants can use the expression

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

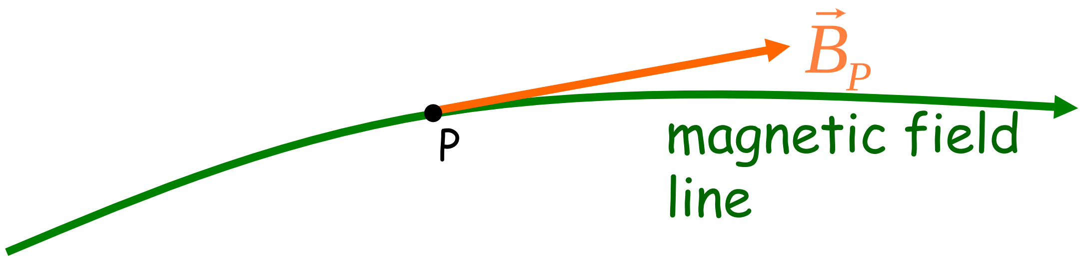
Note : The order of the two vectors in the cross product is important

$$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right)$$

Magnetic Field Lines : In analogy with the electric field lines we introduce the concept of magnetic field lines which help visualize the magnetic field vector $\vec{B}$ without using equations.

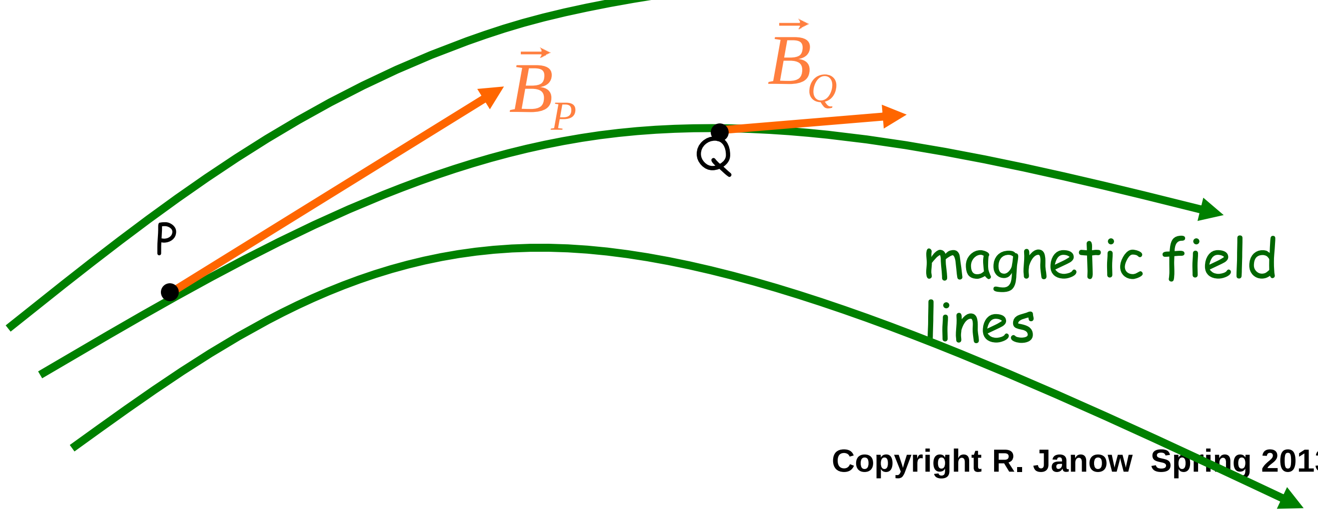
The relation between the magnetic field lines and $\vec{B}$ are:

1. **At any point P the magnetic field vector $\vec{B}$ is tangent to the magnetic field lines**



2. **The magnitude of the magnetic field vector $\vec{B}$ is proportional to the density of the magnetic field lines**

$B_P > B_Q$

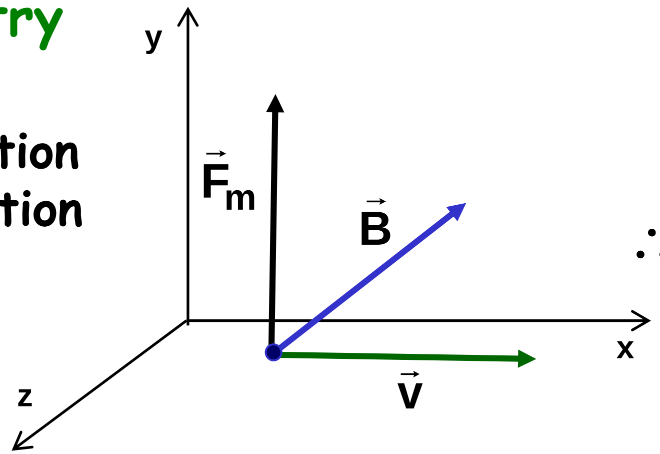


Magnetic force geometry examples:

A simple geometry

- charge +q
- $\underline{B}$ along −z direction
- $\underline{v}$ along +x direction

CONVENTION
● Head ✕ Tail



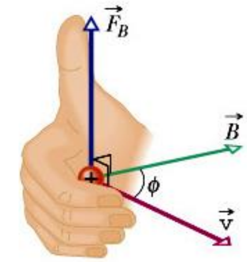
$$\vec{B} = -B_0\hat{k}$$

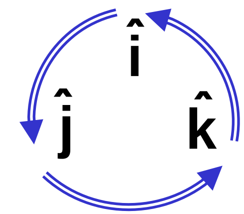
$$\vec{v} = \vec{v}_0\hat{i}$$

$$\hat{i}\times\hat{k} = -\hat{j}$$

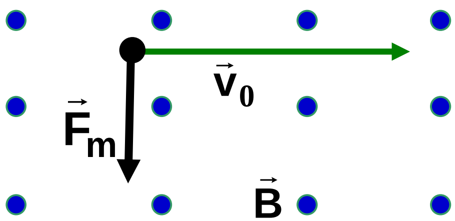
$$\therefore\ \vec{F}_m = q\vec{v}\times\vec{B}$$

$$= q\, v_0\, B_0\ \hat{j}$$

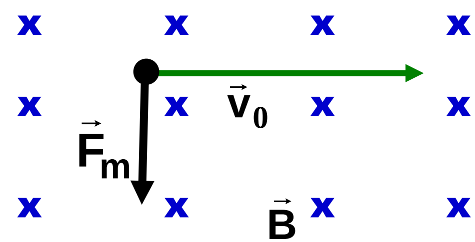




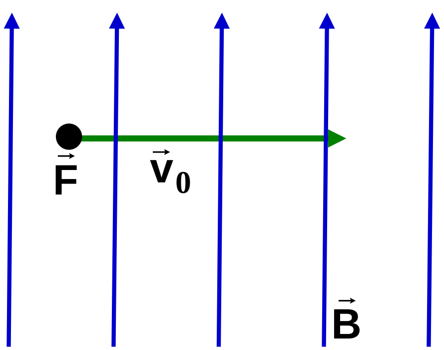
More simple examples



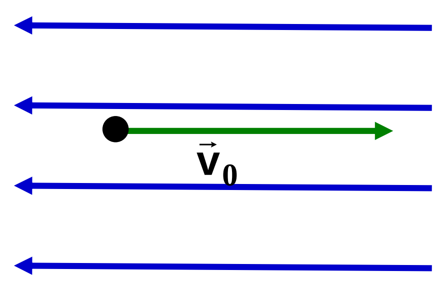
- charge +q
- $\underline{F_m}$ is down
- $\underline{|F_m| = qv_0B_0}$



- charge −q
- $\underline{B}$ into page
- $\underline{F_m}$ is still down
- $\underline{|F_m| = qv_0B_0}$



- charge +q
- $\underline{F_m}$ is out of the page
- $\underline{|F_m| = qv_0B_0}$

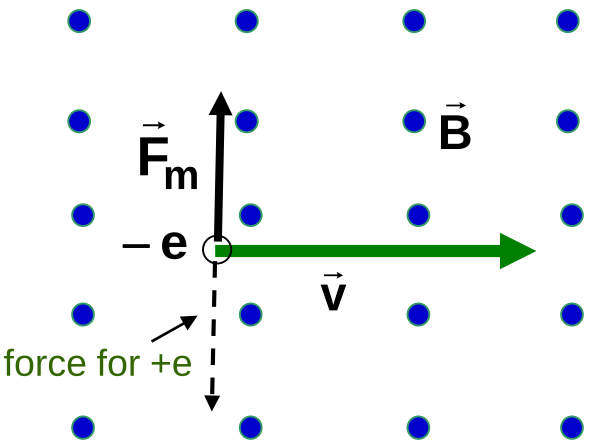


- charge +q or −q
- $\underline{v}$ parallel to $\underline{B}$
- $\underline{|F_m| = qv_0B_0\sin(0)}$

$$\therefore\ \vec{F}_m = 0$$

A numerical example

- Electron beam moving in plane of sketch
- $v = 10^7$ m/s along +x
- $B = 10^{-3}$ T. out of page along +y



a) Find the force on the electron :

$$\vec{F}_m = -e\,\vec{v}\times\vec{B} = -1.6\text{x}10^{-19}\text{x}10^{7}\text{x}10^{-3}\text{x}\sin(90^o)\,\hat{k}$$

$$\vec{F}_m = -1.6\text{x}10^{-15}\,\hat{k}$$

Negative sign means force is opposite to result of using the RH rule

b) Acceleration of electron :

$$\vec{a} = \frac{\vec{F}_m}{m_e} = \frac{-1.6\times10^{-15}\text{ N}}{9\times10^{-31}\text{ Kg}}\hat{k} = -1.76\times10^{+15}\hat{k}\ [\text{m/s}^2]$$

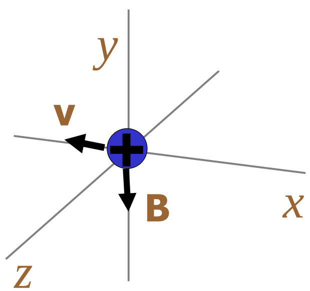
Direction is the same as that of the force

Direction of Magnetic Force

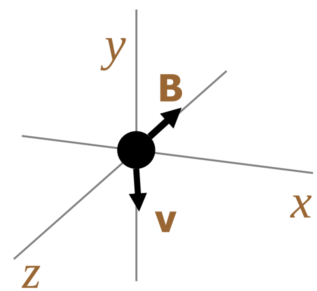
9-2: The figures shows five situations in which a positively charged particle with velocity v travels through a uniform magnetic field B. For which situation is the direction of the magnetic force along the +x axis ?

Hint: Use Right Hand Rule.

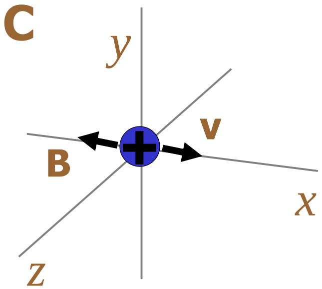
A



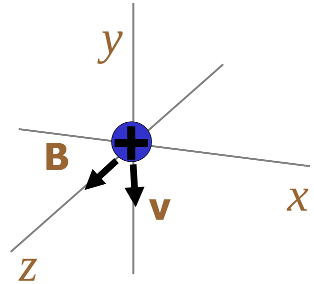
B



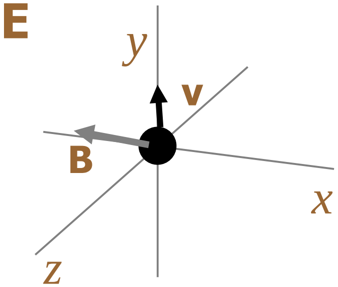
C

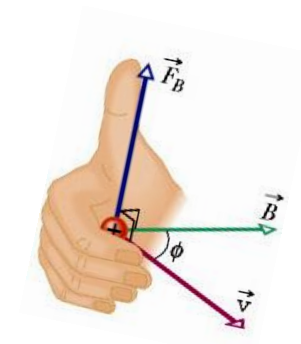


D



E



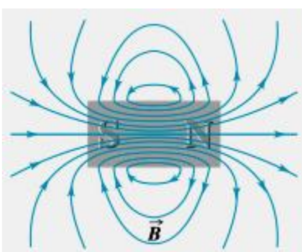


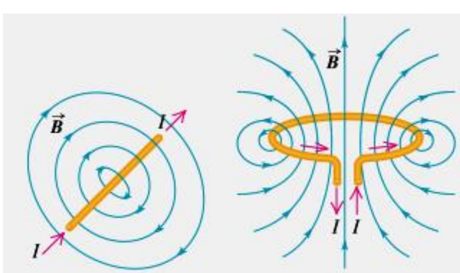
Magnetic Units and Field Line Examples

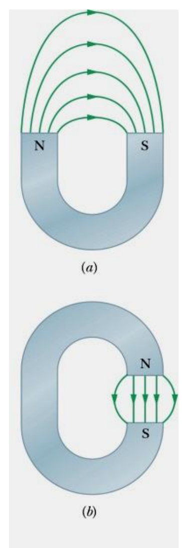
- SI unit of magnetic field: tesla (T)
  - 1T = 1 N/[Cm/s] = 1 N/[Am] = 10^4 gauss

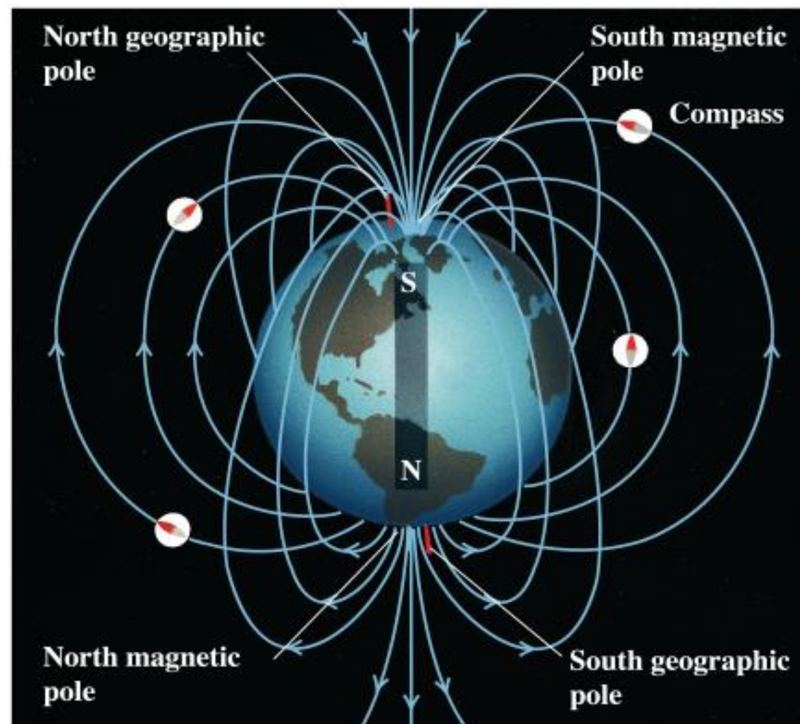
- Magnetic field lines – interpret similarly to E
  - The tangent to a magnetic field line at any point gives the direction of B at that point;
  - The spacing of the lines represents the magnitude of B – the magnetic field is stronger where the lines are closer together, and conversely.

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Inside sunspot	10^{-1} T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T



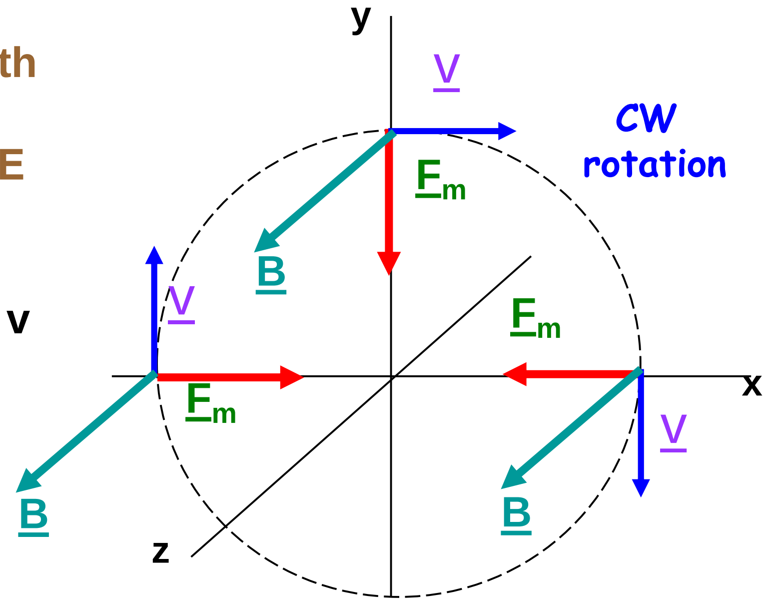






Charged Particles Circle at Constant Speed in a Uniform Magnetic Field: Cyclotron Frequency

- Uniform B in z direction, v in x-y plane tangent to path
- F_B is normal to both v & B ... so Power = F.v = 0.
- Magnetic force can not change particle's speed or KE
- F_B is a centripetal force, motion is UCM
- A charged particle moving in a plane perpendicular to a B field circles in the plane with constant speed v



Set $$F_B = qvB = \frac{mv^2}{r}$$

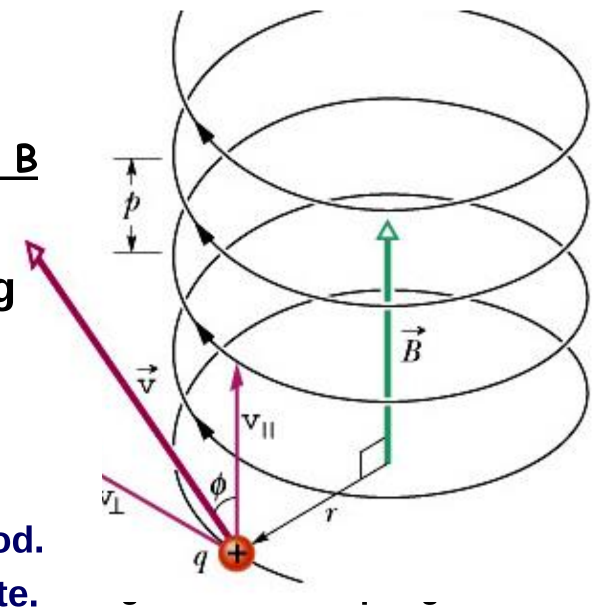
Radius of the path $$r = \frac{mv}{qB}$$

Period $$\tau_c = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Cyclotron angular frequency $$\omega_c \equiv \frac{2\pi}{\tau_c} = \frac{qB}{m}$$

If v not normal to B particle spirals around B

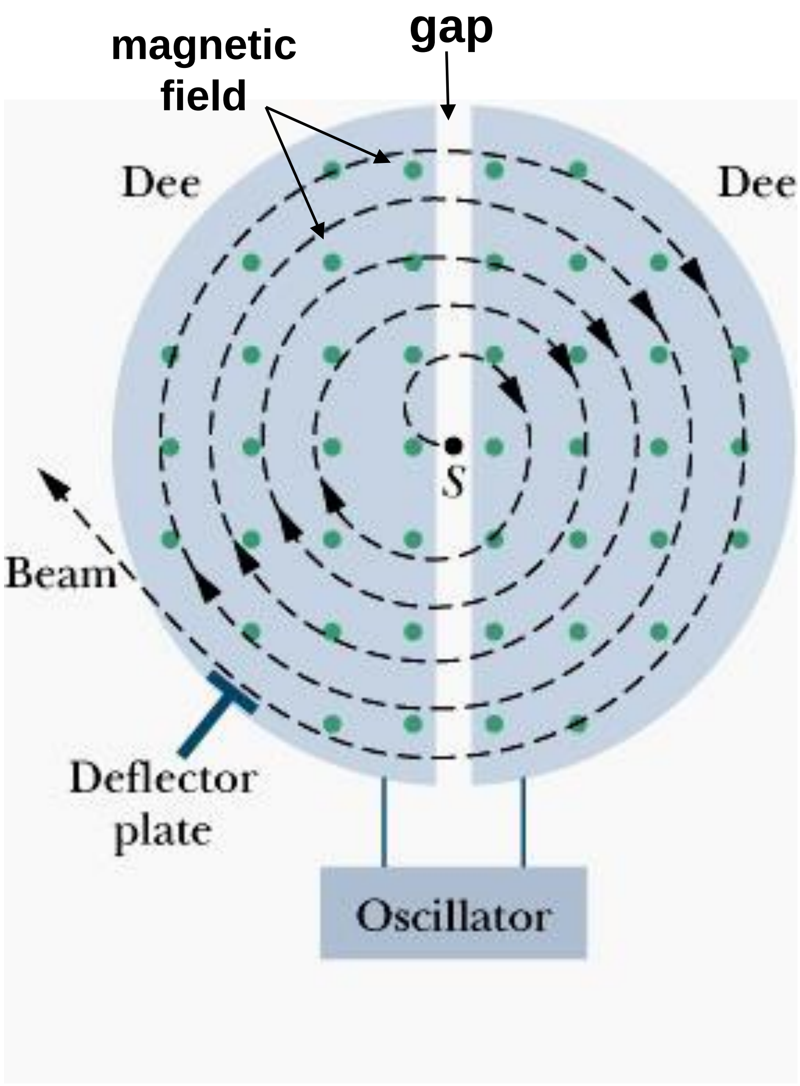
$\vec{v}_\perp$ causes circling
$\vec{v}_{para}$ is constant



- t and ω do not depend on velocity.
- Fast particles move in large circles and slow ones in small circles
- All particles with the same charge-to-mass ratio have the same period.
- The rotation direction for a positive and negative particles is opposite.

Cyclotron particle accelerator

Early nuclear physics research, Biomedical applications



- Inject charged particles in center
- Charged “Dees” reverse polarity as particles cross gap to accelerate them.
- Particles spiral out in magnetic field as they gain KE and are detected

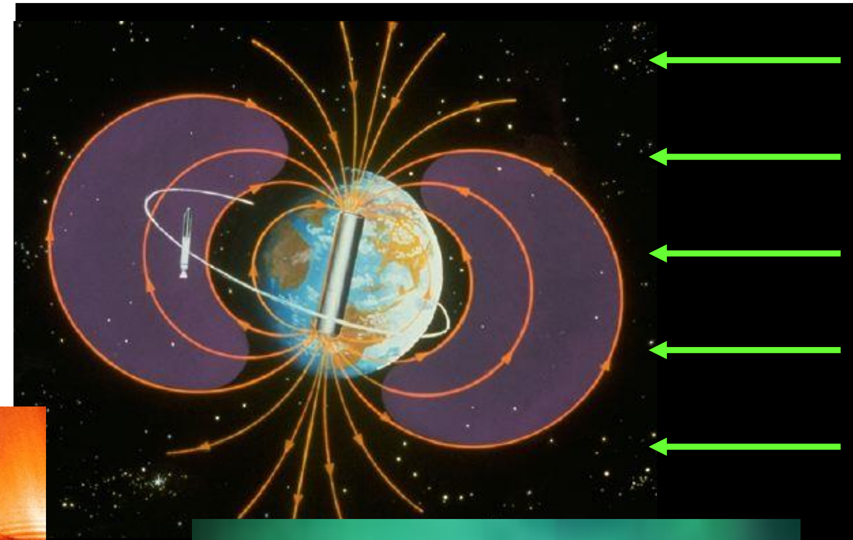
$$r = \frac{mv}{qB}$$

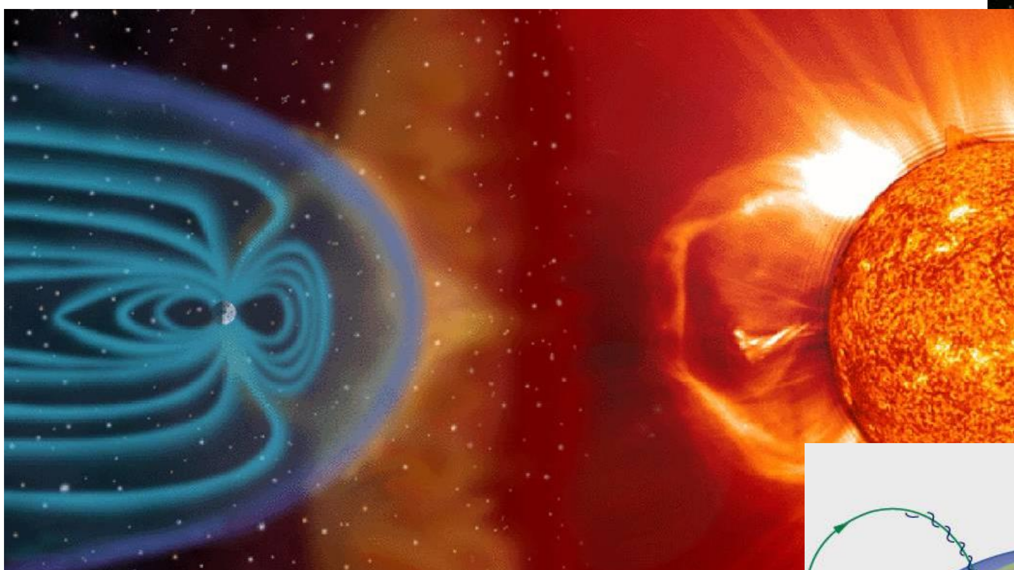
- Frequency of polarity reversal needed does **not** depend on speed or radius of path - it can be constant!

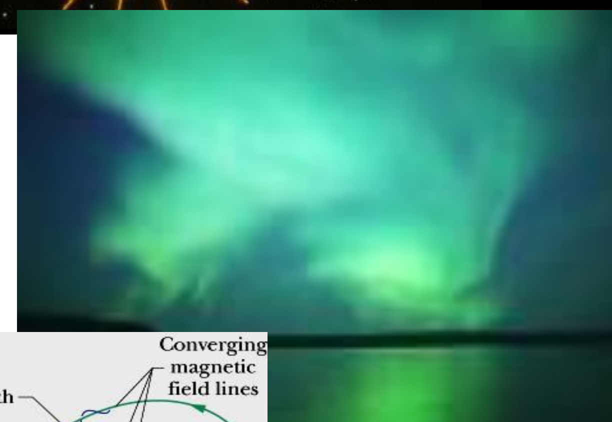
$$\omega_c = \frac{2\pi}{\tau_c} = \frac{qB}{m}$$

Earth's field shields us from the Solar Wind and produces the Aurora

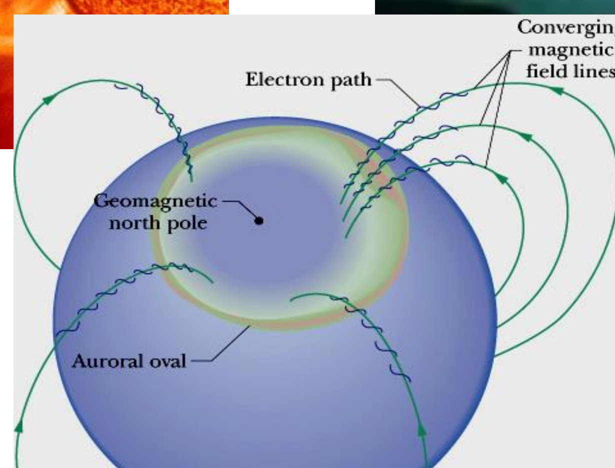
Earth's magnetic field deflects charged solar wind particles (cyclotron effect) protecting the Earth and making life possible (magnetosphere).





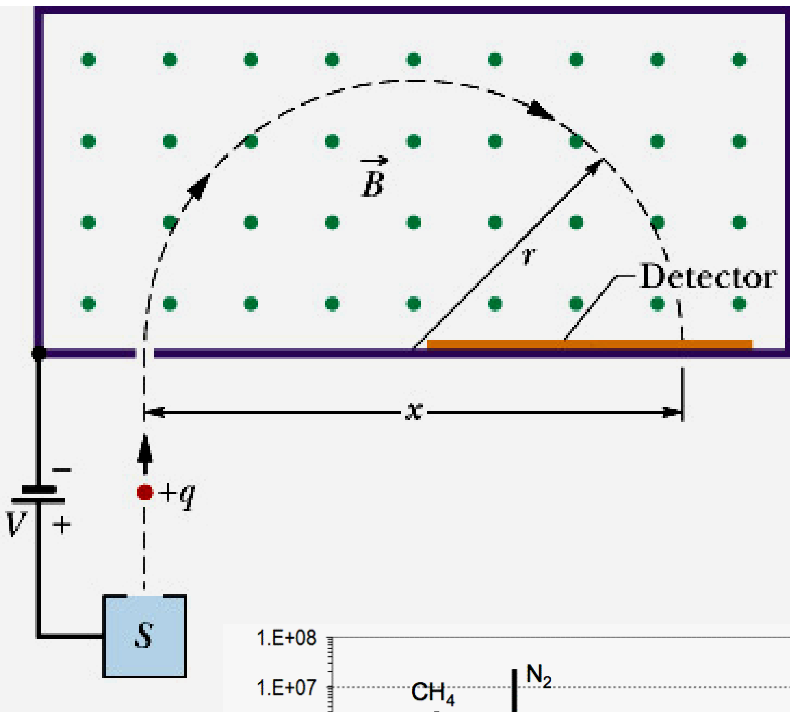


Some solar wind particles spiral around the Earth's magnetic field lines producing the Aurora at high latitudes. They can also be trapped.



Another cyclotron effect device: Mass spectrometer

Separates particles with different charge/mass ratios



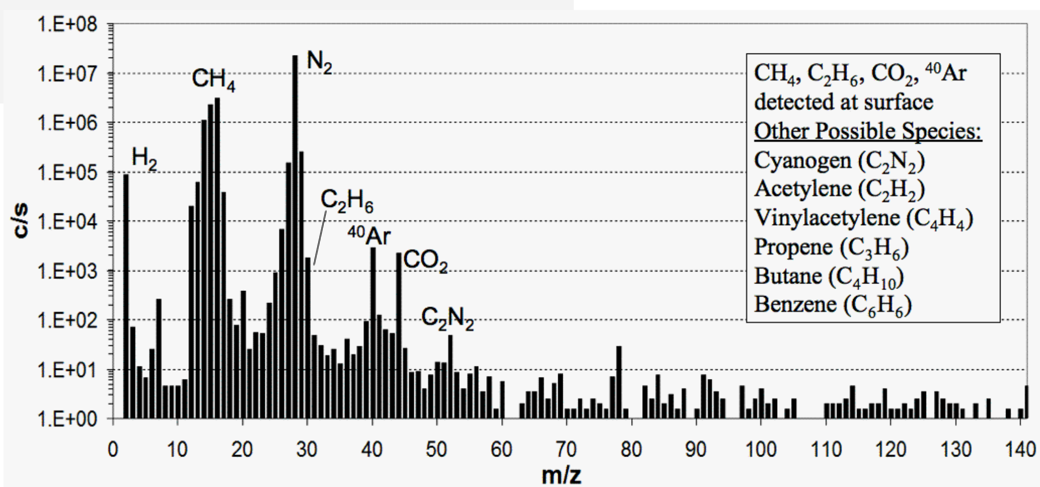
$$v = \sqrt{\frac{2qV}{m}}$$

ionized atoms or molecules accelerated through potential V

$$r = \frac{mv}{qB}$$

ionized isotopes or molecules are separated, forming a "spectrum

$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}} \Rightarrow \boxed{\frac{q}{m} = \frac{2V}{B^2 r^2}}$$

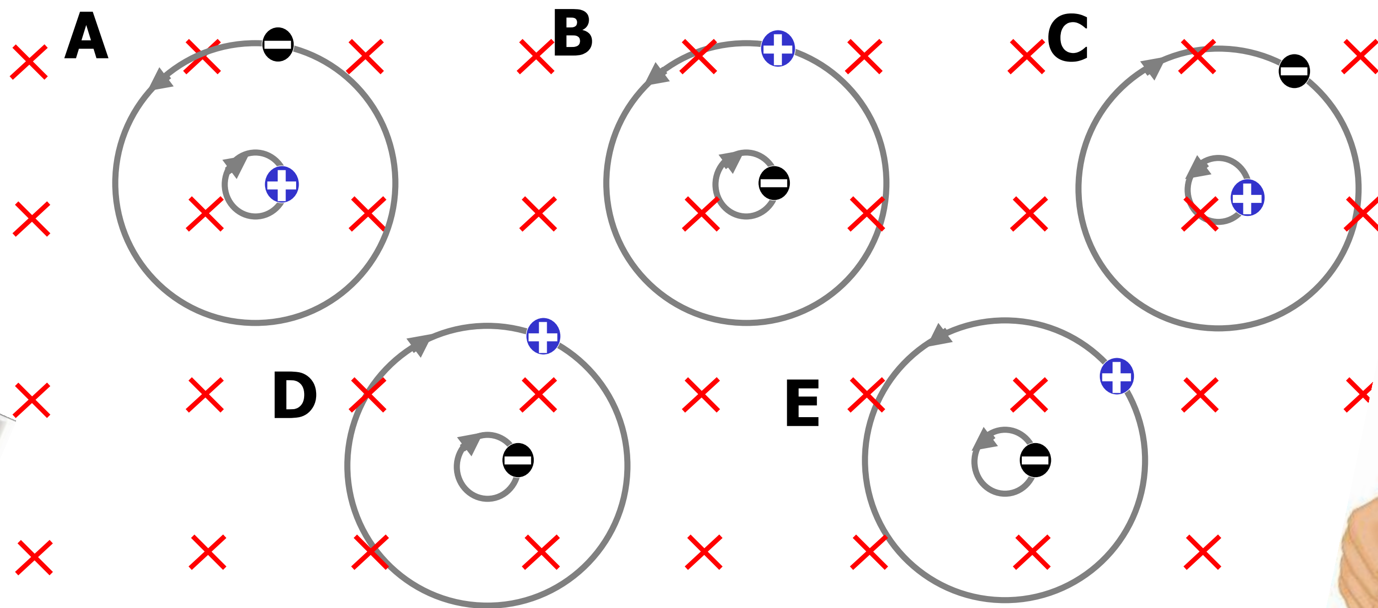


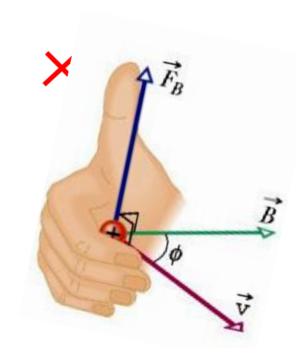
Mass spectrogram for the surface of Titan. Relative abundances are plotted as functions of the ratio of mass m to charge z.

Circulating Charged Particle

9-3: The figures show circular paths of two particles having the same speed in a uniform magnetic field B, which is directed into the page. One particle is a proton; the other is an electron (much less massive). Which figure is physically reasonable?

$$r = \frac{mv}{qB}$$





Charged particle in both E and B fields

Add the electrostatic and magnetic forces

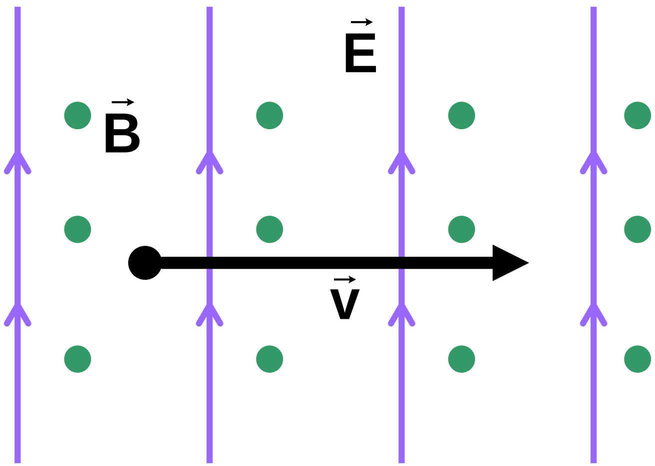
$$\vec{F}_{tot} = q\vec{E} + q\vec{v}\times\vec{B}$$

$\vec{F}_{tot} = m\vec{a}$ also

Does F = ma change if you observe this while moving at constant velocity v'?

Example: Crossed E and B fields

CONVENTION: ● OUT, ✕ IN

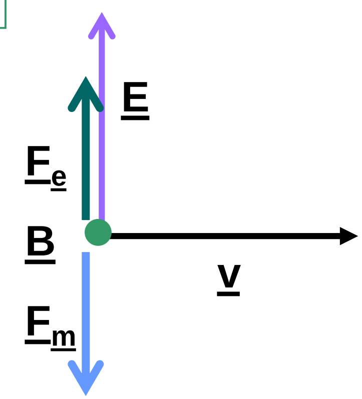


- B out of paper
- E up and normal to B
- + charge
- v normal to both E & B

$$\vec{F}_e = q\vec{E} \quad \text{(up)}$$

$$\vec{F}_m = q\vec{v}\times\vec{B} \quad \text{(down)}$$

FBD of q



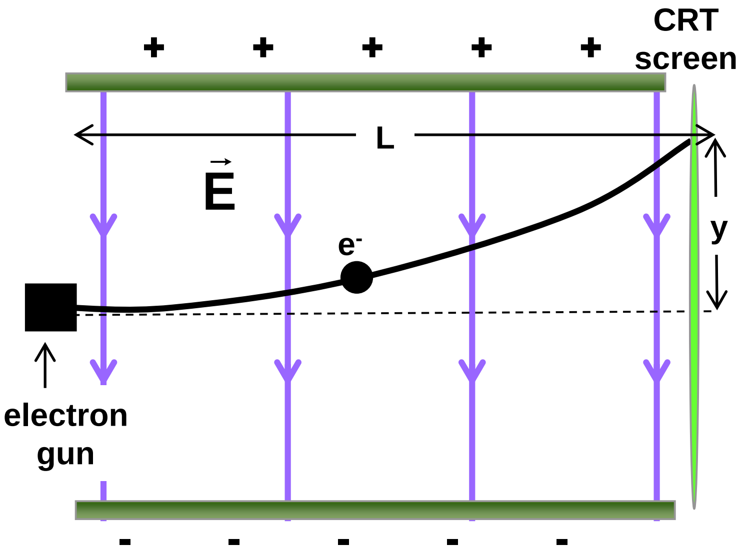
OPPOSED FORCES

Equilibrium when...

$$\vec{F}_{tot} = 0 \;\Rightarrow\; qE = qvB \;\Rightarrow\; v = |E|/|B|$$

- Independent of charge q
- Charges move through fields un-deflected
- Use to select particles with a particular velocity

Measuring e/m ratio for the electron (J. J. Thompson, 1897)



First: . Apply E field only in –y direction
. y = deflection of beam from center of screen (position for E = 0)

$$F_y = qE = ma_y \quad q = -e \quad \Rightarrow \quad \frac{e}{m} = \frac{a_y}{E}$$

measure $y = \frac{1}{2} a_y t^2$

notethat $t = L / v_x$

Next: . Find a_y by measuring y and flight time $t = L / v_x$ (constant)

Use crossed B and E fields to measure v_x

Let B point into the page, perpendicular to both E and Vx

F_M points along –y, opposite to FE (negative charge)

$$\vec{F}_M = -e\vec{v} \times \vec{B} \quad \vec{F}_E = -e\vec{E} \quad \Rightarrow \quad \text{at equilibrium} \quad v_x = E / B$$

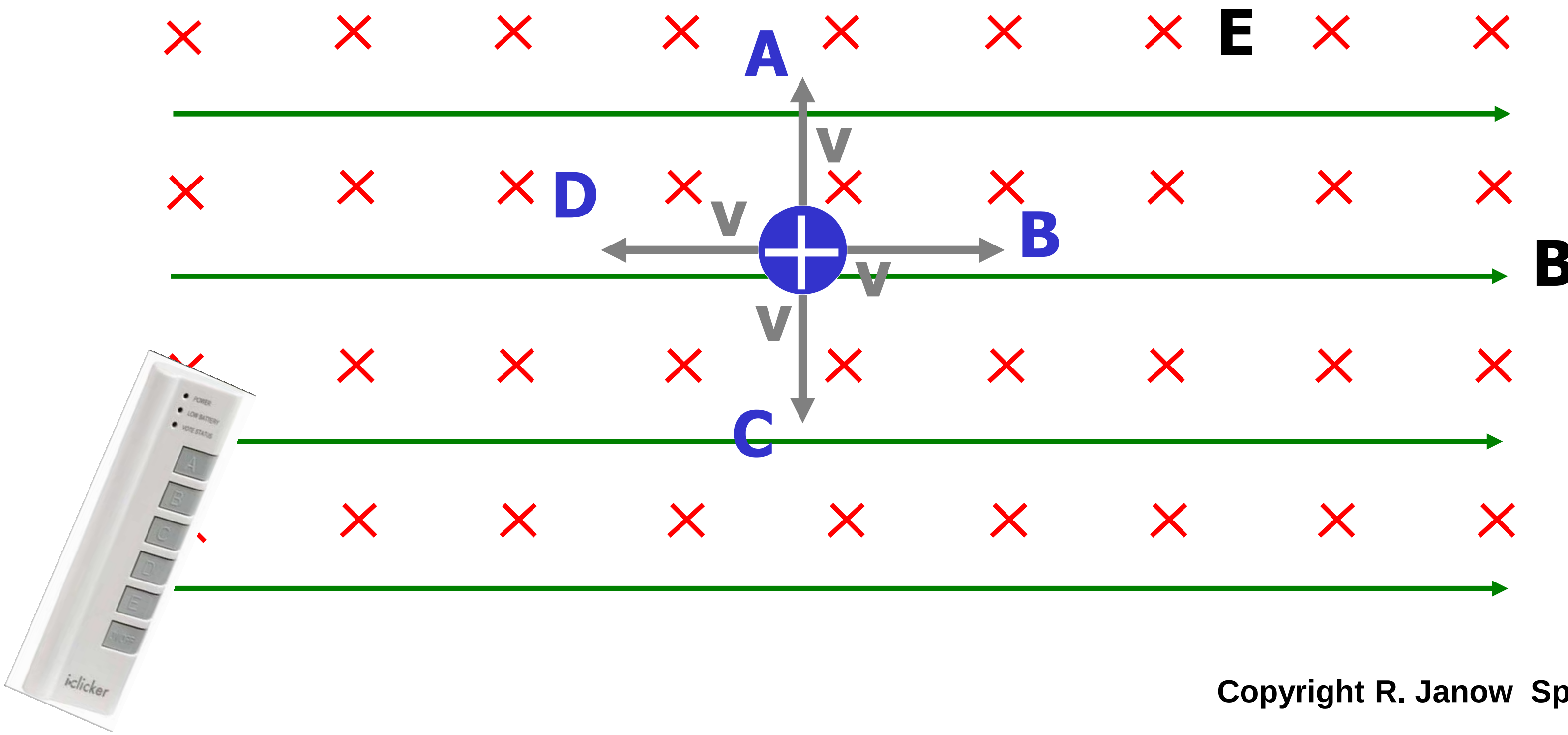
Adjust B until beam deflection = 0 (F_E cancels F_M) to find v_x and time t

$$t = \frac{BL}{E} \quad \Rightarrow \quad a_y = \frac{2y}{t^2} = 2y\frac{E^2}{B^2L^2} \quad \Rightarrow \quad \frac{e}{m} = 2y\frac{E}{B^2L^2} = 1.76 \times 10^{11} \text{ C/kg}$$

Force due to crossed E and B fields

9-4: The figure shows four directions for the velocity vector v of a positively charged particle moving through a uniform electric field E (into the page) and a uniform magnetic field B (pointing to the right). The speed is E/B.

Which direction of velocity produces the greatest magnitude of the net force?

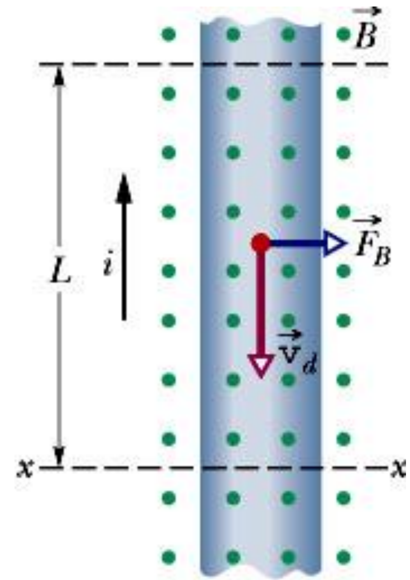


Force on a straight wire carrying current in a B field

Free electrons (negative):

- Drift velocity v_d is opposite to current (along wire)
- Lorentz force on an electron = $-e\underline{v}_d \times \underline{B}$ (normal to wire)

Motor effect: wire is pushed or pulled by the charges



$$\vec{v}_d = -|v_d|\,\hat{n} \qquad \hat{n} \equiv \text{unit vector along current}$$

$$d\vec{F}_m \equiv \text{force due to charges in length } d\vec{x}$$

$$d\vec{F}_m = dq\,\vec{v}_d \times \vec{B}$$

dq is the charge moving in wire whose length $dx = -v_d dt$

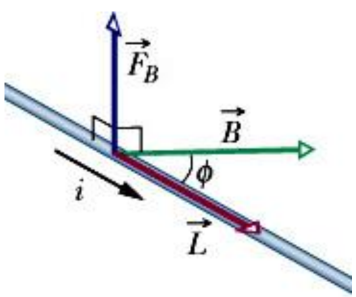
CONVENTION
● OUT ✕ IN

Recall: $dq = -i\,dt$, Note: d$\underline{x}$ is in direction of current (+ charges)

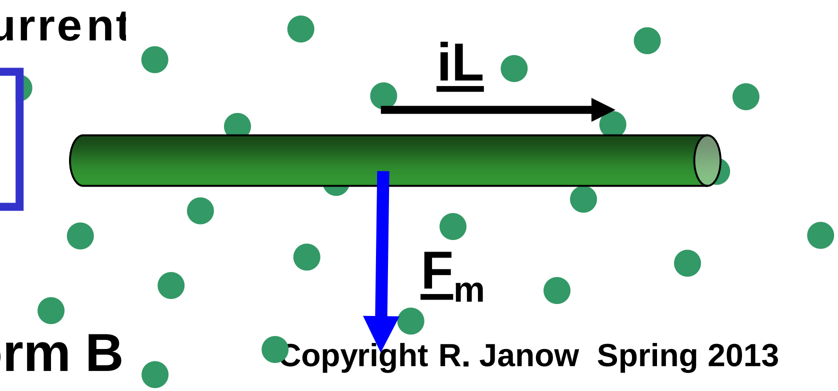
$$d\vec{F}_m = -i\,(\vec{v}_d dt) \times \vec{B} = i\,d\vec{x} \times \vec{B}$$

Integrate along the whole length L of the wire (assume $\underline{B}$ is constant)

$$\vec{L} = \text{length of wire, parallel to current}$$



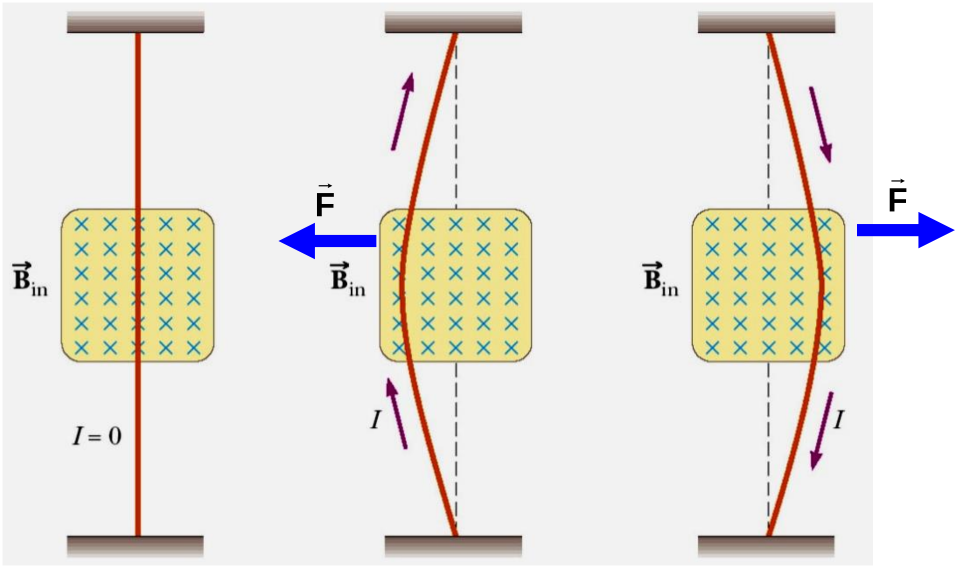
$$\vec{F}_m = i\,\vec{L} \times \vec{B}$$

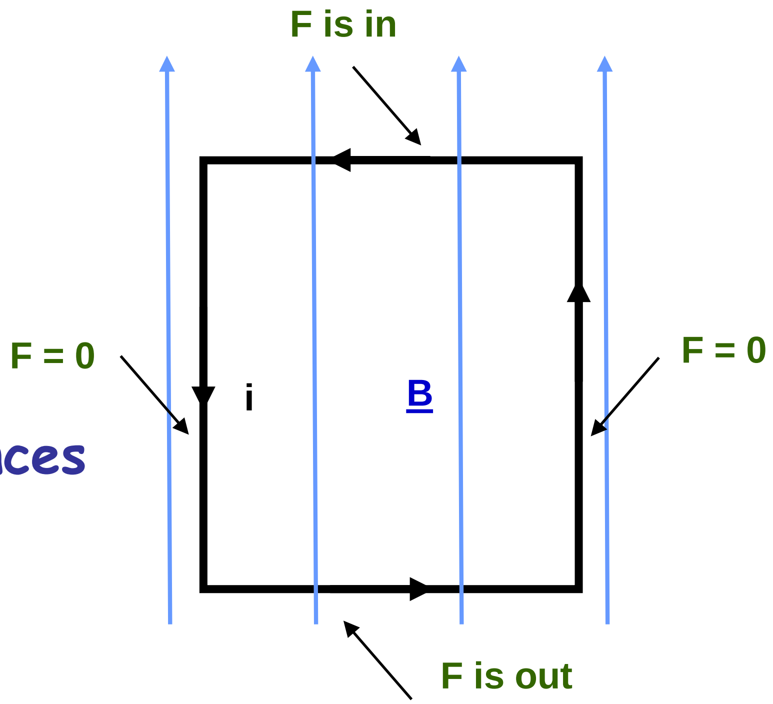


Motor effect on a wire: which direction is the pull?

$$\vec{F}_m = i\vec{L}\times\vec{B}$$

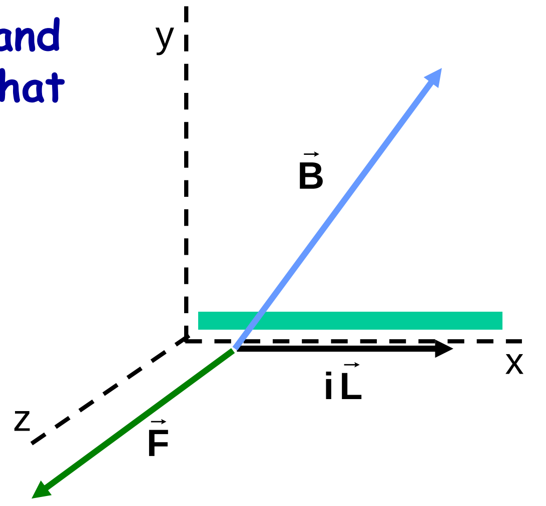
$i\vec{L}$ replaces $q\vec{v}$





A current-carrying loop experiences
A torque (but zero net force)

Example: A 3.0 A. current flows along +x in a wire 1.0 m long aligned parallel to the x-axis. Find the magnitude and direction of the magnetic force on the wire, assuming that the magnetic field is uniform and given by:



$$\vec{B} = 3.0\hat{i} + 5.0\hat{j} \text{ [Tesla]} \qquad i\vec{L} = 3.0 \times 1.0\,\hat{i} \text{ [m]}$$

$$\vec{F}_m = i\vec{L} \times \vec{B} = 3.0\text{x}1.0\hat{i} \text{ x } (3.0\hat{i} + 5.0\hat{j})$$

$$\hat{i} \text{ x } \hat{i} = 0 \qquad \hat{i} \text{ x } \hat{j} = \hat{k}$$

$$\therefore \quad \vec{F}_m = 15.0\,\hat{k} \text{ [N]}$$

Only the component of B perpendicular to the current-length contributes to the force

Example: Same as above, but now the field has a z-component

$$\vec{B} = 3.0\hat{i} + 5.0\hat{j} + 4.0\hat{k} \text{ [Tesla]} \qquad i\vec{L} = 3.0 \times 1.0\,\hat{i} \text{ [m]}$$

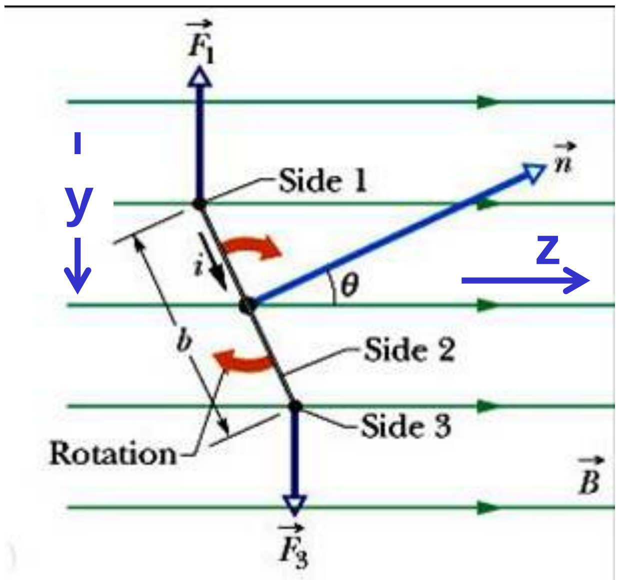
$$\vec{F}_m = i\vec{L} \times \vec{B} = 3.0\text{x}1.0\hat{i} \text{ x } (3.0\hat{i} + 5.0\hat{j} + 4.0\hat{k})$$

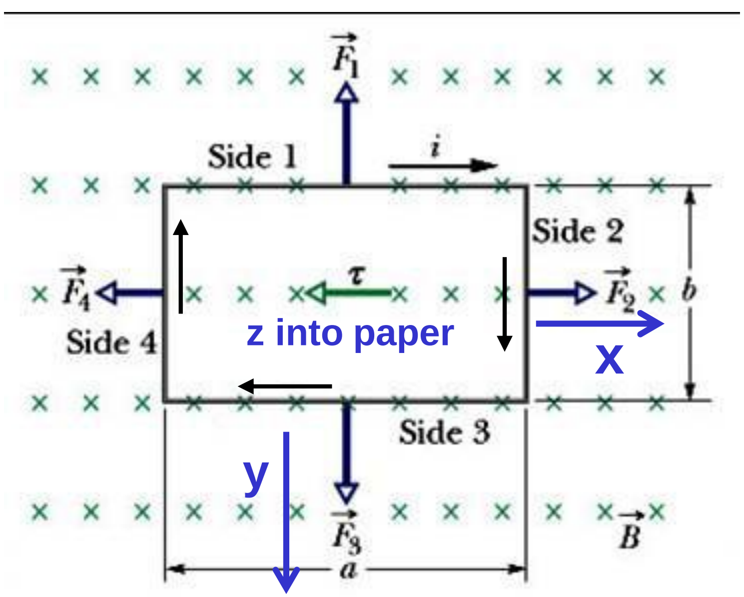
$$\hat{i} \text{ x } \hat{i} = 0 \qquad \hat{i} \text{ x } \hat{j} = \hat{k} \qquad \hat{i} \text{ x } \hat{k} = -\hat{j}$$

$$\therefore \quad \vec{F}_m = 15.0\,\hat{k} - 12.0\hat{j} \text{ [N]}$$

Whatever the direction of the field B, there can never be a force component along the current-length direction

Torque on a current-carrying wire loop in a magnetic field





- Upper sketch: x-axis toward viewer
- Loop can rotate about x-axis
  B field is along z axis

- Apply $\vec{F}_m = i\vec{L} \times \vec{B}$
  to each side of the loop

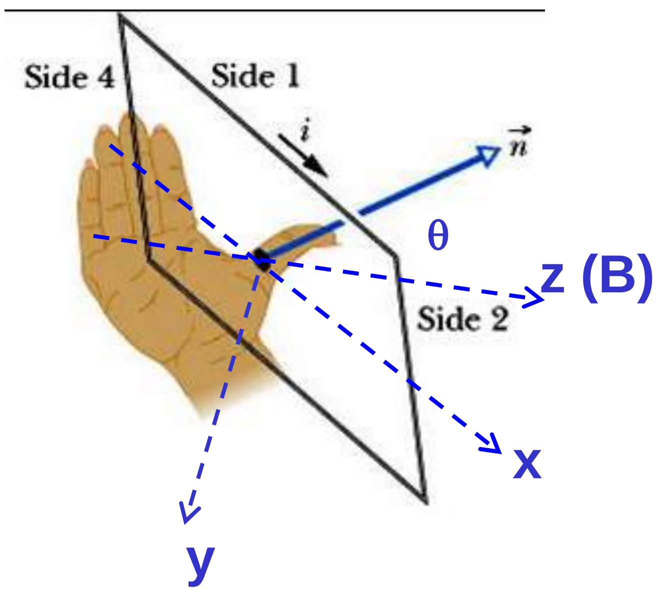
- $|F_1| = |F_3|$ = iaB: Forces cancel again
  but net torque is not zero!

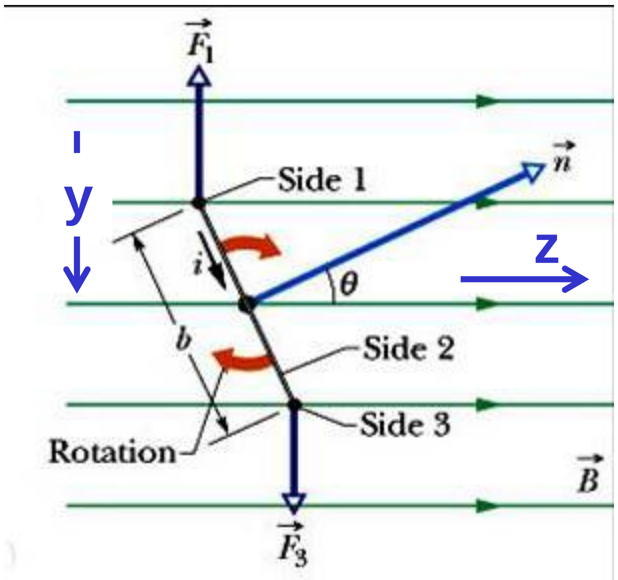
- Moment arms for F_1 & F_3 equal b.sin(q)/2
- Force F_1 produces CW torque equal to
  t_1 = i.a.B.b.sin(q)/2
- Same for F_3
- Torque vector is down into paper along –x rotation axis

$$\text{Net torque } \tau = iabB\sin(\theta)$$

- Lower sketch:
- $|F_2| = |F_4|$ = i.b.Bcos(θ): Forces cancel, same line of action → zero torque

Torque on a current-carrying loop: Motor Effect





torque $\tau = iabB\sin(\theta)$ → $\vec{\tau} = i\,A\,\hat{n}\times\vec{B}$ $A \equiv$ area of loop $= ab$

Maximum torque: $\tau_{max} = iAB \quad \hat{n}\perp\vec{B}$

Sinusoidal variation: $\tau(\theta) = \tau_{max}\sin(\theta)$

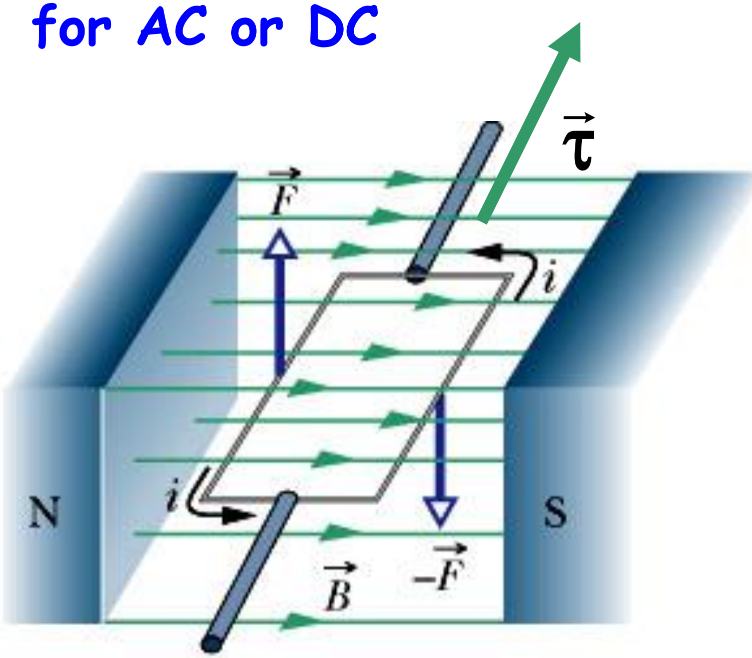
Stable when $\hat{n}$ parallels $\vec{B}$

Restoring torque → oscillations

Field PRODUCED by a current loop is a magnetic dipole field

ELECTRIC MOTOR:

- Reverse current I when torque τ changes sign (θ = 0)
- Use mechanical "commutator" for AC or DC



- Multiple turns of wire increase torque
- N turns assumed in flat, planar coil
- Real motors use multiple coils (smooth torque)

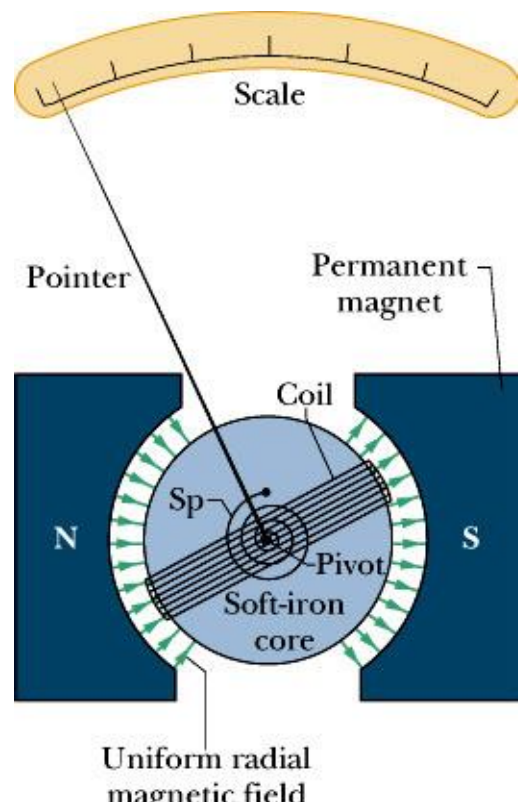
$$\vec{\tau} = NiA\,\hat{n}\times\vec{B} \quad A \equiv \text{area of loop} = ab$$

or

$$\tau = NiA\,B\sin(\theta)$$

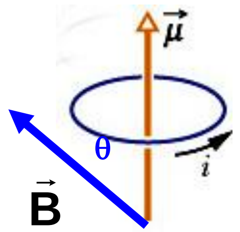
MOVING COIL GALVANOMETER

- Basis of most 19th & 20th century analog instruments: voltmeter, ohmmeter, ammeter, speedometer, gas gauge, ….
- Coil spring calibrated to balance torque at proper mark on scale



Current loops are basic magnetic dipoles

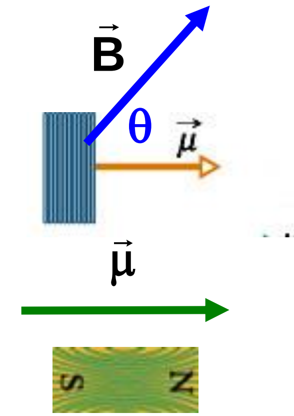
Represent loop as a vector



$$\text{Magnetic dipole moment} \equiv \vec{\mu} \equiv \text{N i A} \hat{n}$$

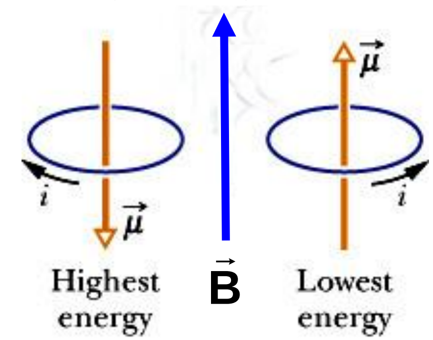
$$\text{Dimensions } [\mu] = \text{ampere-m}^2 = \frac{\text{Newton-m}}{\text{Tesla}} = \frac{\text{Joule}}{\text{Tesla}}$$

N = number of turns in the loop



$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad A \equiv \text{area of loop} = ab$$
$$\tau = \mu B \sin(\theta)$$

$$U_M = \int \tau d\theta = \int \mu B \sin(\theta) d\theta$$



Magnetic dipole moment $\underline{\mu}$ measures

- **strength of response to external $\underline{B}$ field**
- **strength of loop as source of a dipole field**

	ELECTRIC DIPOLE	MAGNETIC DIPOLE
MOMENT	$\lvert p \rvert \equiv qd$	$\lvert \mu \rvert \equiv NiA$
TORQUE	$\vec{\tau}_e = \vec{p} \times \vec{E}$	$\vec{\tau}_m = \vec{\mu} \times \vec{B}$
POTENTIAL ENERGY	$U_e = -\vec{p} \circ \vec{E}$	$U_m = -\vec{\mu} \circ \vec{B}$

SAMPLE VALUES [J / T]	
Small bar magnet	m ~ 5 J/T
Earth	m ~ 8.0×10^{22} J/T
Proton (intrinsic)	m ~ 1.4×10^{-26} J/T
Electron (intrinsic)	m ~ 9.3×10^{-24} J/T

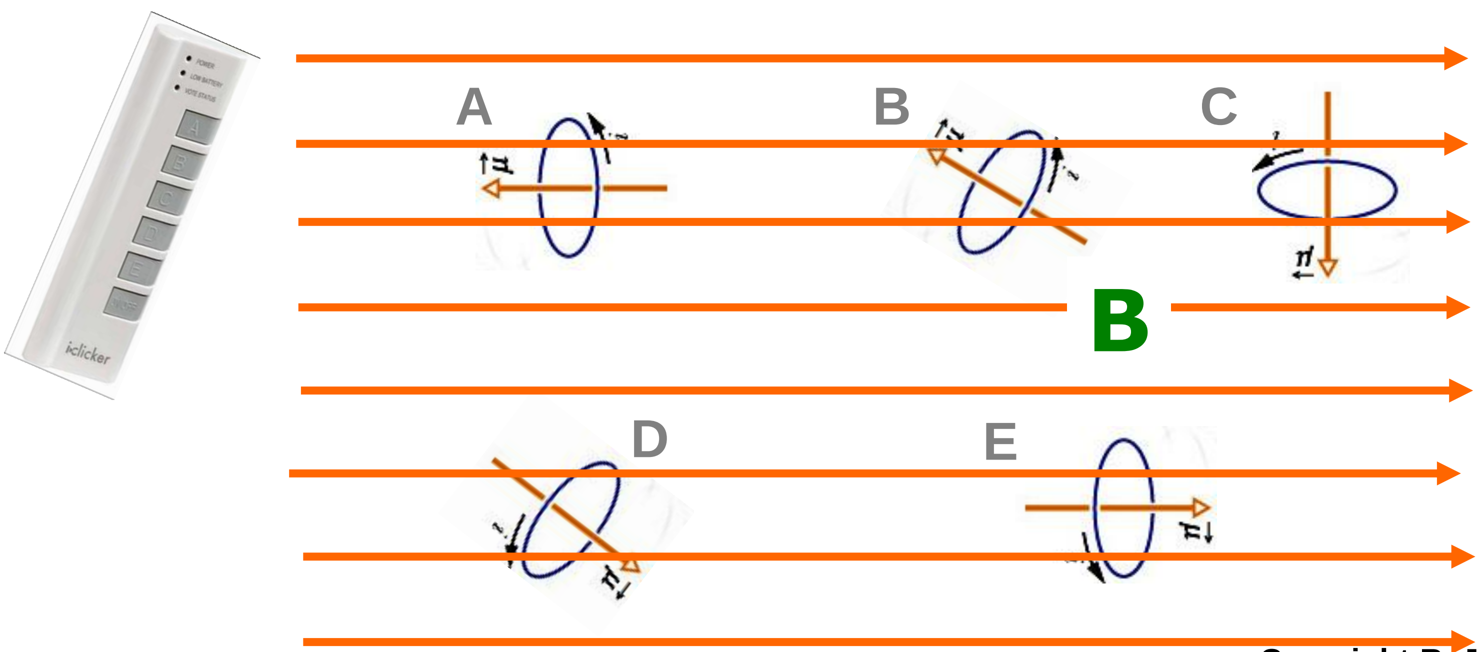
Torque and potential energy of a dipole

9-5: In which configuration is the torque on the dipole at it's maximum value?

$$\vec{\tau}_m = \vec{\mu} \times \vec{B}$$

9-6: In which configuration is the potential energy of the dipole at it's smallest value?

$$U_m = -\vec{\mu} \circ \vec{B}$$

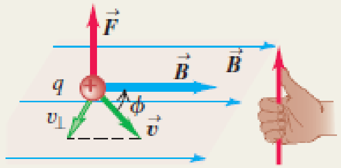


Summary: Lecture 9 Chapter 28 – Magnetic Fields

CHAPTER 27 SUMMARY

Magnetic forces: Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is the tesla ($1\ \text{T} = 1\ \text{N/A}\cdot\text{m}$). (See Example 27.1.)

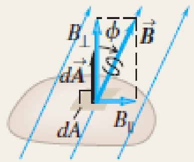
$$\vec{F} = q\vec{v} \times \vec{B} \quad (27.2)$$



Magnetic field lines and flux: A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together the field magnitude is large, and vice versa. Magnetic flux Φ_B through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is the weber ($1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2$). The net magnetic flux through any closed surface is zero (Gauss's law for magnetism). As a result, magnetic field lines always close on themselves. (See Example 27.2.)

$$\Phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A} \quad (27.6)$$

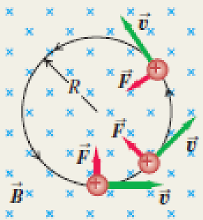
$$\oint \vec{B}\cdot d\vec{A} = 0 \text{ (closed surface)} \quad (27.8)$$



Motion in a magnetic field: The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B and the particle mass m, speed v, and charge q. (See Examples 27.3 and 27.4.)

Crossed electric and magnetic fields can be used as a velocity selector. The electric and magnetic forces exactly cancel when $v = E/B$. (See Examples 27.5 and 27.6.)

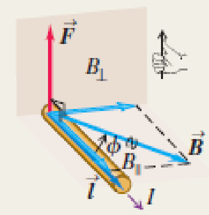
$$R = \frac{mv}{|q|B} \quad (27.11)$$



Magnetic force on a conductor: A straight segment of a conductor carrying current I in a uniform magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{B}$ and the vector $\vec{l}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current-carrying segment $d\vec{l}$. (See Examples 27.7 and 27.8.)

$$\vec{F} = I\vec{l} \times \vec{B} \qquad (27.19)$$

$$d\vec{F} = I\,d\vec{l} \times \vec{B} \qquad (27.20)$$



Magnetic torque: A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can the potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop. (See Examples 27.9 and 27.10.)

$$\tau = IBA\sin\phi \qquad (27.23)$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad (27.26)$$

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi \qquad (27.27)$$