Math 337 —-FINAL EXAM —-December 15, 2010

Show all work and justify all steps of each argument you make.

1)(20 points) Let $A = [v_1 v_2 v_3]$ with $v_1 = (2, -1, 1)^T$, $v_2 = (0, 8, -2)^T$ and $v_3 = (6, 5, 1)^T$ and $b = (10, 3, 3)^T$.

a) Are the columns of A linearly independent? Is A invertible?

b) Find the general solution of $Ax = b$.

c) What is a basis and the dimension of Col(A)? Is b in Col(A)?.

d) What is the rank of A and the dimension of the null space of A?

2) (15 points) Let $u = (-1, 1, 1)^T$, $v = (3, -1, 2)^T$ and $V = span\{v\}$.

a)Find the projection $w = \text{proj}_V u$ of u onto V.

b) Write u as the sum of w and a vector orthogonal to v.

c) Find the distance from u to V.

3) (20 points) Let $A = [x_1 x_2 x_3]$ with $x_1 = (1, -1, -1)^T$, $x_2 = (0, 3, 3)^T$ and $x_3 = (3, 2, 4)^T$.

a) Use the Gram-Schmidt process to find an orthogonal basis for $V = Col(A)$.

b) Find the QR factorization of A. Express R in terms of Q and A, but don't compute it.

c) Use part b) to determine if A is invertible (no computation is required).

4) (20) Let $A = [v_1 v_2 v_3]$ with $v_1 = (7, -3, 2)^T$, $v_2 = (1, 3, 2)^T$ and $v_3 = (-2, 6, 2)^T$.

a) Find the eigenvalues eigenvalue(s) of A.

b) Find bases for the corresponding eigenspaces.

c) Diagonalize A (i.e., write it as $A = PDP^{-1}$). Do not compute P^{-1}. Use it to find det(A).

5) (25 points) a) Write down the matrix A corresponding to the quadratic form $Q(x) = 5x_1^2 + 4x_1x_2 + 2x_2^2$.

b) Is Q positive definite, negative definite, or indefinite?

c) Orthogonally diagonalize A and find A^{10}.

d) Find the change of variable $x = Py$ that transforms Q into a quadratic form with just square terms. Compute this new form.