Math 337 —-Midterm Exam 1—-Fall 2016

1. (20 points) Let $v_1 = (1, 1, -1)^T, v_2 = (2, 3, -4)^T, v_3 = (1, 0, 1)^T$, and $A = [v_1 v_2 v_3]$.

a. Describe the set of all vectors $b = (b_1, b_2, b_3)^T$ for which the system $Ax = b$ has a solution.

b) Solve $Ax = (1, 1, -1)^T$. Indicate free and basic cariables.

c) Is $b = (1, 1, -1)^T$ in the span of the of the columns of A. If yes, write it as a linear combination of these columns. Is this representation unique. Explain.

d) Are the columns of A linearly independent? Explain.

2.(20 points) Let $v_1 = (1, 3, 4)^T, v_2 = (-1, -2, -3)^T, v_3 = v_4 = (-2, -2, -4)^T$, $A = [v_1 v_2 v_3 v_4]$ and $b = (3, -1, 2)^T$.

a) Find the general solution of $Ax = b$ in the form $x = p + x_h$. Indicate basic and free variables.

b) Give a particular (one) solution of Ax=b.

c) Write the solution set of the homogeneus equation Ax=0.

3. (20 points) Let $T : R^4 \to R^2$ and $S : R^2 \to R^3$ be given by $T(x_1, x_2, x_3, x_4) = (x_1 - x_2 + x_3 + x_4, x_1 + x_2 + x_3 - x_4)$ and $S(x_1, x_2) = (-x_1 + x_2, x_1 - 2x_2, 2x_1 - x_2)$.

a) Show that T, S and ST are linear transformations and find their standard matrices. Can you compute TS ?

b) Is $b = (1, 1, 1)^T$ the image of some v under ST? Justify your answer.

c) Is S one to one? onto? Justify your answer.

d) (5 points-extra credit) Show that a linear transformation $L : R^n \to R^n$ is one-to one if and only if it is onto.

4. (20 points) Let $v_1 = (1, 0, -3)^T, v_2 = (0, 1, 4)^T, v_3 = (1, 3, 8)^T$, $A = [v_1 v_2 v_3]$ and $b = (1, 0, 1)^T$.

a) Use row operations to find the inverse of A.

b) Use the inverse to solve the system Ax=b.

c) Find the inverse of $A^{-1}A^T$.

d) Is $A - A^T$ invertible? Justify your answer.

5. (20 points) a) (10 points) Compute the LU decomposition of $A = [v_1 v_2 v_3]$, where $v_1 = (2, 4, -6)^T, v_2 = (1, 1, -2)^T, v_3 = (3, 7, -12)^T$.

b) (10 points) Let the LU decomposition of A be $L = [(1, -1, 4)^T (0, 1, 2)^T (0, 0, 1)^T]$ and $U = [(1, 0, 0)^T (-3, 2, 0)^T (4, 1, 5)^T]$. Use this LU decomposition to solve the system Ax=$(12, -12, 58)^T$ (hint:Solve Ly=b and then Ux=y).