Math 337 —-Midterm Exam 1—-Spring 2016

Give explantions for each answer.

1. (20 points) Let $v_1 = (2, 1, -1)^T$, $v_2 = (-6, -3, 3)^T$, $v_3 = (-1, -1, -1)^T$, $v_4 = (8, 6, 2)^T$ and $A = [v_1 v_2 v_3 v_4]$

a. Is the system $Ax = b$ consistent for all $b = (b_1, b_2, b_3)^T$ in R^3?

b) When consistent, does it have a unique solution? Explain.

c) Can each b in R^3 be written as a linear combination of the columns of A. Explain.

2.(20 points) a) For A given in problem 1), find the general solution of $Ax = (7, 6, 4)^T$ in the form $x = p + x_h$. Indicate basic and free variables.

b) Are the columns of A linearly independent? Explain.

c) Is the linear transformation $T : R^4 \to R^3$ given by $Tx = Ax$ one-to-one? Explain.

3. (20 points) Let $A = [(1, 2, 0)^T (0, -1, 1)^T (1, 0, h)^T]$.

a) Find all value(s) of h for which A is invertible.

b)For $h = 1$, find the inverse of A, and of AA^T.

c) For $h = 1$, is the system $Ax = b$ (uniquely) solvable? If yes, find the solution of $Ax = (1, 1, 1)^T$

4. (20 points) Let $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 + x_2 - 2x_3, 4x_3)$.

a) Show that $T : R^n \to R^m$ is a linear transformation for some m and n and find its standard matrix.

b) Is T one-to-one and onto? Explain.

c) Is $b = (1, 0, 1, 2)^T$ in the range of T? Explain.

5. (20 points) a) Find the standard matrices for the linear transformations T, S,R and RST mapping R^2 into R^2, where T is the reflection through the x_2-axis, S is the rotation by $\pi/2$ and R is the reflection through the line $x_2 = -x_1$-axis.

b) Does the order matter, i.e, is RST=SRT? Explain.

c) Find the images of $v = (1, 2)^T$ under RST and SRT.