Math 337 —-EXAM 2—-Fall 2015

Provide complete explanations for your answers.

1) (25 points) Let $A = [v_1, v_2, v_3]$ with the columns $v_1 = (1, 2, -1)^T$, $v_2 = (3, 8, 3)^T$, $v_3 = (-1, 4, 4)^T$,

a) Find the LU factorisation of A and check explicitely that LU=A.

b) Use part a) to find det(A^2) and det (U^{10}).

c) Use A=LU to find a solution x of $Ax = (-4, 2, 4)^T$. Is this equation uniquely solvable?

2) (25 points) Let $A = [v_1 v_2 v_3 v_4 v_5]$ with the columns are $v_1 = (-1, 0, 0, 0, 1)^T$, $v_2 = (1, 0, 01, -1)^T$, $v_3 = (1, -1, 1, 1, 1)^T$, $v_4 = (0, 0, -1, 0, 1)^T$ and $v_5 = (1, 0, 0, 1, 0)^T$.

a) Compute det(A) by the cofactor expansion.

b) Compute det(A) by reduction to echelon form.

c) Compute the determinants $|3A^2|$, $|2A^{-1}|$, and $|3(A^T)^{-1}|$. Is A invertible? Explain.

3) (25 points) Let $A = [v_1 v_2 v_3 v_4]$ with the columns are $v_1 = (1, 3, 1)^T$, $v_2 = (2, 6, 2)^T$, $v_3 = (-2, -5, 0)^T$, $v_4 = (1, 4, 3)^T$.

a) Find bases and dimensions of Null(A), Col(A) and Row(A) and rank (A).

b) Is $x = (1, 0, 1, 1)^T$ in the Null(A)?

c) What is a basis for the row space of A^T? (No calculation is needed).

4) (10 point) a) Let $A = -A^T$ and of 5×5. Show that det (A)=0.

b) Let A be a 4×6 matrix with dim null(A)=2. Find dim col (A) and dim row (A).

5) (15 points) a) Is the set of matrices $H = [v_1, v_2, v_3]$ with $v_1 = (a, 0, a)^T$, $v_2 = (0, a, 0)^T$ and $v_3 = (a, 0, a)^T$, with a any real number, a subspace of the space of all 3×3 matrices? Explain.

b) Is the set of noninvertible 2×2 matrices H a subspace of the space of all 2×2 matrices? Explain.

c) Is the set H of all continous functions $f \in C[0, 1]$ with $\int_0^1 f(x)dx = 0$ a subspace of $C[0, 1]$? Explain.