

**Math 337 —Midterm Exam 2—Fall 2016**

Provide complete explanations for your answers.

- 1) (20 points) Let  $A = [(0, -1, 8)^T, (1, 3, -1)^T, (-2, 0, 3)^T, (2, 1, 5)^T, (0, 6, 1)^T]$  and  $U = [(1, 0, 0)^T, (0, 1, 0)^T, (0, 0, 1)^T, (1, 0, 1)^T, (-6, 2, 1)^T]$  be the reduced echelon form of A.
- Find bases and dimensions for  $\text{Nul}(A)$ ,  $\text{Col}(A)$  and  $\text{Row}(A)$ .
  - Do the columns of A span  $R^3$ ? Explain.
- 2) (25 points) Let  $A = [(-2, 8, 0, 0, 0)^T, (-1, 4, 1, 0, 0)^T, (0, -3, 5, 2, 0)^T, (0, 0, 0, 1, 4)^T, (0, 0, 0, -1, 2)^T]$ .
- Compute  $\det(A)$  by the cofactor expansion.
  - Compute  $\det(A)$  by reduction to echelon form.
  - Compute  $|2A^2|$ ,  $|3A^{-1}|$ , and  $|2(A^T)^{-1}|$ . Is A invertible? Explain.
- 3) (20 points) a) Let A be n by n matrix with  $\det(2A^2A^T) = 8$ . Find  $\det(A)$ . What's the  $\text{rank}(A)$ ?
- Let A be an m by n matrix. Show that
    - $\dim \text{Nul}(A) + \dim \text{Row}(A) = n$ .
    - $\dim \text{Nul}(A^T) + \dim \text{Col}(A) = m$  (hint: use  $\text{Row}(A^T) = \text{Col}(A)$ ).
- 4) (20 points) a) Let S be a fixed 3 by 3 invertible matrix and  $H = \{ \text{all 3 by 3 matrices } A \mid S^{-1}AS \text{ is a diagonal matrix} \}$ . Is H a subspace of the vector space  $M_{3 \times 3}$  of all 3 by 3 matrices with the usual addition and scalar multiplication.
- Is the set  $D_{3 \times 3}$  of 3 by 3 diagonal matrices a subspace of  $M_{3 \times 3}$ . If yes, find its basis and show that it is a basis.
  - Is  $H = \{(x, y, z) \mid x^2 + 2y^2 - 3z^2 = 0\}$  a subspace of the vector space  $R^3$ ?
- 5) (15) Use Cramer's rule (and no other method) to find  $x_2$  of the solution of the system  $Ax = b$ , where  $A = [(3, -3, 6)^T, (-7, 5, -4)^T, (-2, 1, 0)^T]$  and  $b = (-7, 5, 2)^T$ .