Math 337 —EXAM 2—Spring 2016

Provide complete explanations for your answers.

1) (20 points) Let $A = [v_1, v_2, v_3]$ with the columns $v_1 = (2, 8, 8)^T$, $v_2 = (0, -3, 9)^T$, $v_3 = (-2, -5, -17)^T$.

   a) Find the LU factorisation of A and check explicitely that LU=A.

   b) Given that a matrix A has LU factorization with L=$[(1, -4, 0)^T, (0, 1, -3)^T, (0, 0, 1)^T]$ and U=$[(-4, 0, 0)^T, (3, 4, 0)^T, (1, 0, -4)^T]$, use this LU factorization to find a solution x of $Ax = (1, 4, -36)^T$.

2) (20 points) Let $A = [v_1 v_2 v_3 v_4]$ with the columns $v_1 = (1, 0, 5, 5)^T$, $v_2 = (4, 4, 4, 8)^T$, $v_3 = (3, 1, 5, 0)^T$ and $v_4 = (1, 0, 0, 0)^T$.

   a) Compute det(A) by the cofactor expansion.

   b) Compute det(A) by reduction to echelon form.

   c) Compute $\det(2A^2 A^T)$.

3) (25 points) Let $A = [v_1 v_2 v_3 v_4 v_5]$ with the columns $v_1 = (1, 2, 1)^T$, $v_2 = (2, 1, 5)^T$, $v_3 = (3, 3, 2)^T$, $v_4 = (0, 3, 5)^T$ and $v_5 = (1, -1, 4)^T$.

   a) Is $b = (1, 0, 1)^T$ in the span of the columns of A?

   b) Find bases and dimensions of Null(A), Col(A) and Row(A).

   c) Is $x = (1, 1, 1, 1, 1)^T$ in the Null(A)?

4) (15 points) Use Cramer's rule to find x_1 solution of the system $Ax = (0, 0, 4)^T$, where A=$[(-3, 0, 4)^T, (-1, 1, -2)^T, (0, -2, 0)^T]$.

5) (20 points) Which of the following subsets H are vector subspaces of the given vector spaces. Explain.

   a) $H = \{(x, y, z) \mid x = 2y^2 + 3z^3\}$ in R^3

   b) $H = \{f(t) \mid \int_0^2 f(t)dt = 0\}$ in the space of continuous functions $C([0, 2])$

   c) $H = \{$polynomials of the form $ax^3 + bx + 2 \mid a, b \in R\}$ in the space P_3 of all polynomials of degree ≤ 3.

   d) H is all 2x2 symmetric matrices, that is all A with $A^T = A$ in the space of all 2x2 matrices M_2 with the usual matrix addition and scalar multiplication.