Math 337 —-Final exam—-Spring 2015

1) (20 points) For $Ax = b$, let the reduced echelon form of $[A|b]$ be

$$[(1,0,0)^T, (0,1,0)^T, (3,-2,0)^T, (0,0,1)^T|(5,3,1)^T].$$

a) Write the solution of Ax=b in the form $x = p + x_h$.

b) Does the equation Ax=c have a solution for each c in R^3? Explain.

c) Find bases for the Nul(A), and Row(A) and find their dimentions.

d) Find the projection of $(5,3,1,0)^T$ onto Nul(A) and its distance to Nul(A).

2) (15 points) Let $T : R^3 \to R^3$ be $T(x_1, x_2, x_3) = (x_1 + x_3, 0, x_2)$

a) Show that T is linear and find its standard matrix A.

b) Show that T is not one-to-one by giving an example of $(x_1, x_2, x_3) \neq (0,0,0)$ such that $T(x_1, x_2, x_3) = (0,0,0)$. Explain. In one sentence explain how this can be done using A (no calculation needed).

c) Show that T is not onto by giving an example of $b = (b_1, b_2, b_3)$ such that $T(x_1, x_2, x_3) \neq b$ for any (x_1, x_2, x_3). In one sentence explain how this can be done using A (no calculation needed).

3) (20 points) Let $A = [(7,4,0)^T, (-2,1,0)^T, (0,0,5)^T)]$

a) Find the eigenvalues of A.

b) Find bases of the corresponding eigenspaces.

c) Diagonalize A if possible.

4) (15 points) a) Find k so that $A = [(0,2,2)^T, (1,k,7)^T, (k,-6,4)^T)]$ is invertible

b) For the smallest k, find A^{-1}.

5) (15 points) Let $A = [(1,0,1,0)^T, (1,1,1,,0)^T, (1,-1,0,1)^T]$.

a) Use the Gram-Schmidt process to find an orthonormal basis of the column space of A.

b) Find a QR factorization of A.

6) (15 points) a) Is Q(x)=$3x_1^2 - 4x_1x_2 + 6x_2^2$ positive definite, negative definite or indefinite?

b) Orthogonally diagonalize the matrix corresponding to Q.