Math 337 —-Final exam—-Spring 2016

Provide complete explanations for your answers.

1) (20 points) a) Find the value of k so that the vectors $v_1 = (1, 2, 1)^T$, $v_2 = (2, 5, 3)^T$ and $v_3 = (-1, -4, k)^T$ are linearly dependent.

b) For which value(s) of k, are these vectors a basis for R^3?

c) If $A = [v_1, v_2, v_3]$, for which value(s) of k is the system Ax=b uniquely solvable for each b in R^3? What are these unique solutions?

2) (20 points) a) Let A and B be nXn matrices with detA=4 and detB=5. Show that A, B and AB are invertible matrices and vompute $\det(2(BA)^T(AB)^{-1})$.

b) If A is an nxn matrix whose column space has dimension 10, what's the dimension of the Nul(A)?

c) If A is an nxn matrix with n an odd integer and $A^T = -A$, show that A is invertible.

3) (20 points) Let $v_1 = (3, 0, 0)^T$, $v_2 = (0, 4, 2)^T$ and $v_3 = (0, 1, 5)^T$ and $A = [v_1, v_2, v_3]$.

a) Find the eigenvalues of A.

b) Find bases and dimensions of the corresponding eigenspaces.

c) Diagonalize A and compute also P^{-1}. Find $\det(A^k)$ for a positive integer k?

4) (20 points) Let $v_1 = v_2 = (1, 2, 0)^T$ and $v_3 = v_4 = (2, 5, 3)^T$ and $A = [v_1, v_2, v_3, v_4)]$.

a) Find solutions of $Ax = (1, 1, -3)^T$ in the form $x = p + x_h$.

b) Find a basis for Nul(A).

c) Is the linear map $T : R^3 \to R^3$ given by $Tx = Ax$ one-to-one and onto?

5) (20 points) Let $v_1 = (1, 2, 0)^T$, $v_2 = (2, 4, 2)^T$ and $v_3 = (0, 2, 7)^T$ and $A = [v_1, v_2, v_3]$.

a) Find an orthonormal basis for the Col(A).

b) Find a QR factorization of A.

c) Show that A is symmetric and find the quadratic form whose standard matrix is A.