

## Math 337-final-Spring 2017

Provide complete explanations for your answers.

- 1) (15 points) Let  $A = [(1, 1, -1)^T, (2, 3, -4)^T, (1, 0, 1)^T]$  and  $b = (b_1, b_2, b_3)^T$ .
  - a) Is  $Ax=b$  consistent for all  $b$  in  $R^3$ ? Explain.
  - b) Find the solution of  $Ax = (1, 1, -1)^T$  in the form  $x = p + x_h$ . Explain what  $p$  and  $x_h$  represent.
- 2) (15 points) Let  $A = [(9, 9, 4, 9, 6)^T, (1, 0, 0, 0, 0)^T, (9, 9, 0, 3, 0)^T, (9, 9, 5, 9, 7)^T, (9, 2, 0, 0, 0)^T]$ .
  - a) Compute  $\det(A)$ .
  - b) Are  $A$  and  $2AA^T$  invertible? Explain.
- 3) (15 points) a) For which value(s) of  $\lambda$  is the matrix  $A = [(1, 1, 0)^T, (\lambda, 1, 0)^T, (0, 0, 1)^T]$  invertible?
  - b) For  $\lambda = 2$ , find the inverse of  $A$ .
- 4) (15 points) Let  $T : R^4 \rightarrow R^3$  be given by  $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 - x_3, x_1 + x_4)$ 
  - a) Find the standard matrix  $A$  of  $T$ .
  - b) Find bases and dimensions of the null space and the column space of  $A$ .
- 5) (15 points) Diagonalize the matrix  $A = [(4, 2, 0)^T, (-4, -2, 0)^T, (2, 2, 1)^T]$ , i.e. write in the form  $A = PDP^{-1}$ . Do not compute  $P^{-1}$ .
- 6) (15 points) Find a QR factorization of  $A = [(1, -1, 0, 1, 1)^T, (3, -3, 2, 5, 5)^T, (5, 1, 3, 2, 8)^T]$ .
- 7) (10 points) a) Let  $A$  be a  $3 \times 3$  matrix with  $\det(A)=3$ . Find  $\det(-4A)$ ,  $\det(1/2A^{-1})$ ,  $\det(3A)^3$  and  $\det(2(A^T)^{-1})$ .
  - b) If  $A$  is a  $4 \times 4$  matrix with  $\det(2A^2(A^T)^{-1}) = 4$ , find  $\det A$ .