9.1 For the anharmonic oscillator with the Hamiltonian

\[ H = -\frac{\hbar^2}{2m} \left\{ \frac{d^2}{dx^2} \right\} + \frac{k}{2} x^2 + cx^3 + dx^4 \]

evaluate \( E^1 \) for the first excited state, taking the unperturbed system as the harmonic oscillator.

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HINT: Example p.248 (5th Ed.) shows how to calculate \( E^1 \) for the ground state of the harmonic oscillator. Use the same method, just change the wavefunction to that for the first excited state.

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\[ E_N^1 = H_{NN}^1 = \int (\psi_N^0)^* H^1 \psi_N^0 d\tau, \ N = 1 \text{ for first excited state} \]

\[ H^1 = H - H^0 \]

\[ H^0 = -\frac{\hbar^2}{2m} \left\{ \frac{d^2}{dx^2} \right\} + \frac{k}{2} x^2 \]

\[ H^1 = cx^3 + dx^4 \]

For the harmonic oscillator, \( \alpha = 2\pi \nu m/\hbar = 4\pi^2 \nu m/\hbar \) &

\( \nu = 0 \) is the ground state: \( \psi_0 = c_0 e^{-\alpha x^2/2}, \ c_0 = (\alpha/\pi)^{1/4} \)
Example p. 248 shows that the first order correction to the ground state energy of the anharmonic oscillator is

\[ E_0^1 = H_{00}^1 = 3d/(4\alpha^2) = 3dh^2/[64\pi^4v^2m^2] \]

For the harmonic oscillator

\( v = 1 \) is the first excited state: \( \psi_1 = c_1 x e^{-\alpha x^2/2}, c_1 = (4\alpha^3/\pi)^{1/4} \)

The first order correction to the energy of the first excited state of the anharmonic oscillator is

\[ E_1^1 = H_{11}^1 = \int (\psi_1^0)^* H^1 \psi_1^0 d\tau \]

\[ = \int_{-\infty}^{\infty} (c_1 x e^{-\alpha x^2/2})^2 (cx^3 + dx^4)dx \]

\[ = (c_1)^2 \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} (cx^3 + dx^4)dx \]

\[ = c (c_1)^2 \int_{-\infty}^{\infty} x^5 e^{-\alpha x^2} dx + d (c_1)^2 \int_{-\infty}^{\infty} x^6 e^{-\alpha x^2} dx \]

first term is even \( x \) odd, so integral = 0

\[ E_1^1 = 2d (c_1)^2 \int_{0}^{\infty} x^6 e^{-\alpha x^2} dx \]

\[ = 2d (c_1)^2 (15/2^4)(\pi/\alpha^7)^{1/2} \]
\[
= 2d \left(4\alpha^3/\pi\right)^{1/2} \left(15/2^4\right)(\pi/\alpha^7)^{1/2}
= d15/(4\alpha^2)
= d15/[4(4\pi^2 \nu m/h)^2]
= d15h^2/[64\pi^4 \nu^2 m^2]
\]