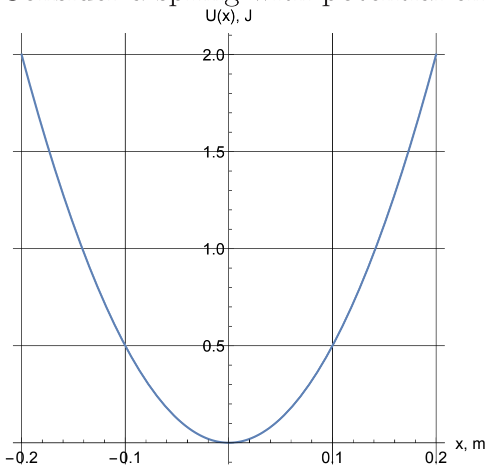
Energy conservation

Conservative forces only

$$\boxed{K + U = E = \text{const}}\,,\; U_s = \frac{1}{2}kx^2\,,\; U_g = mgh\,,\; W = -\Delta U$$

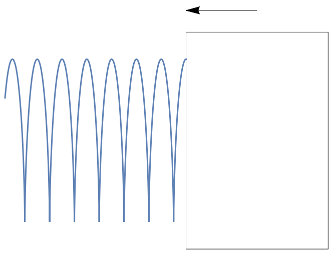
Spring

1. Consider a spring with potential energy shown in the graph below.



(a) Find the spring constant k

(b) A block with mass $M = 1.5\,kg$ approaches the spring with speed $v_0 = 1\,m/s$. Find the maximum compression



(c) Assuming that the block get hooked up to the spring and starts oscillations, find the kinetic energy $K(x)$ and speed $v(x)$ when $x = 5\,cm$

(d) Consider now the same spring held vertically and the same block $M = 1.5\,kg$ is being dropped on the spring (with zero speed) and compresses the spring the distance x until it stops.

i. find x (use energy conservation)

ii. find the work done by gravity

Gravity

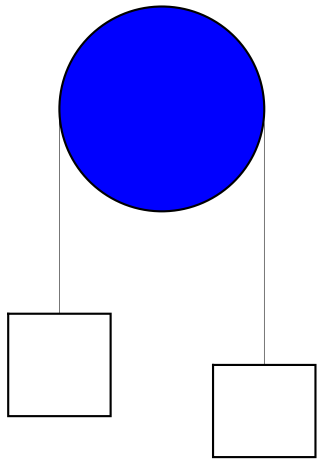
2. An $M = 2\,kg$ projectile is launched from the ground level (origin of coordinates) with initial speed $v_0 = 20\,m/s$. It reaches a point $\vec{r} = 10\hat{i} + 5\hat{j} + 15\hat{k}$.

   (a) find the change in potential energy ΔU

   (b) find the work done by gravity W_g

   (c) find speed v at point $\vec{r}$

3. In the Atwood machine the heavier body on left has mass $M = 1\,kg$, while the lighter body on right has mass $m = 0.9\,kg$. The system is initially at rest, there is no friction and the mass of the pulley is negligible.



   (a) write down the algebraic expression for the full energy E at the initial instant when the larger mass is at elevation h and the smaller mass is at elevation 0

   (b) write down the algebraic expression for the full energy E at the final instant when the larger mass is at elevation 0 and the smaller mass is at elevation h; use the yet unknown speed v when evaluating the full kinetic energy

   (c) Find the speed after M lowers by $h = 50\,cm$ from initial position

4. A skier slides down from a hill which is $H = 30\,m$ high and then, without losing speed, up a hill which is $h = 10\,m$ high. What is his final speed? Ignore friction.

5. An $M = 100\,kg$ crate is released from the top of a frictionless $L = 4\,m$ long ramp, which is makes 30^o with horizontal. The crate slides back to the floor. Do the following:
   (a) find U and E at the top of the ramp
   (b) find K at the bottom of the ramp
   (c) find the speed at the bottom of ramp

With non-conservative forces

$$\boxed{(K_f + U_f) - (K_i + U_i) = W_{non-cons.}}$$

6. Solve the crate problem with $\mu_k = 0.4$. (Note: friction $f = \mu_k mg$, $W_f = -fL$)

7. find the max angle when the crate will still slide

8. Consider problem 1b but with friction ($\mu_k = 0.5$). Find the distance x to first stop.

9. Solve the "policeman's problem" using *only* work-energy considerations. Use $L = 200\,m$ (length of skid marks) and $\mu_k = 0.5$; find v_0 . Assume level road

10. find the time to stop