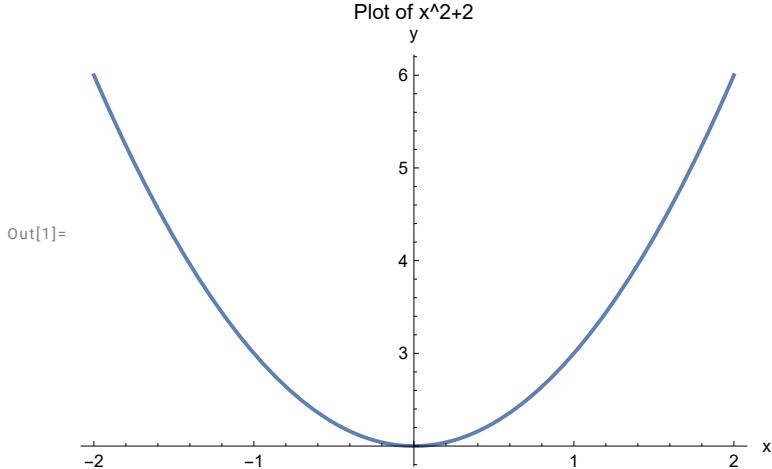


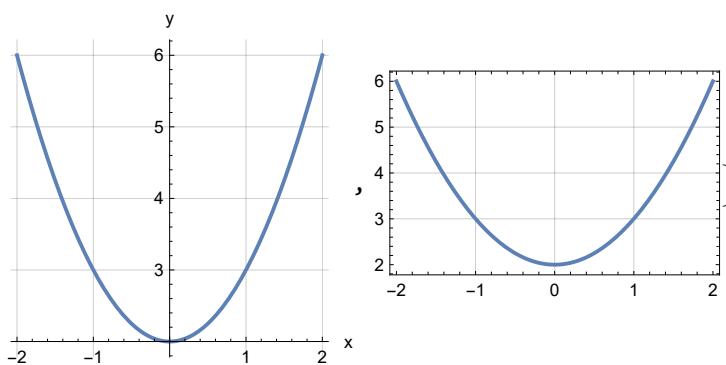
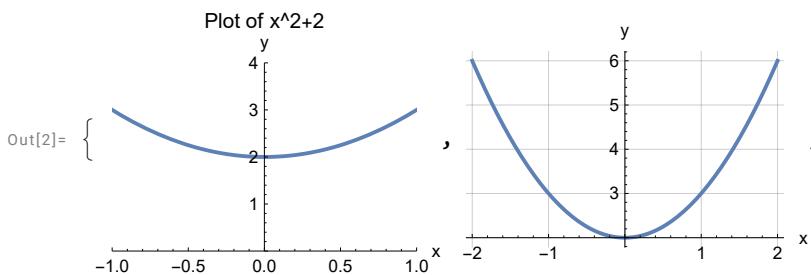
(*Graphics*)

(*In this section, we introduce basic plotting commands and show how to use them.*)

```
In[1]:= Plot[x^2 + 2, {x, -2, 2}, AxesLabel -> {"x", "y"}, PlotLabel -> "Plot of x^2+2"]
```



```
In[2]:= {Plot[x^2 + 2, {x, -2, 2}, AxesLabel -> {"x", "y"},  
PlotLabel -> "Plot of x^2+2", GridLines -> None, PlotRange -> {{-1, 1}, {0, 4}}],  
Plot[x^2 + 2, {x, -2, 2}, AxesLabel -> {"x", "y"}, GridLines -> Automatic],  
Plot[x^2 + 2, {x, -2, 2}, AxesLabel -> {"x", "y"}, GridLines -> Automatic,  
AspectRatio -> 1],  
Plot[x^2 + 2, {x, -2, 2}, AxesLabel -> {"x", "y"}, GridLines -> Automatic, Axes -> None,  
Frame -> True]}
```

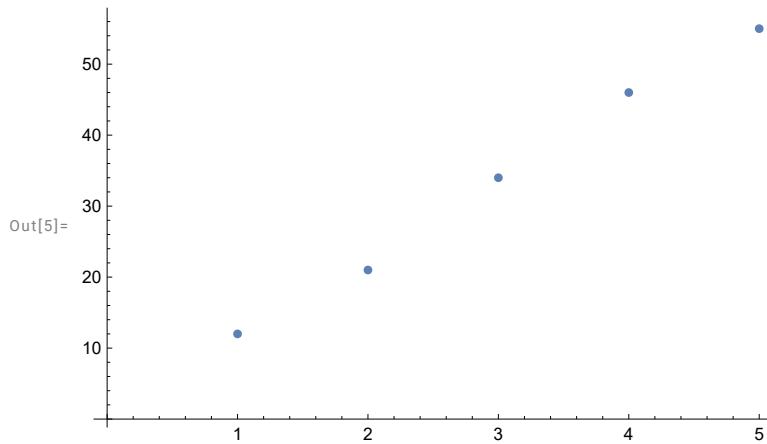


```
In[3]:= (*Plotting a graph using ListPlot command for discrete data:
```

The plot command works on "vectors" (lists) of numerical data. For example, supposed that you have experimental data set listed separately for the X and Y components, such as $Xexp=\{1,2,3,4,5\}$ and the corresponding response, $Yexp=\{12,21,34,46,55\}$, you can type:*)

```
In[4]:= Xexp = {1, 2, 3, 4, 5}; Yexp = {12, 21, 34, 46, 55};
```

```
In[5]:= ListPlot[Yexp] (*since Xexp is a natural list*)
```



```
In[6]:= (*or, create a list of {X,Y}pairs, which is the outcome of most experiments*)
```

```
In[7]:= (*let's have a look*) {Xexp, Yexp} // MatrixForm
```

```
Out[7]//MatrixForm=
```

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 12 & 21 & 34 & 46 & 55 \end{pmatrix}$$

```
In[8]:= data = Transpose[{Xexp, Yexp}]
```

```
Out[8]= {{1, 12}, {2, 21}, {3, 34}, {4, 46}, {5, 55}}
```

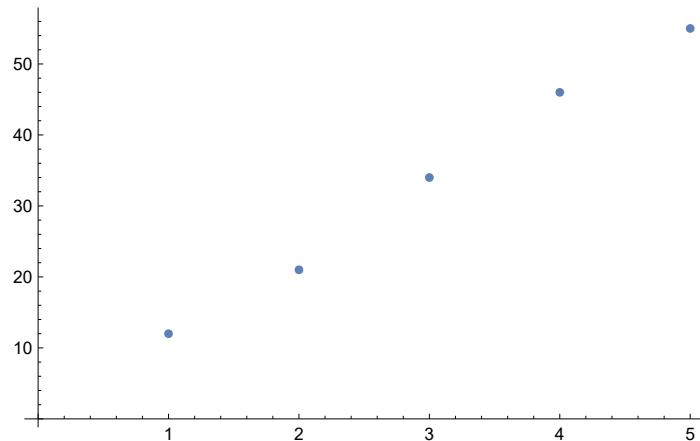
```
In[9]:= MatrixForm[data]
```

```
Out[9]//MatrixForm=
```

$$\begin{pmatrix} 1 & 12 \\ 2 & 21 \\ 3 & 34 \\ 4 & 46 \\ 5 & 55 \end{pmatrix}$$

```
In[10]:= ListPlot[data, AxesOrigin -> {0, 0}] (*gives the same*)
```

```
Out[10]=
```

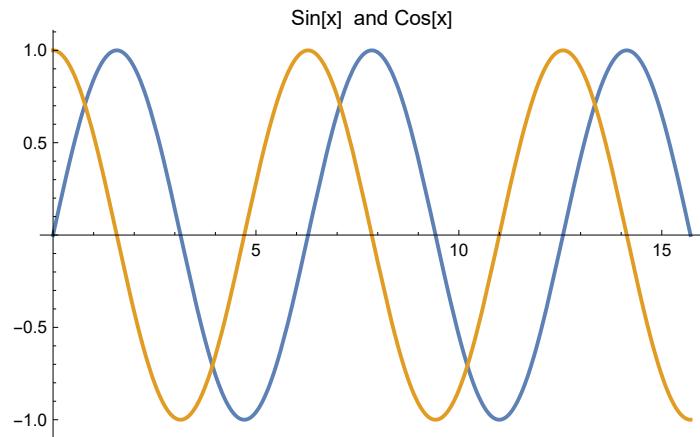


```
In[11]:= (*Plotting Multiple Curves in a plot
```

If you want to plot two or more graphs in a single plot,
 you can list the functions as a list (if they are in the same domain)
 or use the "Show" command to combine separate plots. For example,
 if you want to plot $\sin(x)$ and $\cos(x)$ curves in a single plot:*)

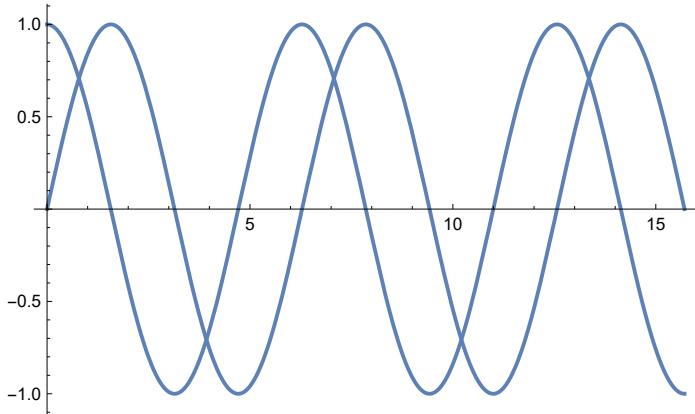
```
In[12]:= Plot[{Sin[x], Cos[x]}, {x, 0, 5 Pi}, PlotLabel -> "Sin[x] and Cos[x]"]
```

```
Out[12]=
```



```
In[13]:= (*or*) Show[Plot[Sin[x], {x, 0, 5 Pi}], Plot[Cos[x], {x, 0, 5 Pi}]]
(*gives the same; you can adjust color/style of curves*)
```

Out[13]=



```
In[14]:= (*similarly, you can define fig1=Plot[Sin[x], {x,0,5Pi}],
fig2=Plot[Cos[x], {x,0,5Pi}] and then use Show[fig1,fig2 - check!]*)
```

In[15]:=

(*Exporting your graphics*)

```
In[16]:= myfig = Show[Plot[Sin[x], {x, 0, 5 Pi}], Plot[Cos[x], {x, 0, 5 Pi}]];
```

```
In[17]:= Export["mySinCosfig.JPEG", myfig, "JPEG"]
(*the file "...JPEG is created outside Mathematica")
```

Out[17]=

mySinCosfig.JPG

```
In[18]:= (*The following example shows you how to show
together a discrete and a continuous plots. In this example,
supposed that you conducted a simple harmonic motion experiment for spring system,
we have experimental data set of mass (mexp) vs.period (Texp) as shown below.*)
mexp = {0.06, 0.08, 0.1, 0.12, 0.16};
Texp = {4.5, 5.2, 5.8, 6.3, 7.2};
```

```
In[20]:= (*we first create pairs of data as you expect from experiment*)
```

```
In[21]:= mydata = Transpose[{mexp, Texp}]
```

Out[21]=

{ {0.06, 4.5}, {0.08, 5.2}, {0.1, 5.8}, {0.12, 6.3}, {0.16, 7.2} }

```
In[22]:= mydata // MatrixForm
```

Out[22]//MatrixForm=

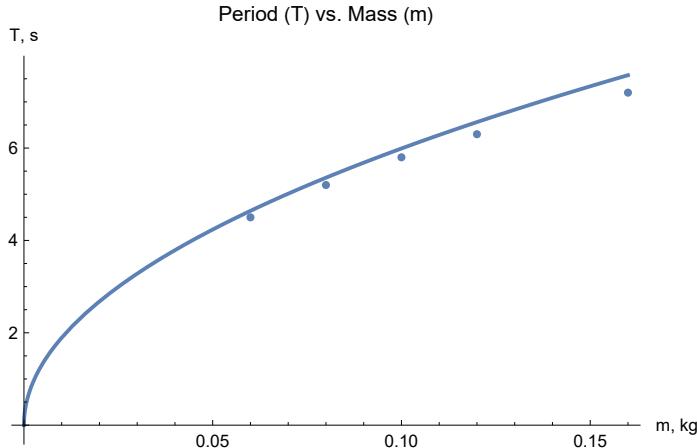
$$\begin{pmatrix} 0.06 & 4.5 \\ 0.08 & 5.2 \\ 0.1 & 5.8 \\ 0.12 & 6.3 \\ 0.16 & 7.2 \end{pmatrix}$$

```
In[23]:= (*The spring constant (k) is 0.11. The equation of simple harmonic motion is given by T= 2Pi Sqrt[m/k]. The code will be as following*)
```

```
In[24]:= Clear[T, m, k]; k = .11; T[m_] := 2 Pi Sqrt[m / k]
```

```
In[25]:= Show[Plot[T[m], {m, 0, 0.16}], ListPlot[mydata], AxesLabel -> {"m, kg", "T, s"}, PlotLabel -> "Period (T) vs. Mass (m)"]
```

Out[25]=



```
In[26]:= (*Note: if possible, for a simple known or assumed theoretical dependence it is better to use variables with a linear dependence; let us repeat the previous example: *)
```

In[27]:=

```
In[28]:= mexp = {0.06, 0.08, 0.1, 0.12, 0.16};  
Texp = {4.5, 5.2, 5.8, 6.3, 7.2};
```

In[30]:= Texp2 = Texp^2

Out[30]=

{20.25, 27.04, 33.64, 39.69, 51.84}

In[31]:= mydata2 = Transpose[{mexp, Texp2}]

Out[31]=

{ {0.06, 20.25}, {0.08, 27.04}, {0.1, 33.64}, {0.12, 39.69}, {0.16, 51.84} }

In[32]:= Clear[T2, m, k]; k = .11; T2[m_] := (2 Pi Sqrt[m / k]) ^2

In[33]:= T2[m]

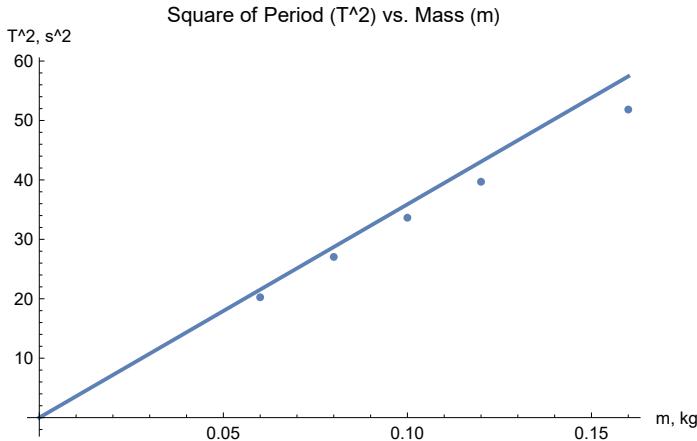
Out[33]=

358.895 m

In[34]:= figTheo = Plot[T2[m], {m, 0, 0.16}]; figExp = ListPlot[mydata2];

```
In[35]:= Show[figTheo, figExp, AxesLabel -> {"m, kg", "T^2, s^2"},  
PlotLabel -> "Square of Period (T^2) vs. Mass (m)"]
```

Out[35]=



```
In[36]:= (*Linear fitting of data*)
```

```
In[37]:= Fexp = {1.9, 3.9, 5.9, 6.9, 7.8};  
Dexp = {0.03, 0.1, 0.16, 0.2, 0.23};
```

```
In[39]:= data = Transpose[{Dexp, Fexp}]
```

Out[39]=

```
{ {0.03, 1.9}, {0.1, 3.9}, {0.16, 5.9}, {0.2, 6.9}, {0.23, 7.8} }
```

```
In[40]:= data // MatrixForm
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 0.03 & 1.9 \\ 0.1 & 3.9 \\ 0.16 & 5.9 \\ 0.2 & 6.9 \\ 0.23 & 7.8 \end{pmatrix}$$

```
In[41]:= LinearModelFit[data, x, x]
```

Out[41]=

$$\text{FittedModel}\left[\boxed{1.01 + 29.7x}\right]$$

```
In[42]:= nfit = Normal[%]
```

Out[42]=

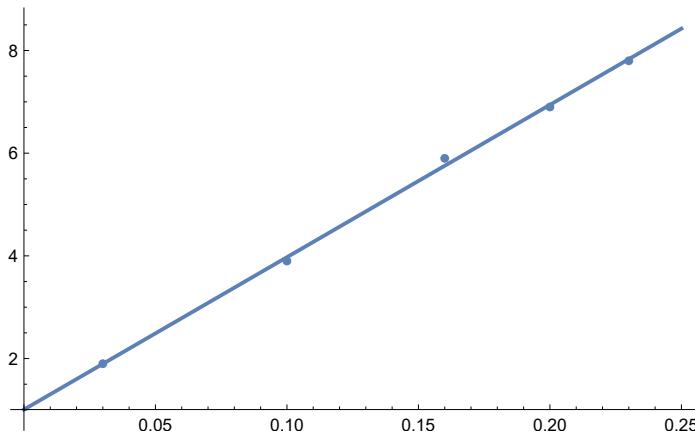
$$1.00591 + 29.6812x$$

```
In[43]:=
```

```
In[44]:=
```

```
In[45]:= Show[Plot[nfit, {x, 0, 0.25}], ListPlot[data]]
```

Out[45]=



» System`

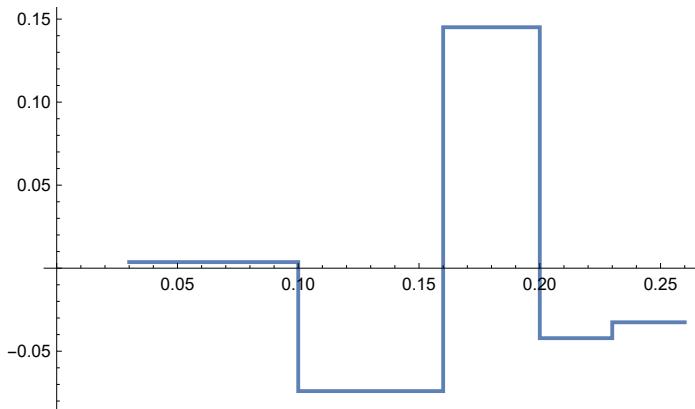
```
In[47]:= dev = Fexp - (nfit /. x → Dexp)
```

Out[47]=

```
{0.00365474, -0.074028, 0.145101, -0.0421462, -0.0325816}
```

```
In[48]:= ListStepPlot[Transpose[{Dexp, dev}]]
```

Out[48]=



```
In[49]:= list = Fexp - Mean[Fexp] * Table[1, {Length[Fexp]}]
```

Out[49]=

```
{-3.38, -1.38, 0.62, 1.62, 2.52}
```

```
In[50]:= devMean = Norm[list]^2
```

Out[50]=

```
22.688
```

```
In[51]:=
```

```
In[52]:= totDev = Norm[dev]^2
Out[52]=
0.0293857

In[53]:= r2 = 1 - totDev / devMean
Out[53]=
0.998705

In[54]:= (*good!*)

In[55]:= (*Graphics of vectors and other objects*)

In[56]:= ?Arrow
Out[56]=
```

Symbol

i

Arrow[{ pt_1 , pt_2 }] is a graphics primitive that represents an arrow from pt_1 to pt_2 .

Arrow[{ pt_1 , pt_2 }, s] represents an arrow with its ends set back from pt_1 and pt_2 by a distance s .

Arrow[{ pt_1 , pt_2 }, { s_1 , s_2 }] sets back by s_1 from pt_1 and s_2 from pt_2 .

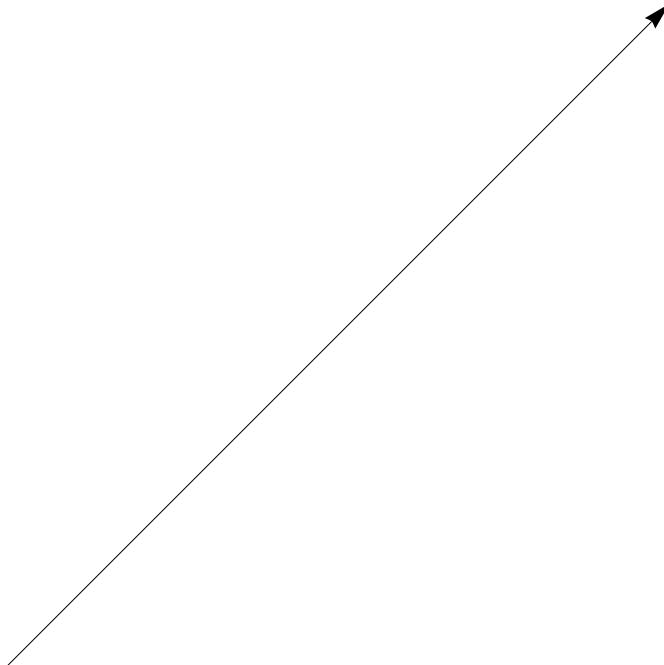
Arrow[*curve*, ...] represents an arrow following the specified *curve*.

▼

```
In[57]:= myarr[xx_] := Arrow[{{0, 0}, xx}] (*always from origin*)
```

```
In[58]:= Graphics[myarr[{1, 1}]]
```

```
Out[58]=
```



```
In[59]:=
```

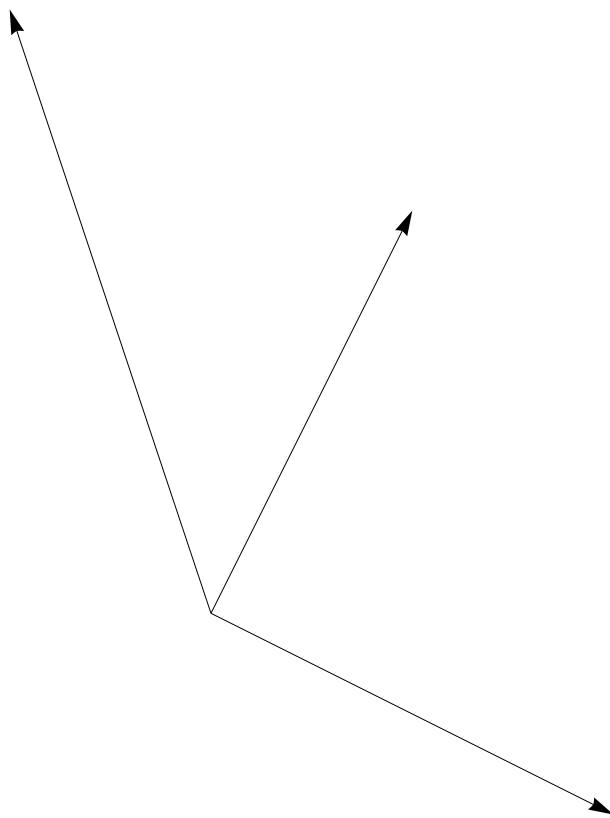
```
In[60]:=
```

```
In[61]:= Clear[A, B, c];
```

```
In[62]:= A = {1, 2}; B = {-1, 3}; c = A - B;
```

```
In[63]:= Graphics[{myarr[A], myarr[B], myarr[c]}]
```

```
Out[63]=
```



```
In[64]:= ?Text
```

```
Out[64]=
```

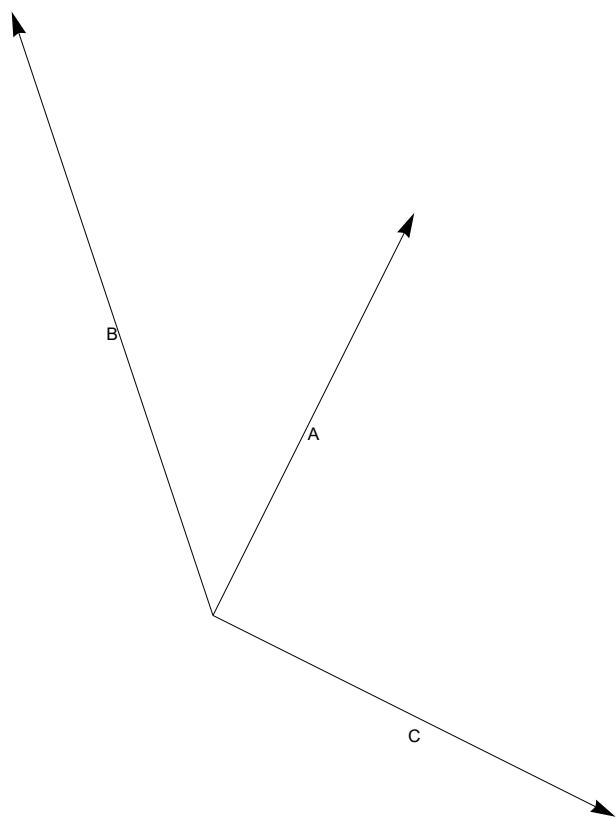
Symbol i

Text[*expr*] displays with *expr* in plain text format.

Text[*expr*, *coords*] is a graphics primitive that displays the textual form of *expr* centered at the point specified by *coords*.

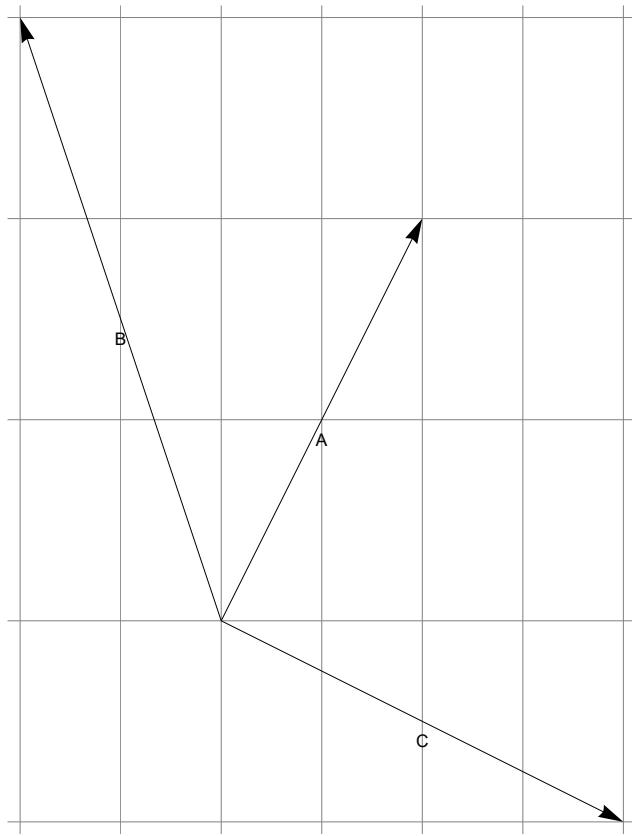
```
In[65]:= Graphics[{myarr[A], myarr[B], myarr[c], Text["A", A / 2 - {0, .1}],  
Text["B", B / 2 - {0, .1}], Text["C", c / 2 - {0, .1}]}]
```

Out[65]=



```
In[66]:= Show[%, GridLines -> Automatic]
```

```
Out[66]=
```



```
In[67]:=
```

```
In[68]:= (*Plotting vector fields*)
```

(*we consider the simplest graphical example
of the gravitational field around a planet: $\mathbf{g} = -G*M / r^2 \mathbf{r}$,
with G - gravitational constant and M - the mass of the planet. For simplicity,
assume $G*M=1$, and we use the vector "field" = $- \hat{\mathbf{r}} / r^3$.

The same structure of field is observed for a negative
electric charge*)

```
Clear[x, y]; vecr = {x, y}; r = Norm[vecr];
```

```
In[71]:= field = -vecr / r^3
```

```
Out[71]=
```

$$\left\{ -\frac{x}{(\text{Abs}[x]^2 + \text{Abs}[y]^2)^{3/2}}, -\frac{y}{(\text{Abs}[x]^2 + \text{Abs}[y]^2)^{3/2}} \right\}$$

```
In[72]:= FullSimplify[%, Assumptions -> x > 0 && y > 0]
```

```
Out[72]=
```

$$\left\{ -\frac{x}{(x^2 + y^2)^{3/2}}, -\frac{y}{(x^2 + y^2)^{3/2}} \right\}$$

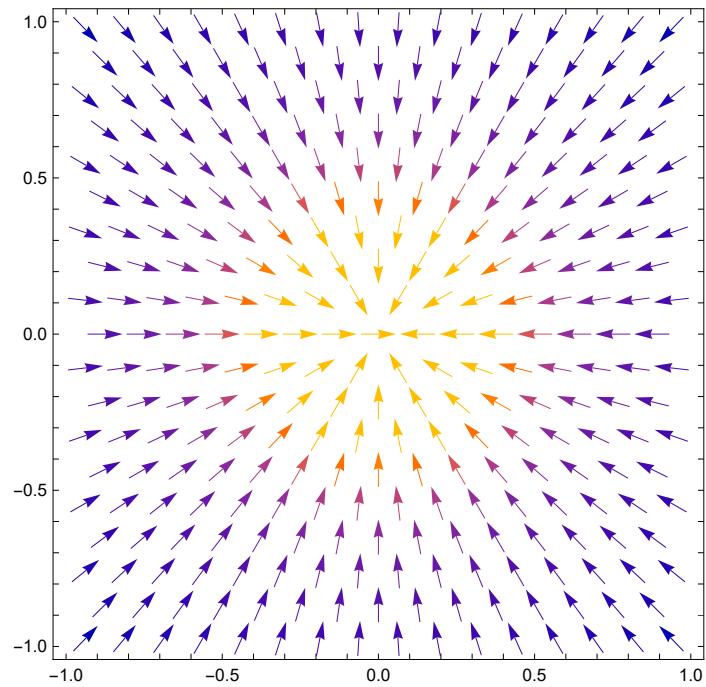
```
In[73]:= field = %
```

```
Out[73]=
```

$$\left\{ -\frac{x}{(x^2 + y^2)^{3/2}}, -\frac{y}{(x^2 + y^2)^{3/2}} \right\}$$

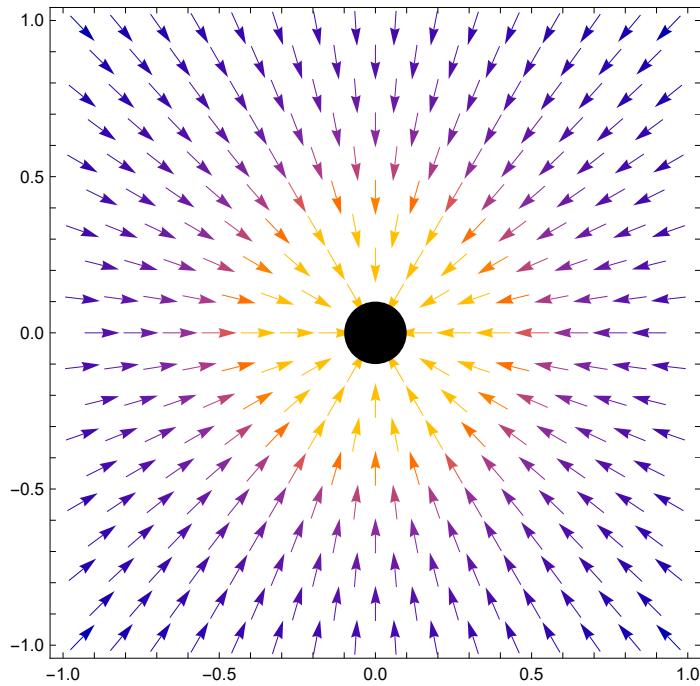
```
In[74]:= VectorPlot[field, {x, -1, 1}, {y, -1, 1}]
```

```
Out[74]=
```



```
In[75]:= Show[%, Graphics[Disk[{0, 0}, .1]]]
```

```
Out[75]=
```



```
In[76]:= ?ListStreamPlot
```

```
Out[76]=
```

Symbol i

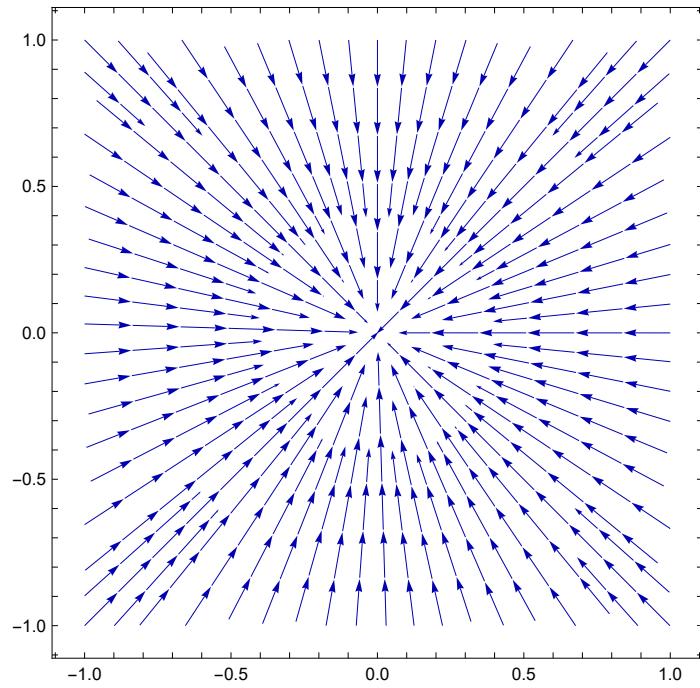
ListStreamPlot[varr] generates a stream plot from an array *varr* of vectors.

ListStreamPlot[{{{x₁, y₁}, {vx₁, vy₁}}, ...}] generates a stream plot from vectors {vx_i, vy_i} given at points {x_i, y_i}.

ListStreamPlot[{data₁, data₂, ...}] plots data for several vector fields.

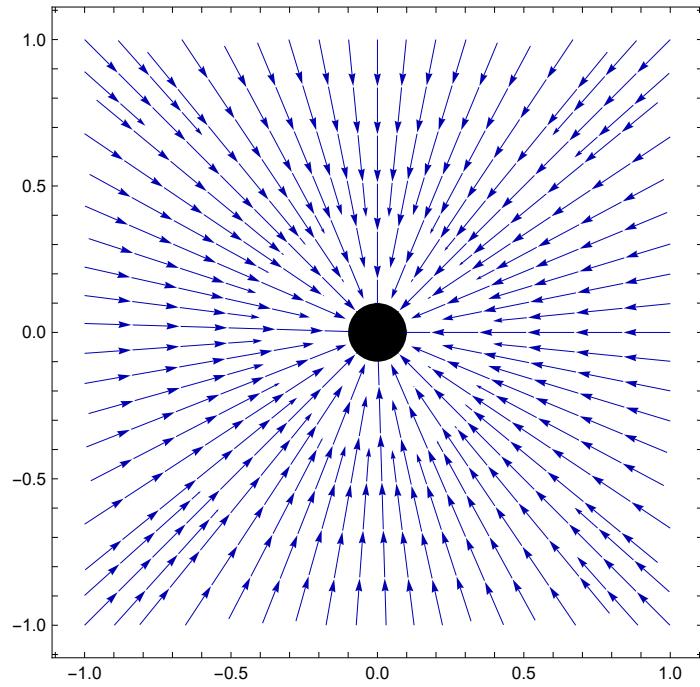
```
In[77]:= StreamPlot[field, {x, -1, 1}, {y, -1, 1}]
```

```
Out[77]=
```



```
In[78]:= Show[%, Graphics[Disk[{0, 0}, .1]]]
```

```
Out[78]=
```



```
In[79]:=
```

```
In[80]:=
```

(*Advanced: Mathematica modelling of Propagation of errors*)

```
myrand := 2 RandomReal[] - 1 (*random number between -1 and 1*)
```

In[105]:=

```
Table[myrand, {10}]
```

Out[105]=

```
{0.817509, -0.17352, -0.336672, 0.444658,
-0.460805, -0.606929, 0.315531, -0.622823, 0.899948, 0.132814}
```

X := x + 0.5 myrand (*X is a random "measurement", scattered around the fixed value of x. The 0.5*myrand is "noise" (experimental error) *)

In[223]:=

```
x = 3; listX = Table[X, {50}] (*is the result of 50 measurements*)
```

Out[223]=

```
{3.13497, 2.78048, 3.22463, 3.43589, 2.5394, 3.45472, 2.87116, 3.13051, 3.21753, 2.73164,
3.31978, 3.08968, 2.92723, 2.77519, 2.81071, 2.79714, 2.99667, 3.03381, 2.71216, 3.04764,
3.09765, 3.29402, 2.83988, 3.08005, 3.25484, 2.57037, 2.54881, 2.6455, 3.17365, 3.3035,
2.74743, 2.71668, 3.37735, 3.10932, 3.20238, 2.73416, 2.51854, 3.07417, 2.68613, 2.79818,
2.72743, 3.25413, 3.27587, 3.47177, 2.63199, 3.10277, 3.101, 3.11136, 2.92625, 2.56043}
```

In[225]:=

```
xmean = Mean[listX] (*close to 3 since 50 is a "big" number*)
```

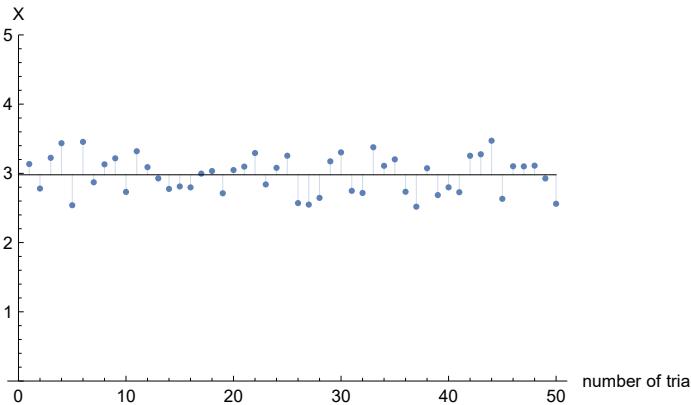
Out[225]=

```
2.97933
```

In[226]:=

```
Show[ListPlot[listX, Filling -> xmean, PlotRange -> {0, 5}],
Graphics[Line[{ {0, xmean}, {50, xmean}}]], AxesLabel -> {"number of trial", "X"}]
```

Out[226]=



(*in the above vertical lines indicate the distance of individual measurement from the mean value of X. The average of those distances, given by MeanDeviation, is "simple error". The average of squares of the distances is "variance", and the root of variance is StandardDeviation*)

```

In[137]:= errx = MeanDeviation[listX]
Out[137]= 0.241687

In[138]:= sigx = StandardDeviation[listX]
Out[138]= 0.280813

In[229]:= Y := y + 0.3 myrand (*another "experimental" variable with smaller average and noise*)
In[230]:= y = 1; listY = Table[Y, {50}];
In[231]:= erry = MeanDeviation[listY]
Out[231]= 0.143192

In[232]:= sigy = StandardDeviation[listY]
Out[232]= 0.16595

In[233]:= listPlus = listX + listY; (* we measure X+Y*)
In[234]:= listMinus = listX - listY; (* we measure X-Y*)
Mean[listPlus] (*close to sum of means*)
Out[235]= 3.944

Mean[listMinus] (*close to the difference*)
Out[236]= 2.01466

In[237]:= errPlus = MeanDeviation[listPlus]
Out[153]= 0.255255

In[154]:= errMinus = MeanDeviation[listMinus]
Out[154]= 0.286256

```

```

errx + erry (*simple errors do not add up!*)
Out[155]=
0.393381

In[250]:= sigPlus = StandardDeviation[listPlus]
Out[250]=
0.278877

In[251]:= 

In[239]:= sigx
Out[239]=
0.280813

In[240]:= sigy
Out[240]=
0.16595

Sqrt[sigx^2 + sigy^2] (*close to sigPlus, but not very since 50 is not too "big"*)
Out[241]=
0.326183

(*we now try a larger number of trials*)

In[242]:= X := x + .2 * myrand
In[243]:= x = 5; listX = Table[X, {100000}];
In[244]:= MeanDeviation[listX]
Out[244]=
0.100037

In[245]:= StandardDeviation[listX]
Out[245]=
0.115464

In[246]:= Y := y + 0.3 myrand
In[247]:= y = 3; listY = Table[Y, {100000}];

In[252]:= listZp = listX + listY;
In[253]:= listZm = listX - listY;

```

```

In[254]:= Mean[listZp]
Out[254]= 7.99915

In[255]:= Mean[listZm]
Out[255]= 2.00034

In[256]:= MeanDeviation[listZp]
Out[256]= 0.172016

In[265]:= sigP = StandardDeviation[listZp]
Out[265]= 0.208115

In[266]:= sigM = StandardDeviation[listZm] (*very close !!*)
Out[266]= 0.208231

In[262]:= sigX = StandardDeviation[listX]
Out[262]= 0.115464

In[263]:= sigY = StandardDeviation[listY]
Out[263]= 0.173217

Sqrt[sigX^2 + sigY^2] (*should be close to sigP and/or to sigM*)
Out[264]= 0.208173

Out[99]= {0.20001, 0.299989, 0.360813, 0.381893}

In[100]:= 
```

```

In[267]:= (* PRODUCT X*Y , RATIO X/Y, etc.*)

Out[200]= 0.0300131

(* PRODUCT X*Y , RATIO X/Y, etc.*)

In[270]:= listZprod = listX * listY;

In[271]:= Length[listZprod]

Out[271]= 100000

In[272]:= listZrat = listX / listY;

In[273]:= sd := StandardDeviation

In[274]:= {sd[listX] / Mean[listX], sd[listY] / Mean[listY],
           sd[listZprod] / Mean[listZprod], sd[listZrat] / Mean[listZrat]}

Out[274]= {0.0230941, 0.0577503, 0.0621907, 0.0624101}

In[211]:= Sqrt[ (%[[1]] ) ^ 2 + (%[[2]] ^ 2) ]

Out[211]= 0.00608818

In[275]:= listZpow = listX^3 * listY^2;

In[276]:= {3 sd[listX] / Mean[listX], 2 sd[listY] / Mean[listY], sd[listZpow] / Mean[listZpow]}

Out[276]= {0.0692822, 0.115501, 0.134518}

```

```
In[277]:= Sqrt[ (%[1]) ^2 + (%[2] ^2) ]  
Out[277]= 0.134686
```

(*General function of X*)

```
In[278]:= listZsin = Sin[listX];
```

```
In[284]:= {sd[listX], sd[listZsin]}  
Out[284]= {0.115464, 0.0330943}
```

```
In[285]:= %[1] Cos[Mean[listX]]  
Out[285]= 0.0327248
```

» System`