PHYS 621, Final Exam Dec., 2007

Name: 4-digit ID:

Take-home exam. Problems should be clearly described (in pen) and returned together with the last homework to the Physics Office, or to the instructor’s office (452T) by noon, Tue., Dec. 18.

Problems are 20 point each, unless specified otherwise; the remaining points are provided by the homework. No partial credit for incomplete or incorrect answers will be given. (but individual questions to a problem, (a), (b), (c), etc. will be credited, even if not all of them are answered). Passing of the exam requires 60 points or more.

1. A ring of radius $R$ carries a constant current $I$ in a plane which is parallel to the $xy$ plane, at $z = a$. The center of the ring is on the $z$-axis. Another identical ring with current $I$ in the same direction is placed in a parallel plane $z = -a$.

(a) find $\vec{B}$ at arbitrary $z$ on the axis
(b) write down the field at large $z$ using the magnetic moment of the rings and compare the results
(c) calculate $d^2B/dz^2$ and find $a = a_0$ when this derivative is zero (this corresponds to Helmholtz coils, used in the Lab to produce a reasonably uniform field near the midpoint between the rings).
(d) Plot the result for some $a < a_0$, $a = a_0$ and $a > a_0$

2. Consider a generalization of the previous problem when you have an infinite number of identical parallel rings with currents $I$, which are located at $z = na$ with $n = 0, \pm 1, \pm 2, \ldots$ (a model for an infinite solenoid).

(a) Write the field at $x, y, z = 0$ in terms of an infinite sum
(b) (optional) try to evaluate the infinite sum explicitly
(c) sketch (or plot) $B(z)$ on the $z$-axis
(d) Consider the limit $a \rightarrow 0$ and compare with the known elementary expression
(e) Consider a semi-infinite system by removing all rings with $n > 0$. Explore $B(z)$ around the end of such a ”solenoid”.

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3. Consider an infinite charged line (charge density $\lambda$) which is placed along the $z$-axis. A current $I$ is also flowing along the same line in the positive $z$ direction.

(a) Find the direction and magnitude of the Poynting vector at any point in space, off the line.

(b) Evaluate the total energy flux per unit length of the line. [If the integral diverges, introduce a reasonable cut-off and discuss the physical reason for it].

(c) Discuss qualitatively which meaning, if any, should be given to an energy flux in a static field.

4. A plane monochromatic wave with frequency $\omega$ and the amplitude of electric field $E_0$ is propagating in the $z$-direction. It encounters a nontransparent screen in the $x, y$ plane.

(a) Using the fact that the momentum density is given by $\vec{J} = \vec{S}/c^2$ ($\vec{S}$ is the Poynting vector) determine the pressure on the screen if the wave is completely absorbed.

(b) Same question if the wave is reflected by the screen (Hint: use the connection between the force and the change in momentum).

5. (optional - solve the following problem only if you missed on the topic of Legendre polynomials on the midterm). A charge $+2q$ is located at the origin and two charges of $-q$ are located on the $z$-axis at $z = \pm a$, respectively.

(a) Find the exact potential everywhere.

(b) Write the potential on the $z$-axis and perform the large-$z$ expansion.

(c) Write expansion of the potential $\Phi(r, \theta)$ in terms of Legendre polynomials.