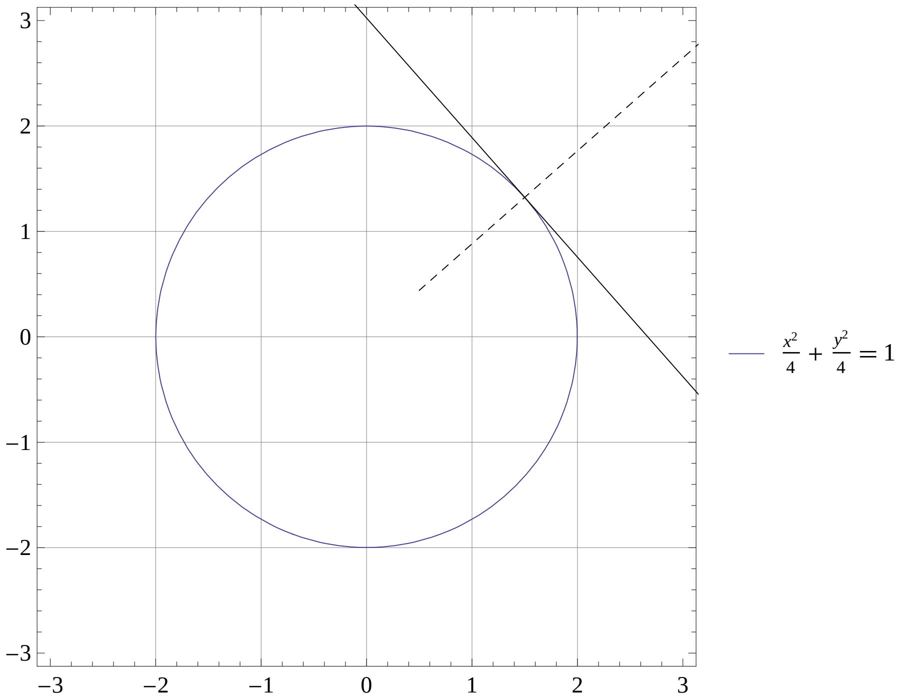
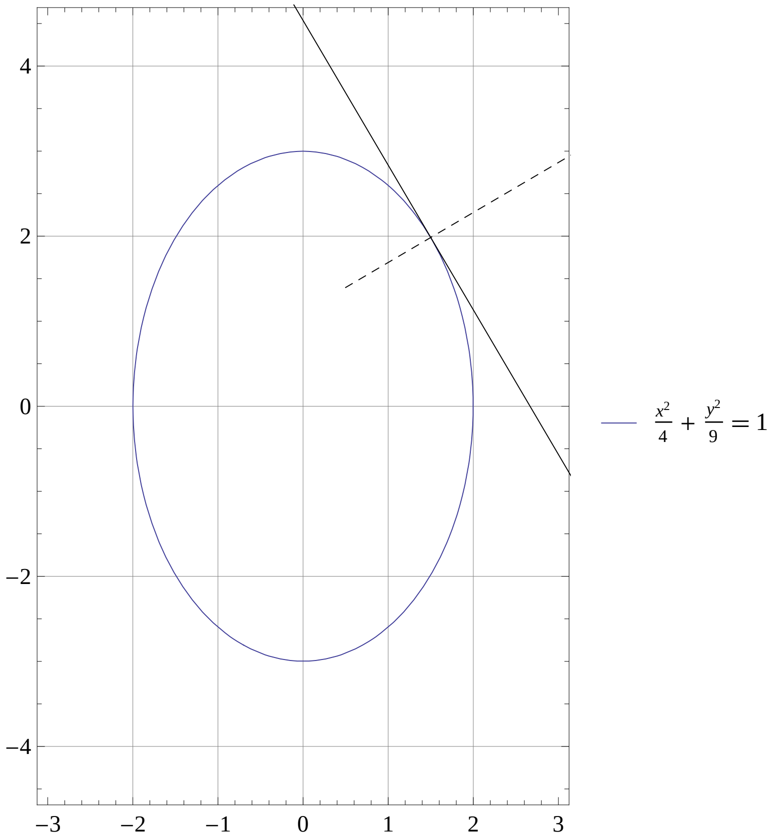
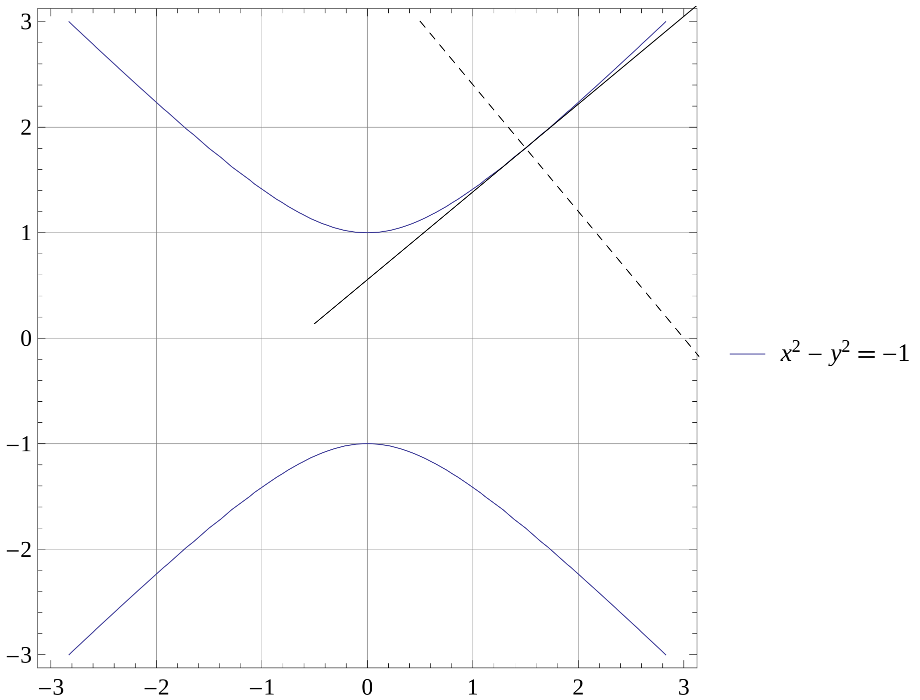
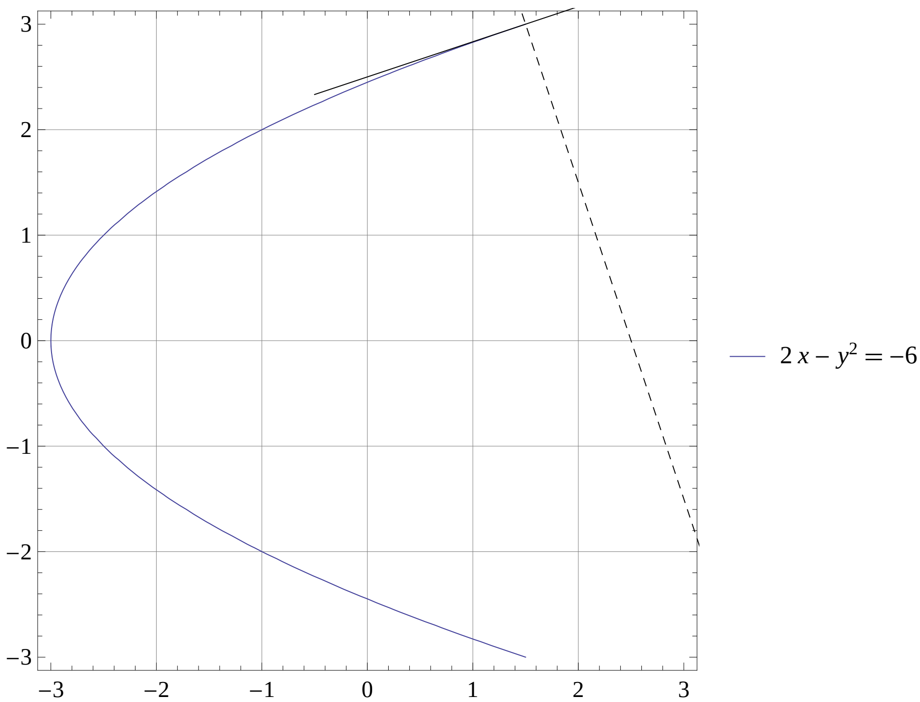
For all examples below construct general equations for the tangent line and the normal at arbitrary point $(x_0\,,\ y_0)$. Evaluate for $x_0 = 1.5$ and compare with plots. *Extra credit.* Evaluate at $x_0 = -1$ and add your lines to the plots.






3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
$x^2 - y^2 = -1$


3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
$2\,x - y^2 = -6$

Extra credit. Galileo's parabola.
Consider

$$y(t) = v_{0y} - \frac{1}{2}gt^2 \tag{1}$$

for the y-component of the projectile motion starting from $y = 0$. Find

$$v_y(t) =$$

$$a_y \equiv \frac{d^2y}{dt^2} =$$

$$t_{\max} =$$

(time to reach $y_{\max}$) and

$$y_{max} =$$

Find the time to get a distance H below zero (i.e., $y(t) = -H$):

$$t =$$

explain the 2nd root. Find

$$v_y(t) =$$

Try algebra first, then use $v_0 = 10\,,\ \alpha = 45^o\,,\ H = 1\,,\ g = 10$ (in metric units -only for math- in physics $g = 9.8\,m/s^2$).

Now include the x-motion:

$$x = v_x t$$

Exclude t, and find $y(x)$. Sketch. Find

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$

Construct a general equations for the tangent line; evaluate when $y = -H$. Compare the slope with v_y/v_x . Find the speed at this level

$$v = \sqrt{v_x^2 + v_y^2} =$$