

*LC* and driven *LC* circuits

1. Consider an *LC* circuit with  $L = 10\text{ mH}$ ,  $C = 0.7\text{ mF}$  and capacitor initially charged to  $10\text{ V}$ .
  - (a) from conservation of energy, find the maximum current
  - (b) find the resonant frequency  $\omega_0$  in *rad/s*
  - (c) derive explicit formulas for the charge  $q(t)$  and the electric energy  $U_C(t)$  in the capacitor
  - (d) derive explicit formulas for the current  $I(t)$  and the magnetic energy  $U_M(t)$  in the inductor
  - (e) verify  $U_M(t) + U_E(t) = \text{const}$  (and determine the *const*)
2. An *LC* circuit is driven by an external AC source with  $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$ . Use  $L = 10\text{ mH}$ ,  $C = 0.7\text{ mF}$ ,  $\mathcal{E}_m = 150\text{ V}$ ,  $f_d = 60\text{ Hz}$ .
  - (a) find  $\omega_d$
  - (b) write down the loop equation in terms of the charge on the capacitor  $q(t) = Q \sin(\omega_d t)$ .
  - (c) find the charge amplitude  $Q$
  - (d) find the current amplitude  $I$
  - (e) suppose,  $L$  can be varied; which value would bring the circuit into resonance?
  - (f) Plot  $I(L)$

