Week12: Chapter 12

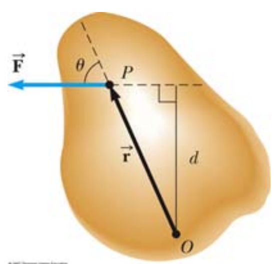
Static Equilibrium
and
Elasticity

Static Equilibrium

- Equilibrium implies the object is at rest (static) or its center of mass moves with a constant velocity (dynamic)
- Static equilibrium is a common situation in engineering
- Principles involved are of particular interest to civil engineers, architects, and mechanical engineers

Torque

- $\vec{\tau} = \vec{\mathbf{F}} \times \vec{\mathbf{r}}$
  - Use the right hand rule to determine the direction of the torque
  - The tendency of the force to cause a rotation about O depends on F and the moment arm *d*



Conditions for Equilibrium

- The net force equals zero
  - $\sum \vec{\mathbf{F}} = 0$
  - If the object is modeled as a particle, then this is the only condition that must be satisfied
- The net torque equals zero
  - $\sum \vec{\tau} = 0$
  - This is needed if the object cannot be modeled as a particle
- These conditions describe the rigid objects in equilibrium analysis model

Translational Equilibrium

- The first condition of equilibrium is a statement of translational equilibrium
- It states that the translational acceleration of the object's center of mass must be zero
  - This applies when viewed from an inertial reference frame

Rotational Equilibrium

- The second condition of equilibrium is a statement of rotational equilibrium
- It states the angular acceleration of the object to be zero
- This must be true for any axis of rotation

Equilibrium Equations

- We will restrict the applications to situations in which all the forces lie in the xy plane
  - These are called coplanar forces since they lie in the same plane
- There are three resulting equations
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma \tau = 0$

Axis of Rotation for Torque Equation

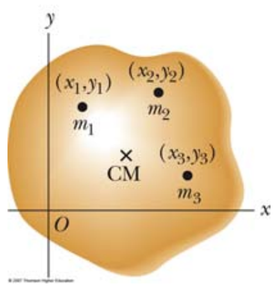
- The net torque is about an axis through any point in the xy plane
- The choice of an axis is arbitrary
- If an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis

Center of Mass

- An object can be divided into many small particles
  - Each particle will have a specific mass and specific coordinates
- The x coordinate of the center of mass will be

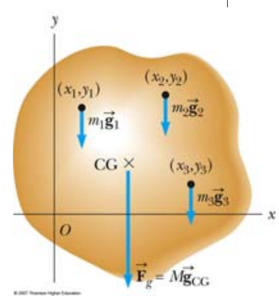
$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

- Similar expressions can be found for the y and z coordinates



Center of Gravity

- All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG)



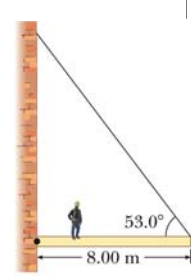
Clicker Question

A meterstick is hung from a string tied at the 25-cm mark. A 0.50-kg object is hung from the zero end of the meterstick, and the meterstick is balanced horizontally. What is the mass of the meterstick?

A. 0.25 kg
B. 0.50 kg
C. 0.75 kg
D. 1.0 kg
E. 2.0 kg

Horizontal Beam Example

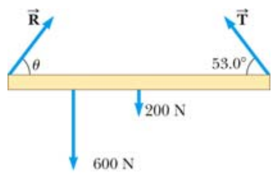
- The beam is uniform
  - So the center of gravity is at the geometric center of the beam
- The person is standing on the beam
- What are the tension in the cable and the force exerted by the wall on the beam?



(a)

Horizontal Beam Example, 2

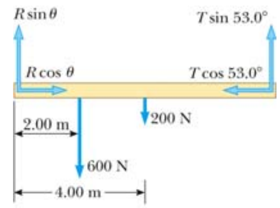
- Analyze
  - Draw a free body diagram
  - Use the pivot in the problem (at the wall) as the pivot
    - This will generally be easiest
  - Note there are three unknowns (T, R, θ)



(b)

Horizontal Beam Example, 3

- The forces can be resolved into components in the free body diagram
- Apply the two conditions of equilibrium to obtain three equations
- Solve for the unknowns



(c)

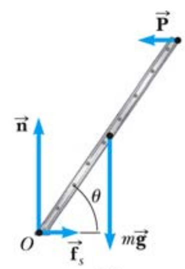
Ladder Example

- The ladder is uniform
  - So the weight of the ladder acts through its geometric center (its center of gravity)
- There is static friction between the ladder and the ground

(a)

Ladder Example, 2

- Analyze
  - Draw a free body diagram for the ladder
  - The frictional force is $f_s = \mu_s n$
  - Let O be the axis of rotation
  - Apply the equations for the two conditions of equilibrium
  - Solve the equations



(b)

Elasticity

- So far we have assumed that objects remain rigid when external forces act on them
  - Except springs
- Actually, objects are deformable
  - It is possible to change the size and/or shape of the object by applying external forces
- Internal forces resist the deformation

Definitions Associated With Deformation

- Stress
  - Is proportional to the force causing the deformation
  - It is the external force acting on the object per unit area
- Strain
  - Is the result of a stress
  - Is a measure of the degree of deformation

Elastic Modulus

- The elastic modulus is the constant of proportionality between the stress and the strain
  - For sufficiently small stresses, the stress is directly proportional to the stress
  - It depends on the material being deformed
  - It also depends on the nature of the deformation

Elastic Modulus, cont

- The elastic modulus, in general, relates what is done to a solid object to how that object responds

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

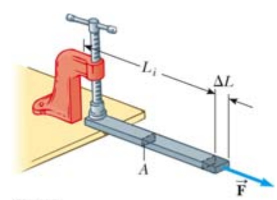
- Various types of deformation have unique elastic moduli

Three Types of Moduli

- Young's Modulus
  - Measures the resistance of a solid to a change in its length
- Shear Modulus
  - Measures the resistance of motion of the planes within a solid parallel to each other
- Bulk Modulus
  - Measures the resistance of solids or liquids to changes in their volume

Young's Modulus

- The bar is stretched by an amount ΔL under the action of the force F
  - See the active figure for variations in values
- The **tensile stress** is the ratio of the magnitude of the external force to the cross-sectional area A



Young's Modulus, cont

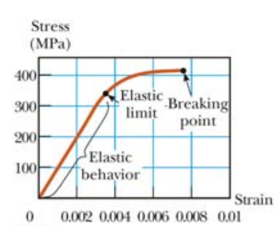
- The **tension strain** is the ratio of the change in length to the original length
- Young's modulus, Y, is the ratio of those two ratios:

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

- Units are N / m^2

Stress vs. Strain Curve

- Experiments show that for certain stresses, the stress is directly proportional to the strain
  - This is the elastic behavior part of the curve

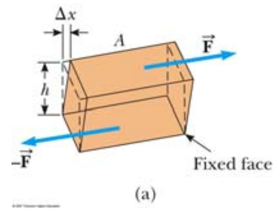


Stress vs. Strain Curve, cont

- The **elastic limit** is the maximum stress that can be applied to the substance before it becomes permanently deformed
- When the stress exceeds the elastic limit, the substance will be permanently deformed
  - The curve is no longer a straight line
- With additional stress, the material ultimately breaks

Shear Modulus

- Another type of deformation occurs when a force acts parallel to one of its faces while the opposite face is held fixed by another force
  - See the active figure to vary the values
- This is called a ***shear stress***



Shear Modulus, cont

- For small deformations, no change in volume occurs with this deformation
  - A good first approximation
- The shear stress is F / A
  - F is the tangential force
  - A is the area of the face being sheared
- The shear strain is Δx / h
  - Δx is the horizontal distance the sheared face moves
  - h is the height of the object

Shear Modulus, final

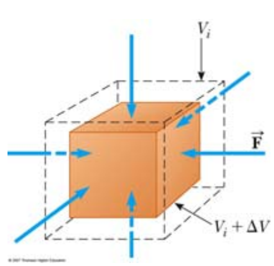
- The shear modulus is the ratio of the shear stress to the shear strain

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

- Units are N / m^2

Bulk Modulus

- Another type of deformation occurs when a force of uniform magnitude is applied perpendicularly over the entire surface of the object
  - See the active figure to vary the values
- The object will undergo a change in volume, but not in shape



Bulk Modulus, cont

- The volume stress is defined as the ratio of the magnitude of the total force, F, exerted on the surface to the area, A, of the surface
  - This is also called the ***pressure***
- The volume strain is the ratio of the change in volume to the original volume

Bulk Modulus, final

- The bulk modulus is the ratio of the volume stress to the volume strain

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V}$$

- The negative indicates that an increase in pressure will result in a decrease in volume

Compressibility

- The compressibility is the inverse of the bulk modulus
- It may be used instead of the bulk modulus

Moduli and Types of Materials

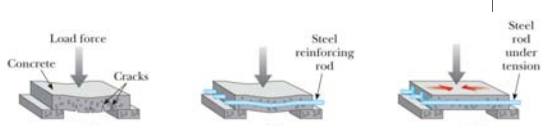
- Both solids and liquids have a bulk modulus
- Liquids cannot sustain a shearing stress or a tensile stress
  - If a shearing force or a tensile force is applied to a liquid, the liquid will flow in response

Moduli Values

TABLE 12.1
Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Prestressed Concrete



- If the stress on a solid object exceeds a certain value, the object fractures
- The slab can be strengthened by the use of steel rods to reinforce the concrete
- The concrete is stronger under compression than under tension

Prestressed Concrete, cont

- A significant increase in shear strength is achieved if the reinforced concrete is prestressed
- As the concrete is being poured, the steel rods are held under tension by external forces
- These external forces are released after the concrete cures
- This results in a permanent tension in the steel and hence a compressive stress on the concrete
- This permits the concrete to support a much heavier load