Week3, Chapter 4

Motion in Two Dimensions

Lecture Quiz

- A particle confined to motion along the *x* axis moves with constant acceleration from $x = 2.0$ m to $x = 8.0$ m during a 1-s time interval. The velocity of the particle at $x = 2.0$ m is 2.0 m/s. What is the acceleration during this time interval?

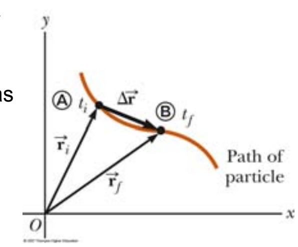
A. 4.0 m/s^2
B. 3.2 m/s^2
C. 6.4 m/s^2
D. 8.0 m/s^2
E. 5.7 m/s^2

Motion in Two Dimensions

- In two- or three-dimensional kinematics, everything is the same as in one-dimensional motion except that we must now use full vector notation
  - Positive and negative signs are no longer sufficient to determine the direction

Position and Displacement

- The position of an object is described by its position vector, $\vec{\mathbf{r}}$
- The **displacement** of the object is defined as the ***change in its position***
- $\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$



Average Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{\mathbf{v}}_{avg} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

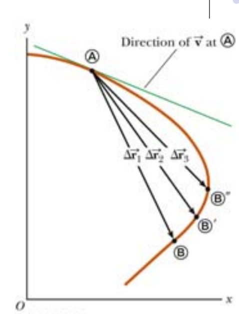
- The direction of the average velocity is the direction of the displacement vector
- The average velocity between points is *independent of the path* taken
  - This is because it is dependent on the displacement, also independent of the path

Instantaneous Velocity

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

  - As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve



Instantaneous Velocity, cont

- The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion
- The magnitude of the instantaneous velocity vector is the speed
  - The speed is a scalar quantity

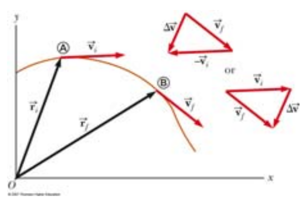
Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

$$\vec{\mathbf{a}}_{avg} \equiv \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Average Acceleration, cont

- As a particle moves, the direction of the change in velocity is found by vector subtraction
  $\Delta \vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$
- The average acceleration is a vector quantity directed along $\Delta \vec{\mathbf{v}}$



Instantaneous Acceleration

- The instantaneous acceleration is the limiting value of the ratio $\Delta \vec{\mathbf{v}} / \Delta t$ as Δt approaches zero

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

  - The instantaneous equals the derivative of the velocity vector with respect to time

Producing An Acceleration

- Various changes in a particle's motion may produce an acceleration
  - The magnitude of the velocity vector may change
  - The direction of the velocity vector may change
    - Even if the magnitude remains constant
  - Both may change simultaneously

Kinematic Equations for Two-Dimensional Motion

- When the two-dimensional motion has a constant acceleration, a series of equations can be developed that describe the motion
- These equations will be similar to those of one-dimensional kinematics
- Motion in two dimensions can be modeled as two *independent* motions in each of the two perpendicular directions associated with the x and y axes
  - Any influence in the y direction does not affect the motion in the x direction

Kinematic Equations, 2

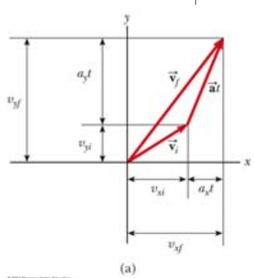
- Position vector for a particle moving in the xy plane $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$
- The velocity vector can be found from the position vector
  $$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$
  - Since acceleration is constant, we can also find an expression for the velocity as a function of time: $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$

Kinematic Equations, 3

- The position vector can also be expressed as a function of time:
  - $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$
  - This indicates that the position vector is the sum of three other vectors:
    - The initial position vector
    - The displacement resulting from the initial velocity
    - The displacement resulting from the acceleration

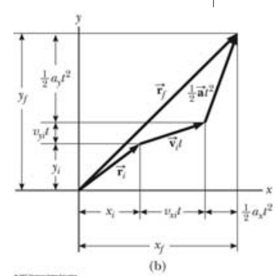
Kinematic Equations, Graphical Representation of Final Velocity

- The velocity vector can be represented by its components
- $\vec{\mathbf{v}}_f$ is generally not along the direction of either $\vec{\mathbf{v}}_i$ or $\vec{\mathbf{a}}$



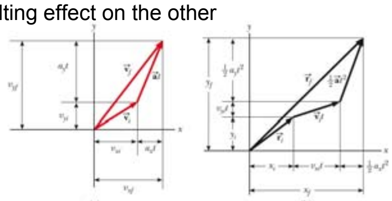
Kinematic Equations, Graphical Representation of Final Position

- The vector representation of the position vector
- $\vec{\mathbf{r}}_f$ is generally not along the same direction as $\vec{\mathbf{v}}_i$ or as $\vec{\mathbf{a}}$
- $\vec{\mathbf{v}}_f$ and $\vec{\mathbf{r}}_f$ are generally not in the same direction



Graphical Representation Summary

- Various starting positions and initial velocities can be chosen
- Note the relationships between changes made in either the position or velocity and the resulting effect on the other



Lecture Quiz

- A boy on a skate board skates off a horizontal bench at a velocity of 10 m/s. One tenth of a second after he leaves the bench, to two significant figures, the magnitudes of his velocity and acceleration are:

A. 10 m/s; 9.8 m/s^2.
B. 9.0 m/s; 9.8 m/s^2.
C. 9.0 m/s; 9.0 m/s^2.
D. 1.0 m/s; 9.0 m/s^2.
E. 1.0 m/s; 9.8 m/s^2.

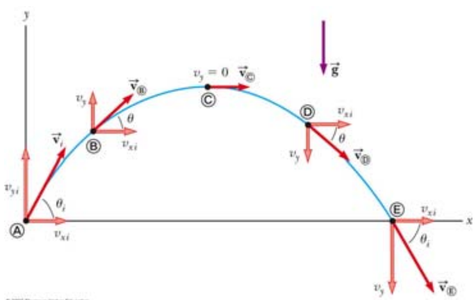
Projectile Motion

- An object may move in both the *x* and *y* directions simultaneously
- The form of two-dimensional motion we will deal with is called **projectile motion**

Assumptions of Projectile Motion

- The free-fall acceleration is constant over the range of motion
  - It is directed downward
  - This is the same as assuming a flat Earth over the range of the motion
  - It is reasonable as long as the range is small compared to the radius of the Earth
- The effect of air friction is negligible
- With these assumptions, an object in projectile motion will follow a parabolic path
  - This path is called the ***trajectory***

Projectile Motion Diagram



Clicker Question

If a baseball player throws a ball with a fixed initial speed, but with variable angles, the ball will move furthest if the angle from horizontal direction is:

A: 0 degrees

B: 30 degrees

C: 45 degrees

D: 60 degrees

E: 90 degrees

Analyzing Projectile Motion

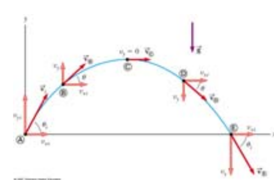
- Consider the motion as the superposition of the motions in the *x*- and *y*-directions
- The actual position at any time is given by:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{g}t^2$$

- The initial velocity can be expressed in terms of its components
  - $v_{xi} = v_i \cos\theta$ and $v_{yi} = v_i \sin\theta$
- The *x*-direction has constant velocity
  - $a_x = 0$
- The y-direction is free fall
  - $a_y = -g$

Effects of Changing Initial Conditions

- The velocity vector components depend on the value of the initial velocity
  - Change the angle and note the effect
  - Change the magnitude and note the effect

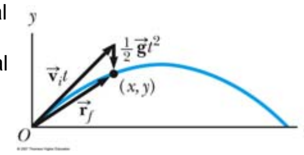


Analysis Model

- The analysis model is the superposition of two motions
  - Motion of a particle under constant velocity in the horizontal direction
  - Motion of a particle under constant acceleration in the vertical direction
    - Specifically, free fall

Projectile Motion Vectors

- $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{g}}t^2$
- The final position is the vector sum of the initial position, the position resulting from the initial velocity and the position resulting from the acceleration

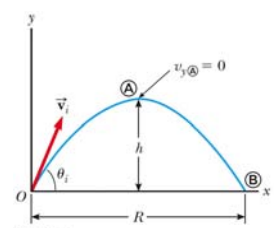


Projectile Motion – Implications

- The *y*-component of the velocity is zero at the maximum height of the trajectory
- The acceleration stays the same throughout the trajectory

Range and Maximum Height of a Projectile

- When analyzing projectile motion, two characteristics are of special interest
- The range, *R*, is the horizontal distance of the projectile
- The maximum height the projectile reaches is *h*



Height of a Projectile, equation

- The maximum height of the projectile can be found in terms of the initial velocity vector:

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

- This equation is valid only for symmetric motion

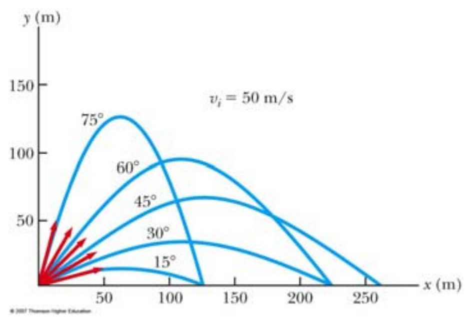
Range of a Projectile, equation

- The range of a projectile can be expressed in terms of the initial velocity vector:

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

- This is valid only for symmetric trajectory

More About the Range of a Projectile

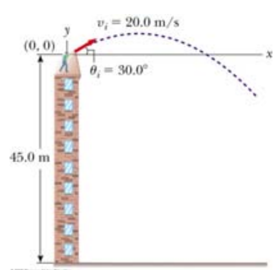


Range of a Projectile, final

- The maximum range occurs at $\theta_i = 45^o$
- Complementary angles will produce the same range
  - The maximum height will be different for the two angles
  - The times of the flight will be different for the two angles

Non-Symmetric Projectile Motion

- Follow the general rules for projectile motion
- Break the *y*-direction into parts
  - up and down *or*
  - symmetrical back to initial height and then the rest of the height
- Apply the problem solving process to determine and solve the necessary equations
- May be non-symmetric in other ways



Uniform Circular Motion

- ***Uniform circular motion*** occurs when an object moves in a circular path with a constant speed
- The associated analysis motion is a *particle in uniform circular motion*
- An acceleration exists since the *direction* of the motion is changing
  - This change in velocity is related to an acceleration
- The velocity vector is always tangent to the path of the object

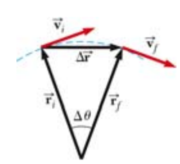
Clicker Question

A particle is undergoing constant-speed circular motion, which of the following statements is correct?

A. The motion velocity is a constant.
B. The velocity is perpendicular to acceleration.
C. The velocity is parallel to the displacement.
D. The acceleration is perpendicular to displacement.
E. The acceleration is perpendicular to the plane of motion.

Changing Velocity in Uniform Circular Motion

- The change in the velocity vector is due to the change in direction
- The vector diagram shows $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \Delta\vec{\mathbf{v}}$



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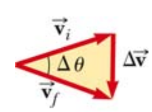


(c)

Centripetal Acceleration

- The acceleration is always perpendicular to the path of the motion
- The acceleration always points toward the center of the circle of motion
- This acceleration is called the ***centripetal acceleration***

Centripetal Acceleration, cont

- The magnitude of the centripetal acceleration vector is given by

$$a_C = \frac{v^2}{r}$$

- The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion

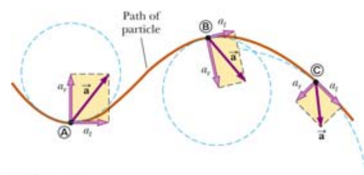
Period

- The ***period***, *T*, is the time required for one complete revolution
- The speed of the particle would be the circumference of the circle of motion divided by the period
- Therefore, the period is defined as

$$T \equiv \frac{2\pi r}{v}$$

Tangential Acceleration

- The magnitude of the velocity could also be changing
- In this case, there would be a ***tangential acceleration***
- The motion would be under the influence of both tangential and centripetal accelerations
  - Note the changing acceleration vectors



Total Acceleration

- The tangential acceleration causes the change in the speed of the particle
- The radial acceleration comes from a change in the direction of the velocity vector

Total Acceleration, equations

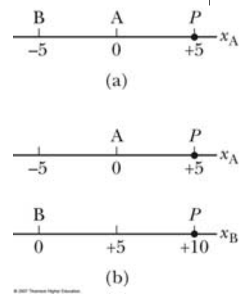
- The tangential acceleration: $a_t = \left|\frac{dv}{dt}\right|$
- The radial acceleration: $a_r = -a_C = -\frac{v^2}{r}$
- The total acceleration:
  - Magnitude $a = \sqrt{a_r^2 + a_t^2}$
  - Direction
    - Same as velocity vector if *v* is increasing, opposite if *v* is decreasing

Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment
- However, the observations seen by each are related to one another
- A *frame of reference* can described by a Cartesian coordinate system for which an observer is at rest with respect to the origin

Different Measurements, example

- Observer A measures point P at +5 m from the origin
- Observer B measures point P at +10 m from the origin
- The difference is due to the different frames of reference being used

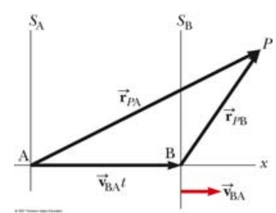


Different Measurements, another example

- The man is walking on the moving beltway
- The woman on the beltway sees the man walking at his normal walking speed
- The stationary woman sees the man walking at a much higher speed
  - The combination of the speed of the beltway and the walking
- The difference is due to the relative velocity of their frames of reference

Relative Velocity, generalized

- Reference frame S_A is stationary
- Reference frame S_B is moving to the right relative to S_A at $\vec{\mathbf{v}}_{AB}$
  - This also means that S_A moves at $-\vec{\mathbf{v}}_{BA}$ relative to S_B
- Define time t = 0 as that time when the origins coincide



Relative Velocity, equations

- The positions as seen from the two reference frames are related through the velocity
  - $\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA}t$
- The derivative of the position equation will give the velocity equation
  - $\vec{\mathbf{u}}_{PA} = \vec{\mathbf{u}}_{PB} + \vec{\mathbf{v}}_{BA}$
    - $\vec{\mathbf{u}}_{PA}$ is the velocity of the particle P measured by observer A
    - $\vec{\mathbf{u}}_{PB}$ is the velocity of the particle P measured by observer B
- These are called the **Galilean transformation equations**

Acceleration in Different Frames of Reference

- The derivative of the velocity equation will give the acceleration equation
- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a *constant velocity* relative to the first frame.