Week 6, Chapter 7 Sect 1-5

Work and Kinetic Energy

Lecture Quiz

- The frictional force of the floor on a large suitcase is least when the suitcase is

A. pushed by a force parallel to the floor.
B. dragged by a force parallel to the floor.
C. pulled by a force directed at an angle θ above the floor.
D. pushed by a force directed at an angle θ into the floor.
E. turned on its side and pushed by a force parallel to the floor.

Introduction to Energy

- The concept of energy is one of the most important topics in science and engineering
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations
- Energy is not easily defined

Energy Approach to Problems

- The energy approach to describing motion is particularly useful when Newton's Laws are difficult or impossible to use
- An approach will involve changing from a particle model to a system model
  - This can be extended to biological organisms, technological systems and engineering situations

Systems

- A *system* is a small portion of the Universe
  - We will ignore the details of the rest of the Universe
- A critical skill is to identify the system

Valid System Examples

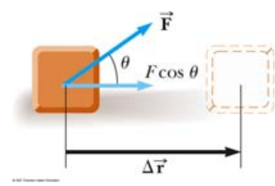
- A valid system may
  - be a single object or particle
  - be a collection of objects or particles
  - be a region of space
  - vary in size and shape

Work

- The work, W, done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and the displacement vectors

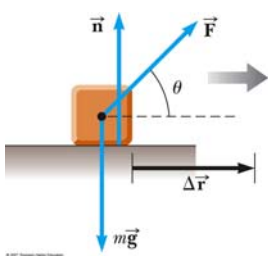
Work, cont.

- $W = F \Delta r \cos \theta$
  - The displacement is that of the point of application of the force
  - A force does no work on the object if the force does not move through a displacement
  - The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application



Work Example

- The normal force and the gravitational force do no work on the object
  - $\cos \theta = \cos 90° = 0$
- The force $\vec{F}$ is the only force that does work on the object



More About Work

- The system and the agent in the environment doing the work must both be determined
  - The part of the environment interacting directly with the system does work on the system
    - Work **by** the environment **on** the system
      - Example: Work done by a hammer (interaction from environment) on a nail (system)
- The sign of the work depends on the direction of the force relative to the displacement
  - Work is positive when projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the same direction as the displacement
  - Work is negative when the projection is in the opposite direction

Units of Work

- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter
  - J = N · m

Work Is An Energy Transfer

- This is important for a system approach to solving a problem
- If the work is done on a system and it is positive, energy is transferred to the system
- If the work done on the system is negative, energy is transferred from the system

Work Is An Energy Transfer, cont

- If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary
  - This will result in a change in the amount of energy stored in the system

Clicker Question

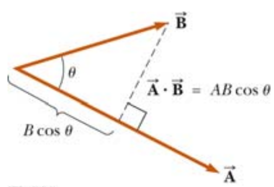
- Assuming that the earth is moving around the sun with a uniform circular motion. The work done by the gravitation force on earth is:

A. Positive
B. Negative
C. Zero
D. Impossible to determine

Scalar Product of Two Vectors

- The scalar product of two vectors is written as $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}$
  - It is also called the dot product
- $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \equiv A\,B\cos\theta$
  - θ is the angle *between A* and *B*
- Applied to work, this means

  $W = F\Delta r\cos\theta = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$



Scalar Product, cont

- The scalar product is commutative
  - $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$
- The scalar product obeys the distributive law of multiplication
  - $\vec{\mathbf{A}}\cdot\left(\vec{\mathbf{B}}+\vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}}+\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}$

Dot Products of Unit Vectors

- $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$

  $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$
- Using component form with vectors:

  $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

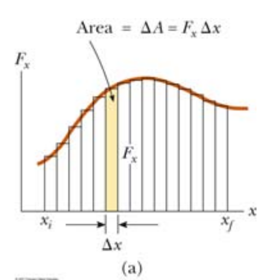
  $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$

  $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$

Work Done by a Varying Force

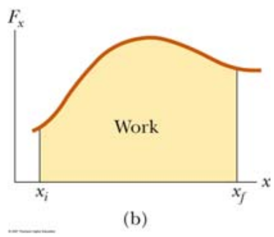
- Assume that during a very small displacement, Δx, F is constant
- For that displacement, $W \sim F\,\Delta x$
- For all of the intervals,

  $W \approx \sum_{x_i}^{x_f} F_x\Delta x$



Work Done by a Varying Force, cont

- $\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$
- Therefore, $W = \int_{x_i}^{x_f} F_x dx$
- The work done is equal to the area under the curve between x_i and x_f



Work Done By Multiple Forces

- If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

- In the general case of a net force whose magnitude and direction may vary

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum \vec{\mathbf{F}} \right) d\vec{\mathbf{r}}$$

Work Done by Multiple Forces, cont.

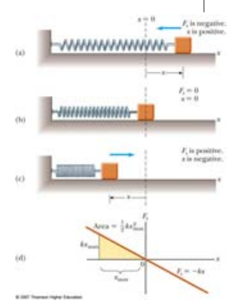
- If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

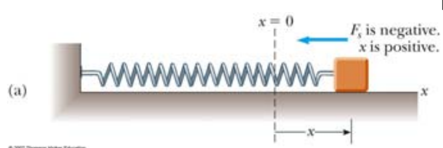
  - Remember work is a scalar, so this is the algebraic sum

Work Done By A Spring

- A model of a common physical system for which the force varies with position
- The block is on a horizontal, frictionless surface
- Observe the motion of the block with various values of the spring constant



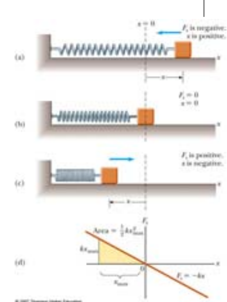
Hooke's Law



- The force exerted by the spring is
  $F_s = - kx$
  - x is the position of the block with respect to the equilibrium position ($x = 0$)
  - k is called the spring constant or force constant and measures the stiffness of the spring
- This is called Hooke's Law

Hooke's Law, cont.

- When x is positive (spring is stretched), F is negative
- When x is 0 (at the equilibrium position), F is 0
- When x is negative (spring is compressed), F is positive



Hooke's Law, final

- The force exerted by the spring is always directed opposite to the displacement from equilibrium
- The spring force is sometimes called the *restoring force*
- If the block is released it will oscillate back and forth between $-x$ and x

Clicker Question

- The work done by a spring to be compressed from x=0 to x=a is: $W_0 = -ka^2/2$. What is the work done by the same spring to be compressed from x=a to x=2a?

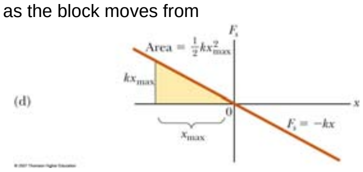
A. W_0
B. $2W_0$
C. $3W_0$
D. $4W_0$
E. $5W_0$

Work Done by a Spring

- Identify the block as the system
- Calculate the work as the block moves from $x_i = -x_{max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_x dx = \int_{-x_{max}}^{0} (-kx)dx = \frac{1}{2}kx_{max}^2$$

- The total work done as the block moves from $-x_{max}$ to x_{max} is zero



Work Done by a Spring, cont.

- Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$
- The work done by the spring on the block is

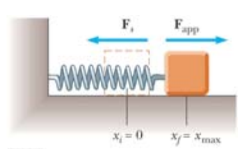
$$W_s = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

  - If the motion ends where it begins, W = 0

Spring with an Applied Force

- Suppose an external agent, F_{app}, stretches the spring
- The applied force is equal and opposite to the spring force
- $F_{app} = -F_s = -(-kx) = kx$
- Work done by F_{app} is equal to $-\frac{1}{2} kx^2_{max}$
- The work done by the applied force is

$$W_{app} = \int_{x_i}^{x_f} (kx)dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$



Kinetic Energy

- Kinetic Energy is the energy of a particle due to its motion
  - $K = \frac{1}{2} mv^2$
    - K is the kinetic energy
    - m is the mass of the particle
    - v is the speed of the particle
- A change in kinetic energy is one possible result of doing work to transfer energy into a system

Kinetic Energy, cont

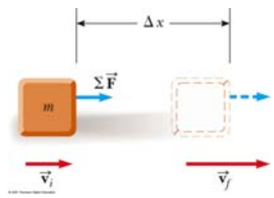
- Calculating the work:

$$W = \int_{x_i}^{x_f} \sum F\, dx = \int_{x_i}^{x_f} ma\, dx$$

$$W = \int_{v_i}^{v_f} mv\, dv$$

$$\sum W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = K_f - K_i = \Delta K$$



Clicker Question

In a symmetric projectile motion, the initial speed is fixed. When the object lands, the kinetic energy would be maximum, if the angle of this motion from horizontal is:

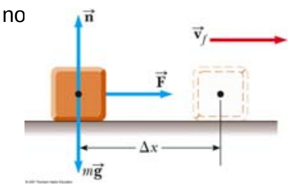
A. 0
B. 90 degrees
C. 30 degrees
D. 45 degrees
E. Does not matter

Work-Kinetic Energy Theorem

- The Work-Kinetic Energy Theorem states $\Sigma W = K_f - K_i = \Delta K$
- When work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.
  - The speed of the system increases if the work done on it is positive
  - The speed of the system decreases if the net work is negative
  - Also valid for changes in rotational speed

Work-Kinetic Energy Theorem – Example

- The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement
- $W = F\,\Delta x$
- $W = \Delta K = ½\, mv_f^2 - 0$



Lecture Quiz

- When a car goes around a circular curve on a level road,

A. no frictional force is needed because the car simply follows the road.
B. the frictional force of the road on the car increases when the car's speed decreases.
C. the frictional force of the road on the car increases when the car's speed increases.
D. the frictional force of the road on the car increases when the car moves to the outside of the curve.
E. there is no net frictional force because the road and the car exert equal and opposite forces on each other.