PHYS 777
Plasma Physics and Magnetohydrodynamics

2004 Fall

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Lecture 1

Introduction & Single Particle Motions of Plasma

Plasma:

Quasi neutral gas of charges and neutral particles which exhibits collective behavior.

Saha Equation for Ionization:

$$\frac{n_i}{n_n} = 2.4 \times 10^{15} \frac{T^{\frac{3}{2}}}{n_i} e^{\frac{-U_i}{KT}}$$

U_i : ionization energy

T : temperature

n_i : number density of ionized gas

n_n : number density of neutral gas

Motion of Neutral Gas:

Maxwell's Distribution

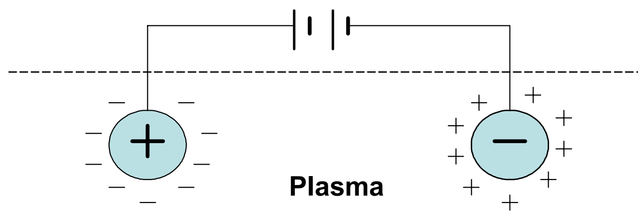
$$f(v_x, v_y, v_z) = A \exp\left[\frac{-\frac{1}{2}m(V_x^2 + V_y^2 + V_z^2)}{KT}\right]$$

$$A = n\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}$$

$$E_{Av} = \frac{3}{2}KT$$

Debye Length and Shielding

Plasma can shield out electric potential applied to it.



If T=0

+ will attract same amount of — from plasma.

− will attract same amount of + from plasma

Shielding is perfect.

If Tπ 0

Thermal motion may make some local charge imbalance. The scale of this imbalance is called **Debye length**.

Thermal energy ï potential energy

$$\lambda_D = \left(\frac{KT_e}{4\pi n e^2}\right)^{\frac{1}{2}}$$

T \` λ \`

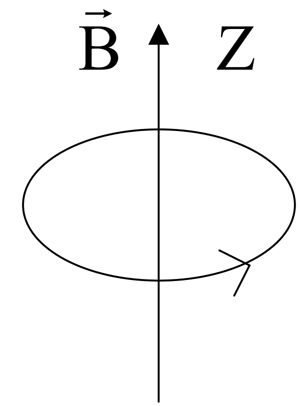
n \` λ ↓

$$\lambda_D = 6.9\left(\frac{T}{n}\right)^{\frac{1}{2}} \qquad T \text{ in } {}^{o}k$$

$$= 740\left(\frac{T}{n}\right)^{\frac{1}{2}} \qquad T \text{ in } eV$$

For plasma λ_D **Ü** other scales of interest as charge imbalance can be introduced in the scale of λ_D

Single Particle Motions



Uniform B field — cyclotron gyration

Lorentz Force = centrifugal force

$$m\vec{a} = q\vec{v}\times\vec{B} = m\frac{d\vec{v}}{dt}$$

$$mv_x = qBv_y \qquad mv_y = -qBv_x \qquad mv_z = 0$$

$$\text{Then:} \quad \ddot{v}_x = -(\frac{qB}{m})^2 v_x$$

$$\ddot{v}_y = -(\frac{qB}{m})^2 v_y$$

oscillation equation

Frequency of Simple Harmonic Motion

$$\omega_C = \frac{|q| B_G}{mc} \qquad \text{Cyclotron Frequency}$$

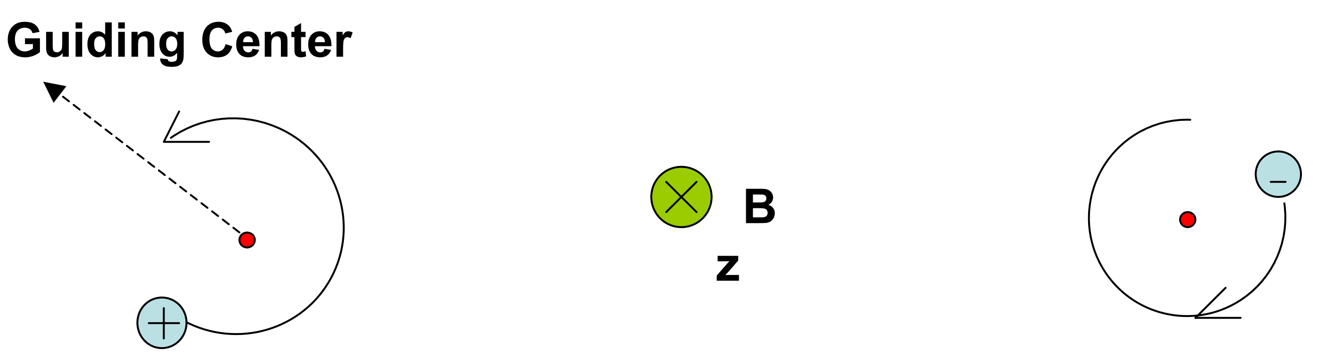
final solution

$$x - x_o = r_2 \, Sin \, \omega_c t$$

$$y - y_o = \pm r_2 \, Cos \, \omega_c t$$

$$r_2 = \frac{v}{\omega_c} = \frac{mvc}{qB} \quad \text{(Larmor Radius)}$$

$x_o \quad y_o$ **coordinates of guiding center**



Any z direction motion will make the orbit helix

Adding Uniform $\vec{E}$

Assume $E_y = 0 \quad E_x \neq 0 \quad E_z \neq 0$

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

for constant E,

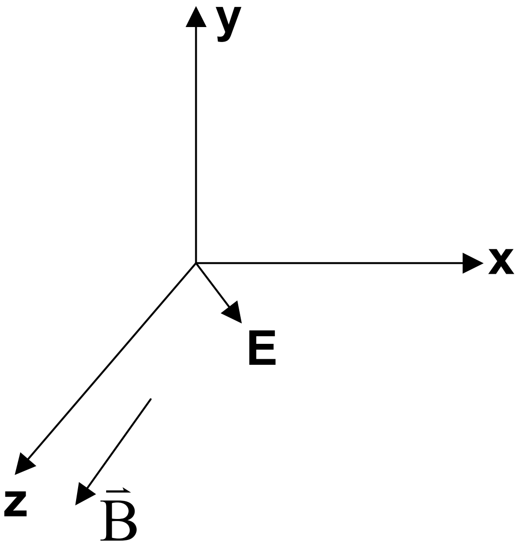
$$\ddot{v}_x = -\omega_c^2 v_x$$

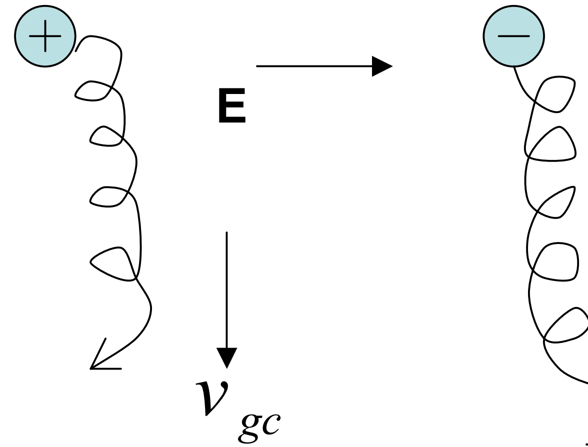
$$\ddot{v}_y = \mp\omega_c(\frac{q}{m}E_x \pm \omega_c v_c) = -\omega_c^2(\frac{E_x}{B} + v_y)$$

$$v_x = v_\perp e^{+i\omega_c t}$$

$$v_y = \pm i v_\perp e^{i\omega_c t} - \frac{E_x}{B}$$

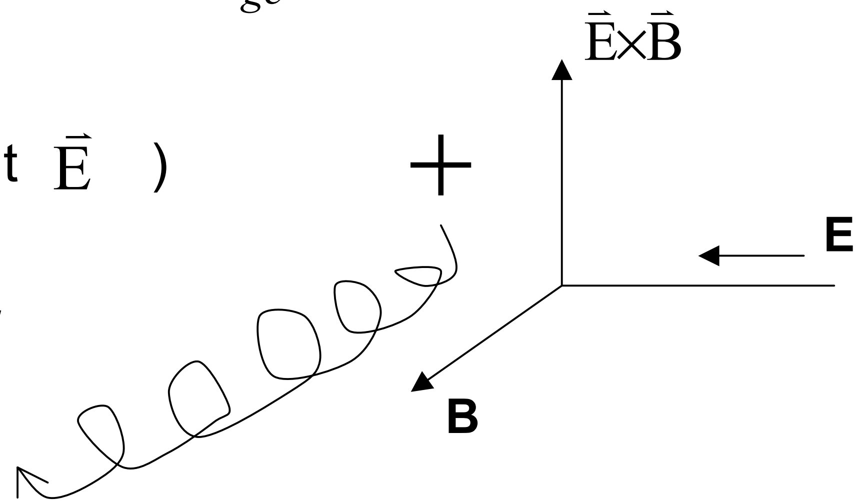
The Larmor motion is the same, but guiding center is drifting





General Formula: (constant $\vec{E}$)

$$\vec{v}_{\perp} gc = \frac{\vec{E}\times\vec{B}}{B^2} = \vec{v}_E$$



slanted helix with changing pitch

As $\vec{F}_E = q\vec{E}$

We can further generalize :

$$\vec{v}_{gc} = \frac{\vec{F}\times\vec{B}}{qB^2}$$

Gravitational Field

$$\vec{v}_f = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}$$

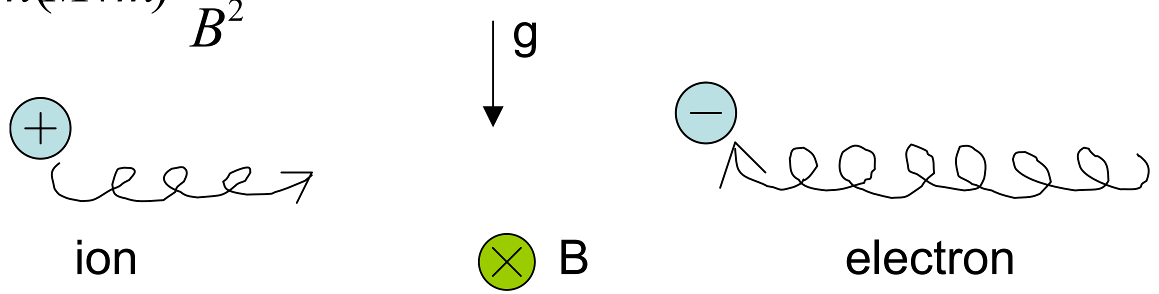
$$\vec{v}_g = \frac{m}{q}\frac{\vec{g}\times\vec{B}}{B^2}$$

(+) & (−) charges

drift in opposite direction

Net current is generated

$$\vec{j} = n(M+m)\frac{\vec{g}\times\vec{B}}{B^2}$$



Non-uniform B & E Fields

$\nabla B \perp B$: Grad - B drift

$$F_y = \mp q v_\perp r_L \frac{1}{2}\frac{\partial B}{\partial y}$$

$$v_{gc} = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2} = \frac{1}{q}\frac{F_y}{|B|}\hat{x} = \mp\frac{v_\perp r_L}{B}\frac{1}{2}\frac{\partial B}{\partial y}\vec{x}$$

$$\vec{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\vec{\nabla}B}{B^2}$$

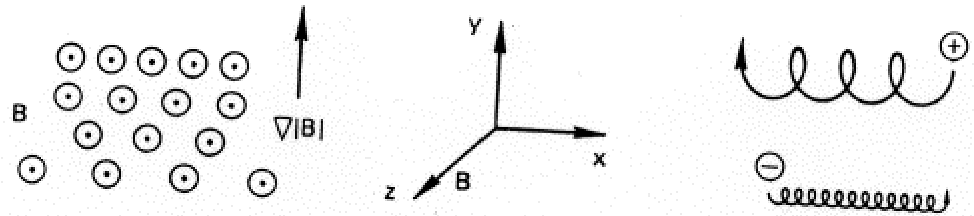


FIGURE 2-5 The drift of a gyrating particle in a nonuniform magnetic field.

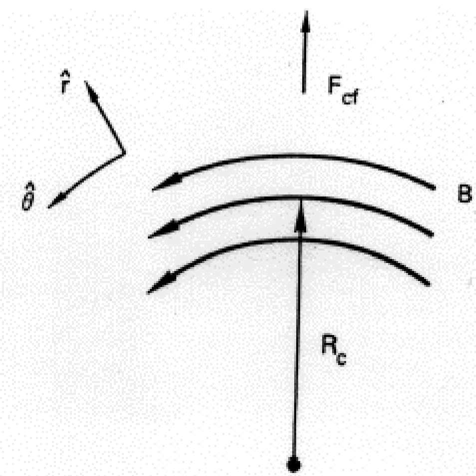
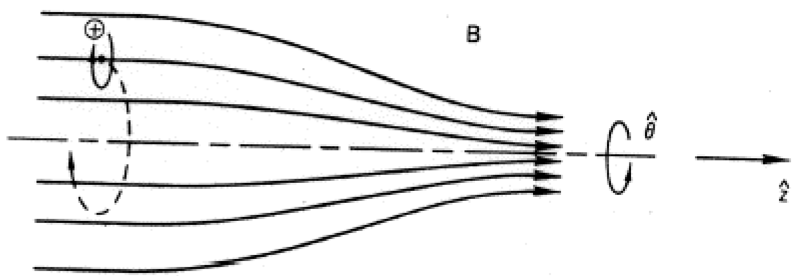


FIGURE 2-6 A curved magnetic field.



Drift of a particle in a magnetic mirror field. FIGURE 2-7

Curved $\vec{B}$: Curvature Drift

Centrifugal force $\vec{F}_{cf} = \frac{mv_{\parallel}^2}{R_C}\hat{r} = mv_{\parallel}^2 \frac{\vec{R}_C}{R_C^2}$

$$\vec{v}_R = \frac{1}{q}\frac{\vec{F}_{cf}\times\vec{B}}{B^2} = \frac{mv_{\parallel}^2}{qB^2}\,\frac{\vec{R}_C\times\vec{B}}{R_C^2}$$

Magnetic Mirrors $\nabla\vec{B} \,||\, \vec{B}$

$$\vec{\nabla}\cdot\vec{B} = 0$$

So $\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \qquad (B_\theta = 0 \quad \frac{\partial}{\partial\theta} = 0)$

if $\frac{\partial B_z}{\partial z}$ does not vary much with r.

$$rB_r = -\int_0^r r\frac{\partial B_z}{\partial z} = -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0}$$

$$B_r = -\frac{1}{2}r\left[\frac{\partial B_z}{\partial z}\right]_{r=0}$$

Lorentz force $\vec{F} = (\vec{v}\times\vec{B})q$

$$F_r = q(vB_z)$$

$$F_\theta = q(-v_r B_z + v_z B_r)$$

$$F_z = -qv_\theta B_r = \frac{1}{2}qv_\theta r\frac{\partial B_z}{\partial z}$$

magnetic moment of gyrating particle $\mu = \frac{\frac{1}{2}mv_\perp^2}{B}$

so $\bar{F}_z = \overline{(\frac{1}{2} q v_\theta r \frac{\partial B_z}{\partial z})}$

$$= -\frac{1}{2}\frac{m v_\perp^2}{B}\frac{\partial B_z}{\partial z} = -\mu \frac{\partial B_z}{\partial z}$$

$$\vec{F}_\parallel = -\mu \nabla_\parallel B$$

prove that μ is invariant $\frac{d\mu}{dt} = 0$

motion along B:

$$m\frac{dv_\parallel}{dt} = -\mu \frac{\partial B}{\partial S}$$

$$m v_\parallel \frac{dv_\parallel}{dt} = \frac{d}{dt}(\frac{1}{2} m v_\parallel^2) = -\mu \frac{\partial B}{\partial S}\frac{dS}{dt} = -\mu \frac{dB}{dt}$$

(where $\frac{dS}{dt} /\!/ v_\parallel$)

energy of particles is conserved

$$\frac{d}{dt}(\frac{1}{2} m v_\parallel^2 + \frac{1}{2} m v_\perp^2) = \frac{d}{dt}(\frac{1}{2} m v_\parallel^2 + \mu B) = 0$$

$$-\mu \frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

$$\frac{d\mu}{dt} = 0$$

$B\uparrow$, $v_\perp \uparrow$ $v_\parallel \downarrow$

magnetic mirror occurs if B in the throat is

very high, $v_{\|} = 0$ ----- reflected

Plasma Oscillation

If electrons are displaced from ions, an $\vec{E}$ field will be generated to restore position of electrons. Electrons will overshoot and oscillate around their equilibrium positions. Oscillation frequency is called plasma frequency.

Three equations (next lecture)

motion $$mn_e[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \nabla)\vec{v}_e] = -en_e\vec{E}$$

continuity $$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0$$

$$\varepsilon_0 P \cdot \vec{E} = e(n_i - n_e)$$

linearize equations
$$n_e = n_0 + n_1$$
$$v_e = v_0 + v_1$$
$$E = E_0 + E_1$$

oscillation equation: $$\frac{\partial^2 v_e}{\partial t^2} = -\omega_P^2 v_e$$

$$\omega_P = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m}}$$ plasma frequency

Homework

Chen Book

1-5, 1-6 and 2-12