Lecture 4 Waves

A simple wave: Sound wave in the air.—restoring pressure balance

In the Sun, possible waves:

- Alfven wave: magnetic tension
- Sound wave: pressure
- Gravity wave: gravity force
- or combination of above

Basic Equations

$$\frac{\partial \rho}{\partial t} + \rho \nabla \bullet \overrightarrow{V} = 0$$

$$\rho \frac{\partial \overrightarrow{V}}{\partial t} = -\nabla \rho + (\nabla \times \overrightarrow{B}) \times \frac{\overrightarrow{B}}{\mu} - \rho g \hat{z} - 2\rho \overrightarrow{\Omega} \times \overrightarrow{V}$$

$$\frac{\partial}{\partial t}\left(\frac{P}{\rho r}\right) = 0$$ adiabatic law

$$\frac{\partial \overrightarrow{B}}{\partial t} = \nabla \times (\nabla \times \overrightarrow{B})$$

$\nabla \bullet \vec{B} = 0$ $\quad j = \frac{1}{\mu} \nabla \times \vec{B}$ $\quad T = \frac{mP}{k\rho}$

If $\uparrow\uparrow B_0 \quad \uparrow^z$ $\quad \rho_0(z) = \rho_0 e^{-\frac{z}{\Lambda}}$ $\quad P = P_0 e^{-\frac{z}{\Lambda}}$

$\Lambda = \frac{P_0}{\rho_0 g}$ Scale height 150Km in photosphere, 10^8m in corona

Let's consider small departure from equilibrium

$$\rho = \rho_0 + \rho_1,\ \mathrm{v} = \mathrm{v}_1,\ \mathrm{P} = \mathrm{P}_0 + P_1,\ \vec{\mathrm{B}} = \vec{\mathrm{B}}_0 + \vec{B}_1$$

Linearize equation

$$\frac{\partial \rho_1}{\partial t} + (v_1 \bullet \nabla)\rho_0 + \rho_0(\nabla \bullet \vec{v}_1) = 0$$

$$\rho_0 \frac{\partial v_1}{\partial t} = -\nabla \vec{P}_1 + (\nabla \times \vec{B}_1) \times \frac{B_0}{nl} - \rho_1 g \hat{z} - 2\rho_0 \vec{\Omega} \times \vec{v}_1$$

$$\frac{\partial \rho_1}{\partial t} + (v_1 \bullet \nabla) P_0 - C_s^2 (\frac{\partial \rho_1}{\partial t} + (\vec{v}_1 \bullet \vec{\nabla})\rho_0) = 0$$

$$\frac{\partial B_1}{\partial t} = \nabla \times (\vec{v}_1 \times \vec{B}_0)$$

$$\nabla \bullet \vec{B}_1 = 0$$

Where $C_s^2 = \frac{\gamma p_0}{\rho_0} = \frac{\gamma kT}{m}$

Remove everything except $\vec{v}_1$

$$\frac{\partial^2 \vec{v}_1}{\partial t^2} = C_s^2 \nabla(\nabla \bullet \vec{v}_1) - (\gamma - 1) g \hat{z} (\nabla \bullet \vec{v}_1) - g \nabla v_{1z} - 2\Omega \times \frac{\partial v_1}{\partial t} + [\nabla \times (\nabla \times (v_1 \times B_0))] \times \frac{\vec{B}_0}{\mu \rho_0}$$

Seek plane-wave solution

$$v_1(\vec{r}, t) = \vec{v}_1 e^{i(\vec{k} \bullet \vec{r} - \omega t)}$$

$\vec{k}$ wavenumber vector

ω frequency $\quad \mathrm{T} = \frac{2\pi}{\omega} \quad \lambda = \frac{2\pi}{\mathrm{k}}$

If assume ρ_0 is a locally constant (WKB approx)

$$\omega^2 \vec{v}_1 = C_s^2 \vec{k} (\vec{k} \bullet \vec{v}_1) + i(\gamma - 1) g \hat{z} (\vec{k} \bullet \vec{v}) + igk v_{1z} - 2i\omega \Omega \times \vec{v}_1 + [\vec{k} \times (\vec{k} \times (\vec{v}_1 \times \vec{B}))] \times \frac{\vec{B}}{\mu \rho_0} \qquad (4.16)$$

Main objective: drive $\omega = \omega(\vec{k})$ dispersion relation

$\vec{v}_p = \frac{\omega}{k} \vec{k}$ phase velocity

$\mathrm{v_{gx}} = \frac{\partial \omega}{\partial k_x}, \ \mathrm{v_{gy}} = \frac{\partial \omega}{\partial k_y}, \ \mathrm{v_{gz}} = \frac{\partial \omega}{\partial k_z}$ group velocity

Sound wave

$g = B_0 = \Omega = 0 \qquad \omega^2 \vec{v}_1 = C_s^2 \vec{k}(\vec{k} \bullet \vec{v}_1) \qquad \omega^2 = k^2 C_s^2 \qquad \omega = kC_s \qquad v_p = v_g = C_s$

$\gamma = \frac{5}{3} \quad m = 0.5 m_p \quad C_s = 166 T^{\frac{1}{2}} \; m/s$

$C_s = 10$ Km/s photosphere

200 Km/s corona

Sound wave is longitudinal wave

Magnetic wave

Alfven speed $v_A = \frac{B_0}{(\mu\rho_0)^{1/2}} = 2.8 \times 10^{16} \frac{B_0}{n_0^t}$

In corona above sunspots $n_0 = 10^{16} m^{-3}$, $B_0 = 10G$, $v_A = 300 km/s$

In photospheric network $n_0 = 10^{23} m^{-3}$, $B_0 = 10^3 G$, $v_A = 10 km/s$

Let's ignore $\Omega, P \& G$ to derive Alfven wave

Look at fig. 4.1 Alfven wave could be longitudinal or transverse, or can be propogate Obliquely. (4.16) becomes $\omega^2 v_1 = [\vec{k} \times (\vec{k} \times (v \times \hat{B}_0))] \times \hat{B}_0 v_A^2$

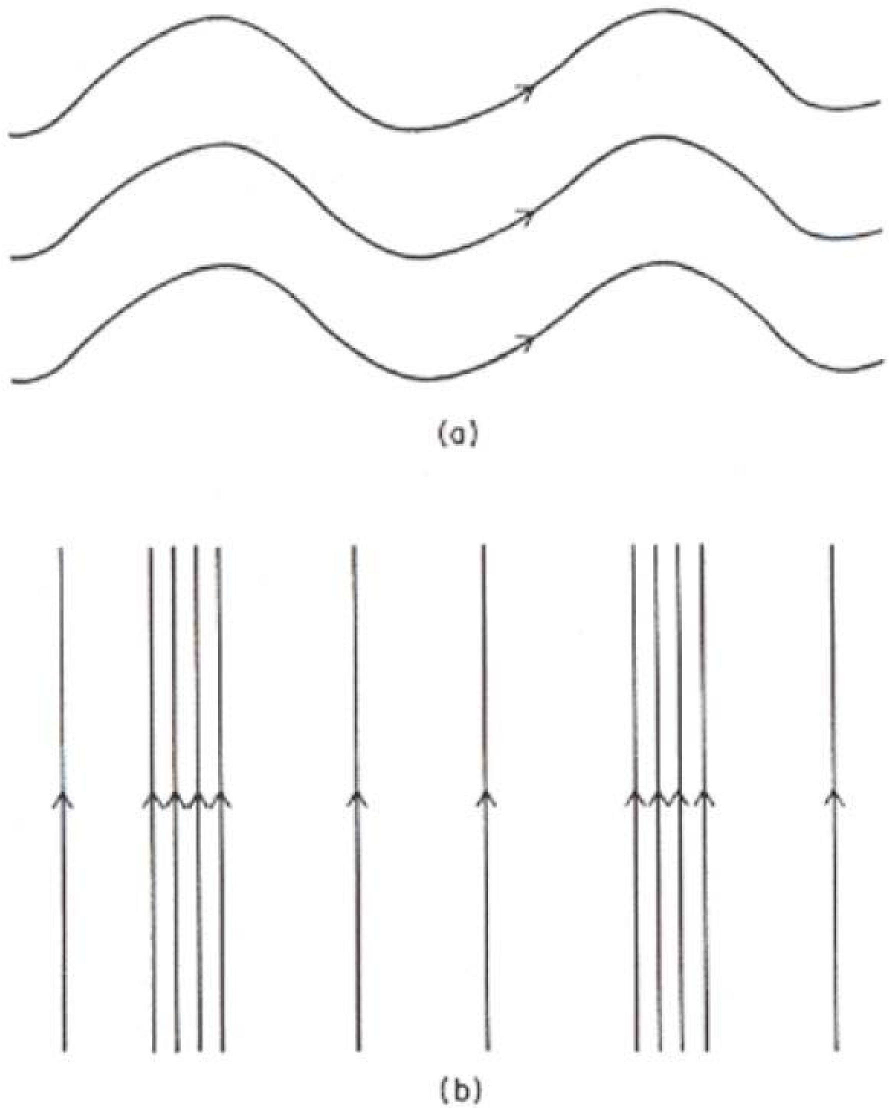

Fig. 4.1. (a) The ripples of magnetic field lines caused by an Alfvén wave propagating along the field. (b) The compression and rarefaction of magnetic field lines due to a compressional Alfvén wave propagating across the field.

expand $\omega^2 v_1 / v_A^2 = (\vec{k} \bullet \vec{B}_0)^2 \vec{v}_1 - (\vec{k} \bullet \vec{v}_1)(\vec{k} \bullet \vec{B}_0)\vec{B}_0 + [(\vec{k} \bullet \vec{v}_1) - (\vec{k} \bullet \vec{B}_0)(\vec{B}_0 \bullet \vec{v}_1)]\vec{k}$

Assume angle between $\vec{B}_0$ & $\vec{k}$ is θ_B

$$\omega^2 \vec{v}_1 / v_A^2 = k^2 \cos^2 \theta_B \vec{v}_1 - (\vec{k} \bullet \vec{v}) k \cos\theta_B \vec{B}_0 + [(\vec{k} \bullet \vec{v}_1) - k\cos\theta_B (\vec{B}_0 \bullet \vec{v}_1)]\vec{k}$$

$\vec{\nabla} \bullet \vec{B} = 0 \rightarrow \vec{k} \bullet \vec{B}_1 = 0$ $\vec{B}_1$ normal to $\vec{k}$

also $\vec{B}_0 \bullet \vec{v}_1 = 0$ Perturbed velocity normal to $\vec{B}_0$

Eq. 4.21 becomes $(\bullet \vec{k})$

$$(\omega^2 - k^2 v_A^2)\vec{k} \bullet \vec{v}_1 = 0 \qquad (4.23)$$

Two solutions

$\vec{k} \bullet \vec{v}_1 = 0$ $\omega = k v_A \cos\theta_B$ shear Alfven wave

$v_p = v_A \cos\theta_B$

Wave propagate fastest in $\vec{B}_0$ direction, not at all in a direction normal to $\vec{B}_0$

Other properties

$\vec{v}_g = v_A \vec{B}_0$ energy is carried at v_A in $\vec{B}_0$ direction

$\vec{v}_1 = -\frac{\vec{B}_1}{(\mu\rho_0)^{1/2}}$ $\vec{B}_1$ and $\vec{v}_1$ in same direction lying in a plane parallel to wave from

$\vec{B}_0 \bullet \vec{B}_1 = 0$ magnetic field perturbation is normal to $\vec{B}_0$

Second solution to (4.23)

$\omega = k v_A$ compression Alfven wave

$\vec{v}_g = v_A \vec{k}$ energy is propagate isotropically

$\theta = 90^\circ$, $v_1 \parallel k$ longitudinal wave,----pressure

$\theta = 0^\circ$, $v_1 \perp k$ transverse wave, ----tension

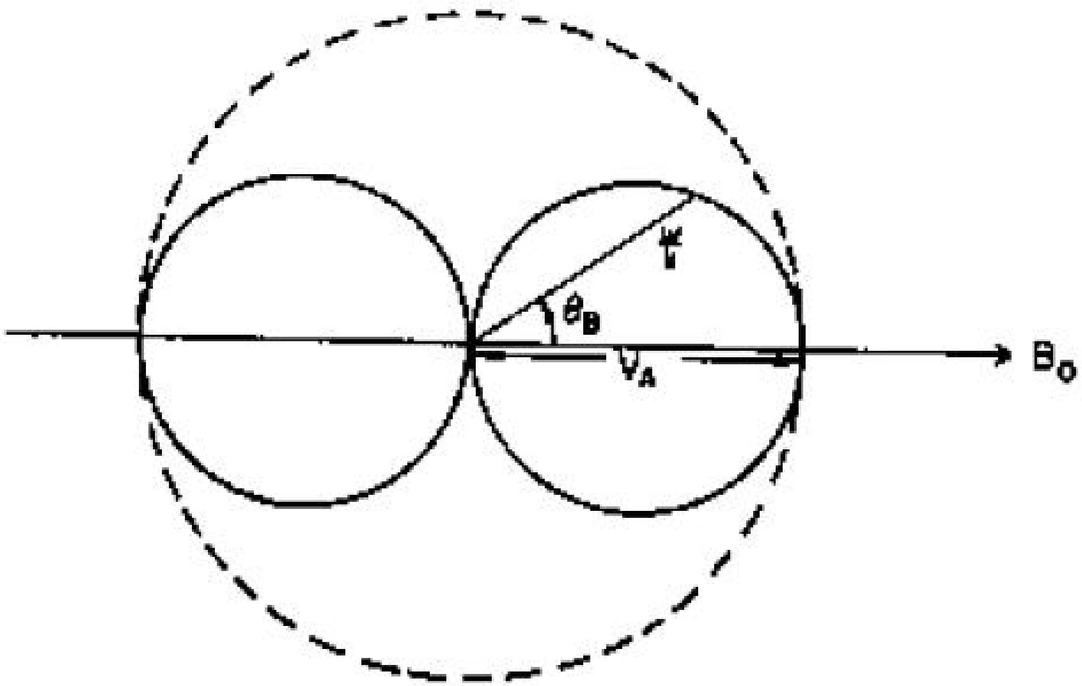

Fig. 4.2. A polar diagram for Alfvén waves (solid curve) and compressional Alfvén waves (dashed curve). The length of the radius vector at an angle of inclination θ_B to the equilibrium magnetic field ($\mathbf{B}_0$) is equal to the phase speed (ω/k) for waves propagating in that direction.

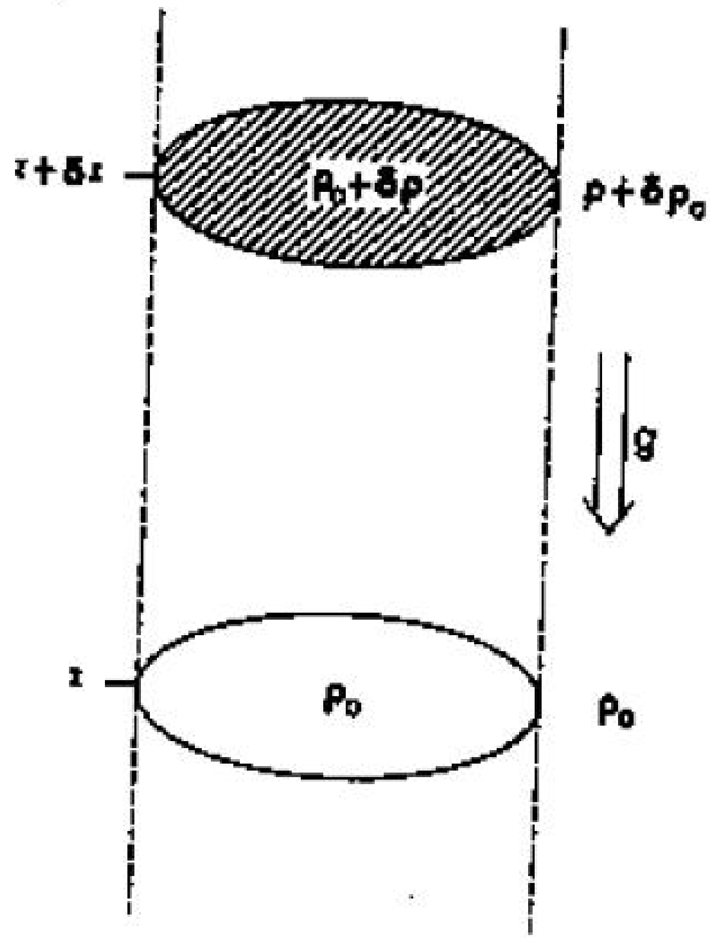

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Fig. 4.5. An element of plasma moves vertically from a height z, where the external density is ρ_0, to a height $z + \delta z$, where the external density is $\rho_0 + \delta\rho_0$.

Internal Gravity wave

consider displacement of a blob of plasma, displace vertically from equilibrium. It remains in Pressure equilibrium with surrounding. Density change inside the blob are adiabatic

At equilibrium position $\frac{d\rho_0}{dz} = -\rho_0 g$

Outside the blob at z+dz $\delta\rho_0 = -\rho g \delta z$ $\delta\rho_0 = -\frac{d\rho_0}{dz} dz$

Adiabatic $\frac{P}{\rho^r} = const$ $C_S^2 = \frac{rP_0}{\rho_0} = \frac{rkT}{m}$ $\delta P = C_S^2 \delta\rho$

New density inside the blob differs from ambient density at new height, the blob experiences a Bouyancy force:

$$g(\delta\rho_0 - \delta\rho) = -N^2 \rho_0 dz$$

$$N^2 = -g\left(\frac{1}{\rho}\frac{d\rho_0}{dz} + \frac{g}{C_S}\right) = \frac{g}{T_0}\left[\frac{dT_0}{dz} - \left(\frac{dT}{dz}\right)_{ad}\right]$$

N: Brunt---Vaisala frequency

$$\left(\frac{dT}{dz}\right)_{ad} = -(r-1)\frac{T_0 g}{C_S^2} \qquad N^2 = \frac{(r-1)g^2}{C_S^2} \quad (T_0 \text{ constant})$$

In the presence of horizontal magnetic fields $N^2 = -g\left(\frac{1}{\rho_0}\frac{d\rho_0}{dz} + \frac{g}{C_S^2 + v_A^2}\right)$

If T is constant $N^2 = \frac{g}{C_S^2}\left(r - \frac{C_S^2}{C_S^2 + v_A^2}\right)$

If the only force is due to buoyancy $F = m\frac{d^2 z}{dt^2}$ $\rho_0 \frac{d^2 \delta z}{dt^2} = -N^2 \rho_0 \delta z$

This is a simple harmonic motion W=N, this is true only if N>0

If N<0 $-(\frac{dT_0}{dz}) > -(\frac{dT}{dz})_{ad} \rightarrow$ convective instability exponentially growing solution

Go back to equation 4.15, assuming $\omega << kC_S$ derive dispersion $\omega^2 C_S^2 = (r-1)g^2(\frac{1-k_z}{k^2})$

This is called internal gravity wave ---- not surface
N-1~50s only propagate horizontally

$$\omega = N(1-\frac{k_z^2}{k^2})^{\frac{1}{2}}$$

$$v_{gz} = \frac{\partial \omega}{\partial k_z} = -\frac{\omega k_z}{k^2}$$ carry energy downword

Intertial waves

Consider coriolis force alone $\frac{\partial v_1}{\partial t} = -2\vec{\Omega} \times \vec{v}_1$ $\omega = \pm \frac{2(\vec{k} \bullet \vec{\Omega})}{k}$ 4-10

Effect: cause a small frequency splitting of Alfven wave $\omega^2 = \omega_A^2(1 \pm \frac{\omega_z}{\omega_A})$

Magneto accoustic waves

Let's consider P&B, ignore g& Ω. 4.21 becomes

$$\omega^2 \vec{v}_1 / v_A^2 = k^2 \cos^2 \theta_B \vec{v}_1 - (\vec{k} \bullet \vec{v}_1) k \cos\theta_B \widehat{B}_0 + [(1+\frac{C_S^2}{v_A^2})(\vec{k} \bullet \vec{v}_1) - k\cos\theta_B(\widehat{B}_0 \bullet \vec{v})]\vec{k}$$

Take scalar product with $\vec{k}$ & $\vec{B}_0$ in turn

$$(-\omega^2 + k^2 C_S^2 + k^2 v_A^2)(\vec{k} \bullet \vec{v}_1) = k^3 v_A^2 \cos\theta(\widehat{B}_0 \bullet \vec{v}_1)$$

$$k\cos\theta_B C_S^2(\vec{k} \bullet \vec{v}) = \omega^2(\widehat{B}_0 \bullet \vec{v}_1)$$

Dispersion relation for magnetoacoustic waves

$$\omega^4 - \omega^2 k^2 (C_S^2 + v_A^2) + C_S^2 v_A^2 k^4 \cos^2\theta_B = 0$$

Solution
$$\frac{\omega}{k} = [\frac{1}{2}(C_S^2 + v_A^2) \pm \frac{1}{2}\sqrt{C_S^4 + v_A^4 - 2C_S^2 v_A^2 \cos^2\theta_B}\,]^{\frac{1}{2}}$$

+: higher frequency fast magnetoacoustic wave

-: lower frequency slow magnetoacoustic wave

If $v_A = 0$ slow wave disappears
fast wave becomes sound wave

If $C_S = 0$ slow wave disappears
fast wave becomes compression Alfven wave

If $\beta = \frac{P}{P_0} = \frac{2\mu P_0}{B^2} >> 1$

fast: $\frac{\omega}{k} \approx C_S$ slow: $\frac{\omega}{k} \approx v_A \cos\theta_B$

Acoustic gravity wave

compressibility and buoyancy force together, both acoustic and gravity modes occur

Dispersion relation
$$\omega^2(\omega^2 - N_S^2) = (\omega^2 - N^2 \sin\theta'_g) k'^2 C_S^2$$

$$N_S = \frac{rg}{2C_S} = \frac{C_s}{2\Lambda} \qquad N = \frac{(r-1)^{1/2} g}{C_S}$$ Brunt - Vaisala frequency

$$\sin^2\theta_g = 1 - \frac{k_z'^2}{k^2} \qquad \vec{k}' = \vec{k} + i\frac{rg}{2C_S^2}\hat{z}$$

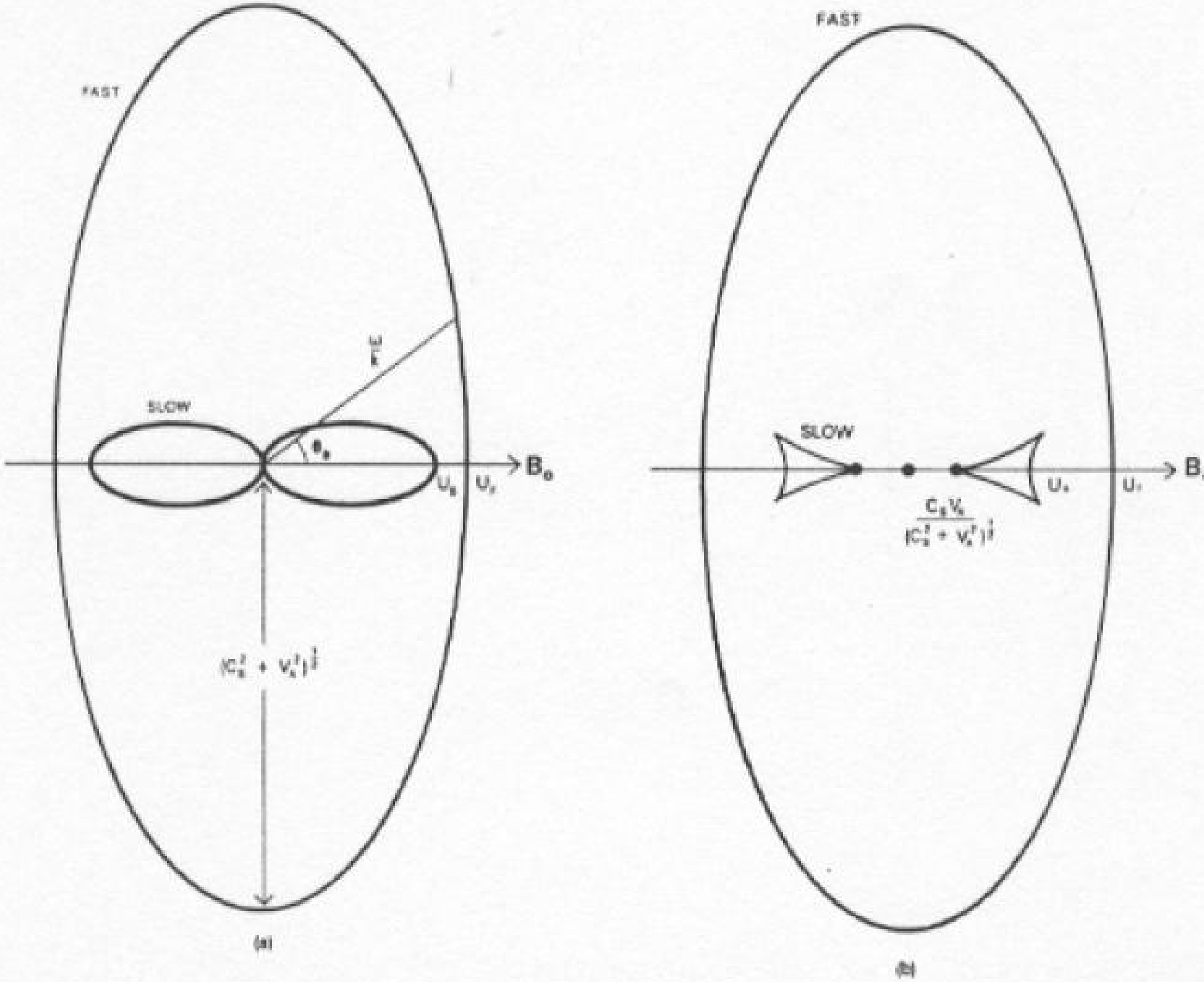

Fig. 4.7. Polar diagrams for fast and slow magnetoacoustic waves propagating at an angle θ_B to the equilibrium magnetic field. The speeds u_s and u_f are the slower and faster, respectively, of the Alfvèn speed (v_A) and sound speed (c_s). (a) shows the phase velocities and (b) the group velocities.

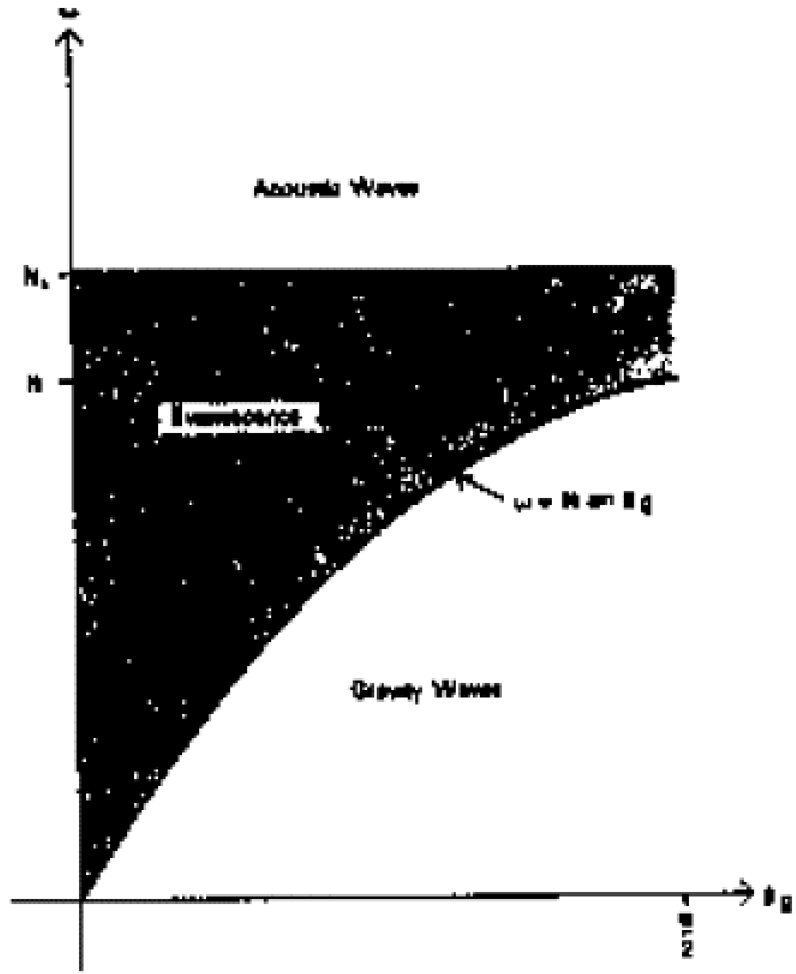

Fig. 4.8. The allowable domains for the propagation of acoustic-gravity waves of frequency ω at an angle θ_g to the vertical. In the shaded region, disturbances cannot propagate.

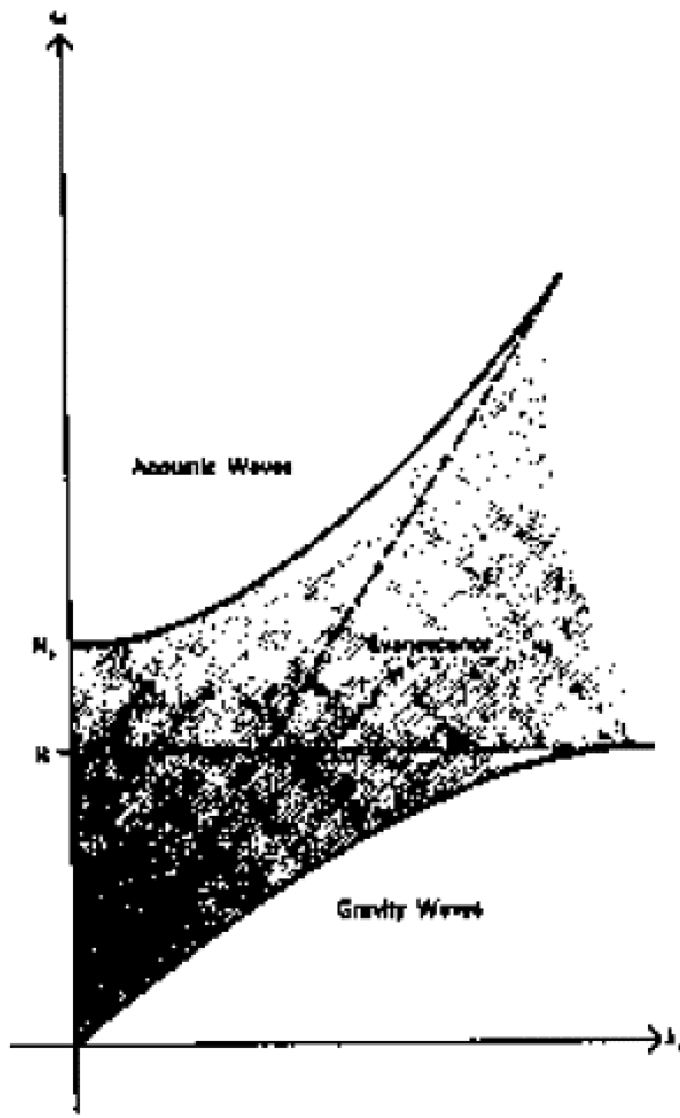

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Fig. 4.9. A diagnostic diagram indicating the allowable regions for the vertical propagation of waves of frequency ω and horizontal wavenumber k_x. Disturbances in the shaded region are non-propagating (evanescent). The asymptotes $\omega = N$ and $\omega = k_x c_s$ are indicated by dashed lines.

Magnetoacoustic-gravity wave

$$\omega^4 - \omega^2 k^2 (C_S^2 + v_A^2) + k^2 C_S^2 N^2 \sin\theta_g + k^4 C_S^2 v_A^2 \cos^2\theta_B = 0$$

If N=0 $\quad \omega^4 - \omega^2 k^2 (C_S^2 + v_A^2) + k^4 C_S^2 v_A^2 \cos^2\theta_B = 0 \quad$ magnetoacoustic wave

If $C_S^2 >> v_A^2 \quad \omega^2 = k^2 C_S^2 \quad$ acoustic wave

$\omega^2 = N^2 \sin^2\theta_g + k^2 v_A^2 \cos^2\theta_B \quad$ magneto-gravity wave

Five minute oscillation (Helioseismology)

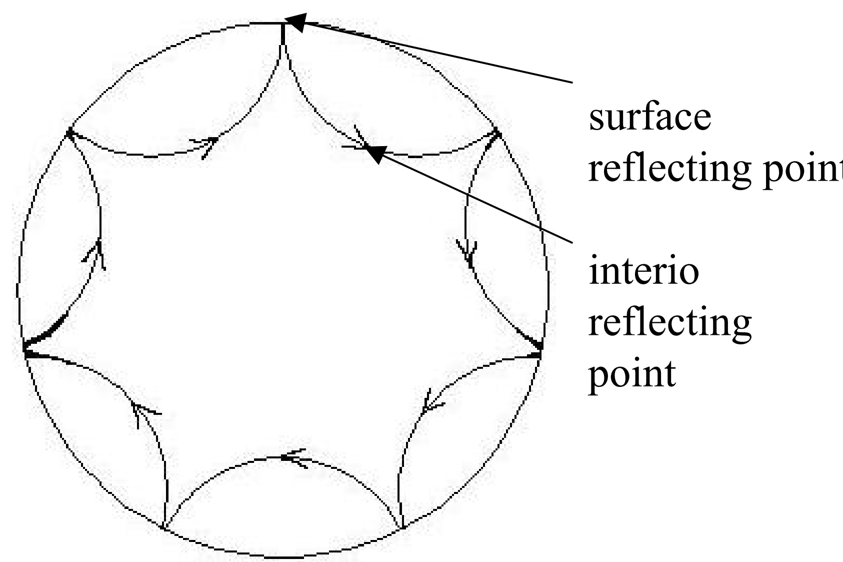
Observations:

periods:	~5 min	
duration:	23~50 min	Dopplergrams
velocity:	0.1~0.3 km/s	or
wavelength:	5000 to 10000 km	Intensity maps
k- ω diagram:	ridges	

Theory: trapped acoustic standing wave

Application:

- probe sound speed inside the Sun
  - get ρ, P, T as a function of depth.
- probe rotation speed inside the Sun
- probe magnetic fields before emergence to surface



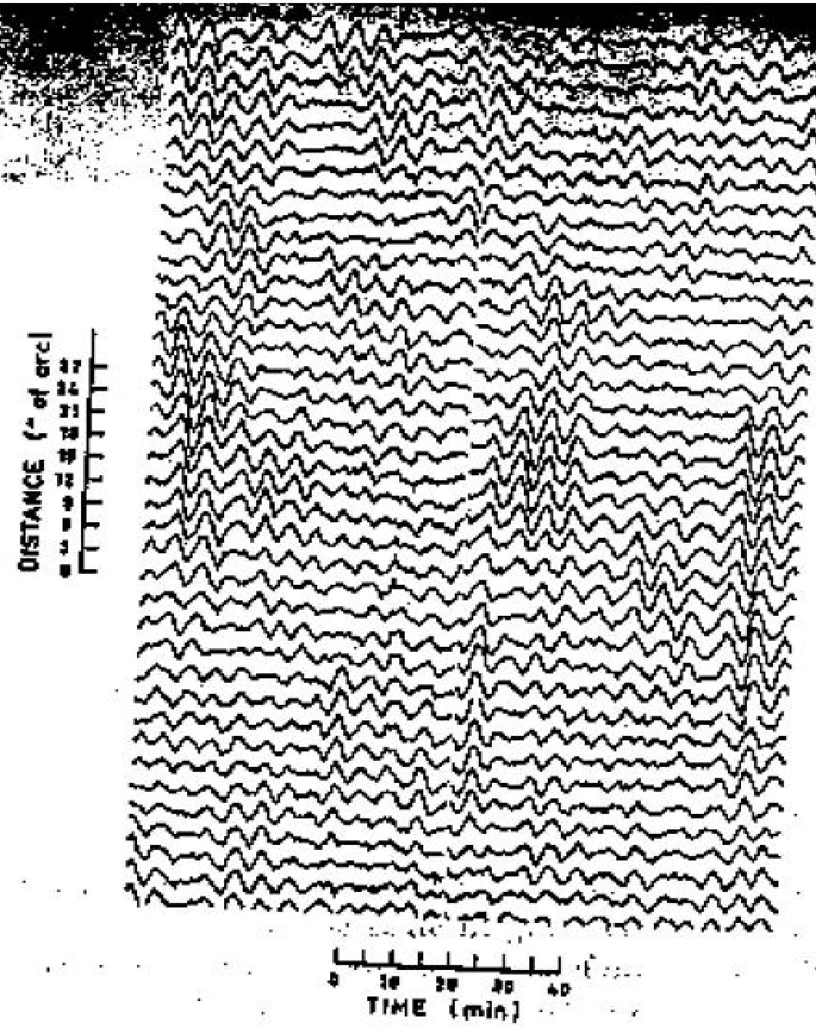
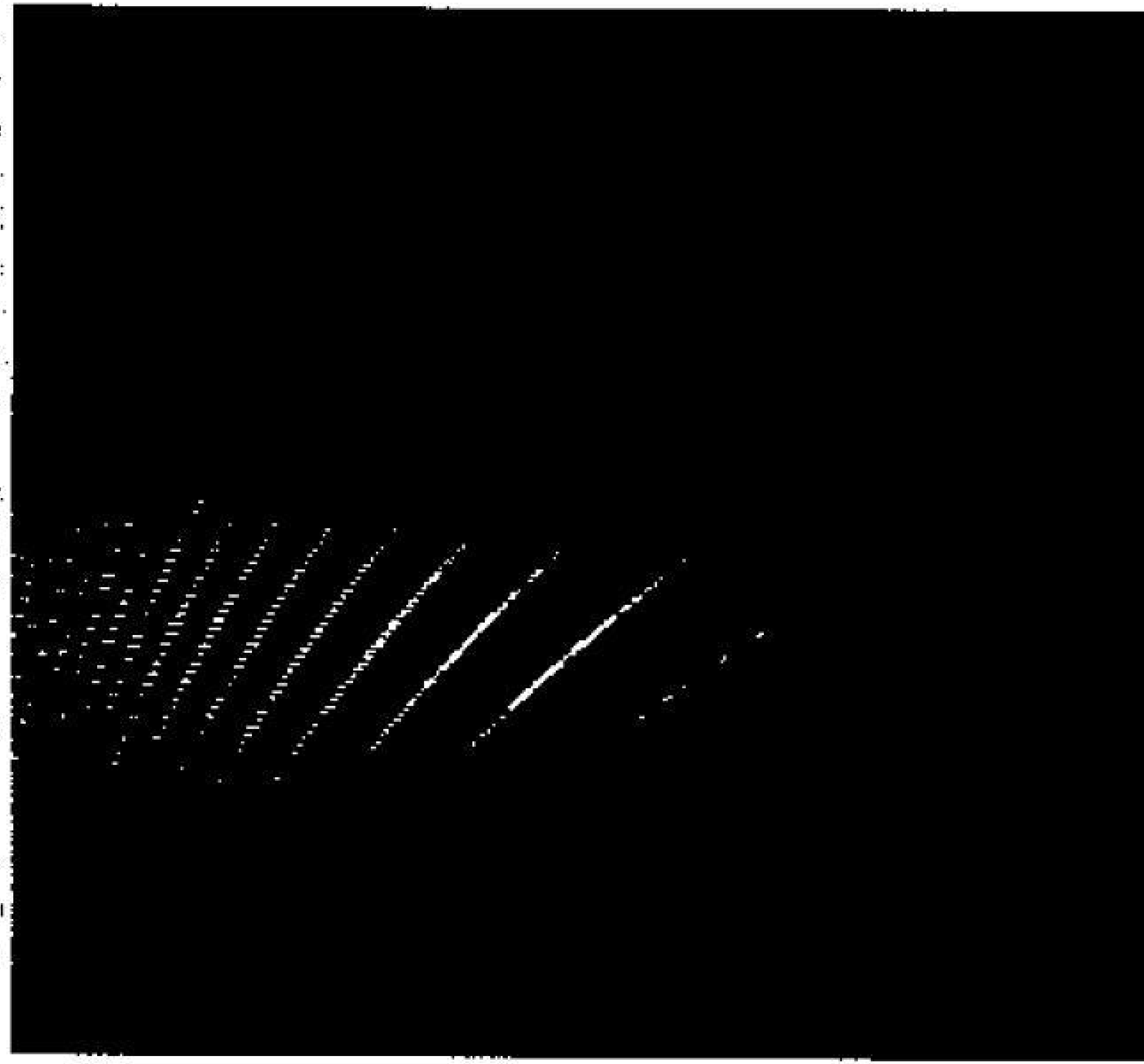


Fig. 4.10. The observed vertical velocity as a function of time at many photospheric locations, each separated by 3 arc sec (about 2200 km). The velocity scale is such that the distance between adjacent curves corresponds to 0.4 km s^{-1} (from Musman and Rust, 1970).



6.2. A "$k - \omega$" plot of spherical eigenvalue l (which corresponds to oscillation scale) vs. frequency in the solar brightness oscillations in a 6 Å band at the K line. These remarkable results were obtained by Harvey and Duvall in a 50-hr run at the South Pole. The modes have been averaged over the azimuthal eigenvalue m and the individual l mode frequencies are clearly separated. The abscissa (l) ranges from 6 (left) to 250 (right). The ordinate (frequency) ranges from 2238 to 4679 (top) μHz. The frequency resolution of 5 μHz is not good enough to resolve the modes of $l > 100$. (KPNO)

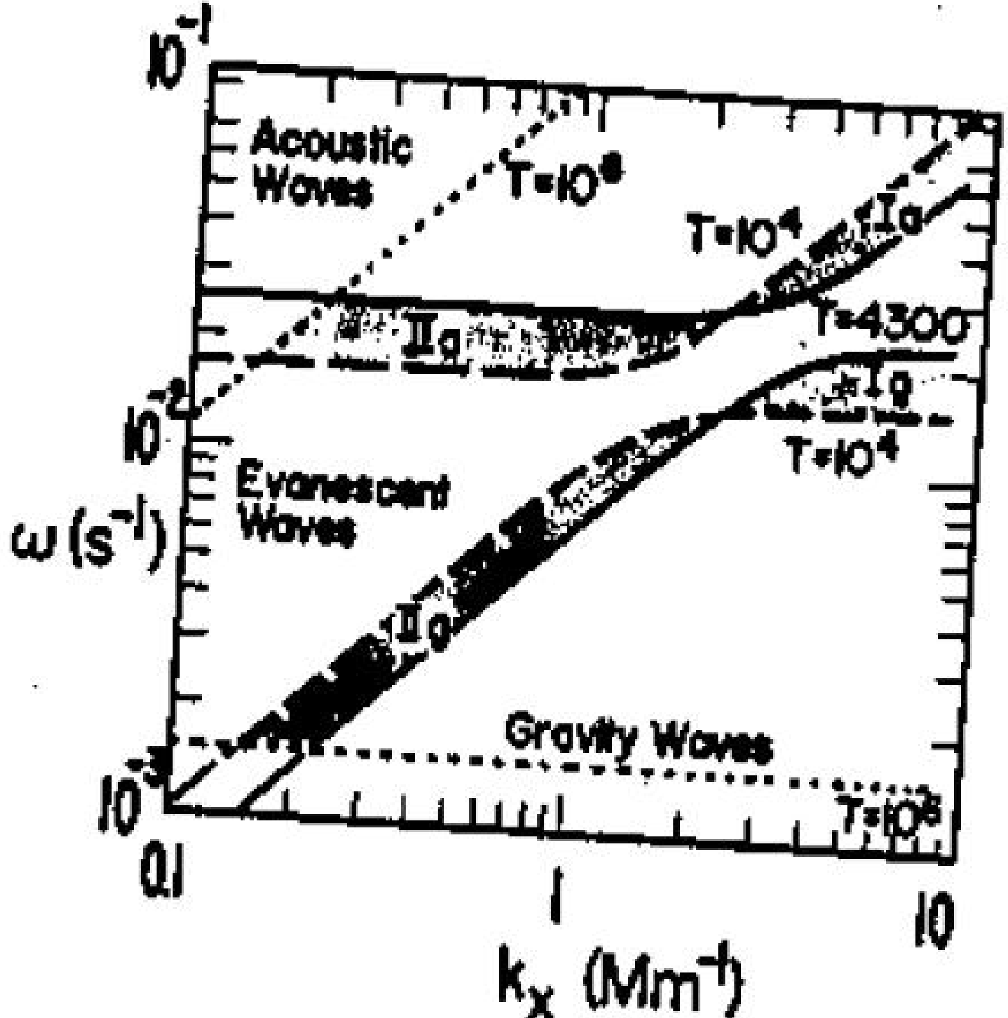


Fig. 4.11. Diagnostic diagrams for acoustic-gravity waves at the temperatures 4300 K (solid), 10^4 K (dashed) and 10^6 K (dotted). The shaded regions indicate ranges of frequency ω and horizontal wave-number k_x for which the waves are trapped in the solar atmosphere (after Stein and Leibacher, 1974).

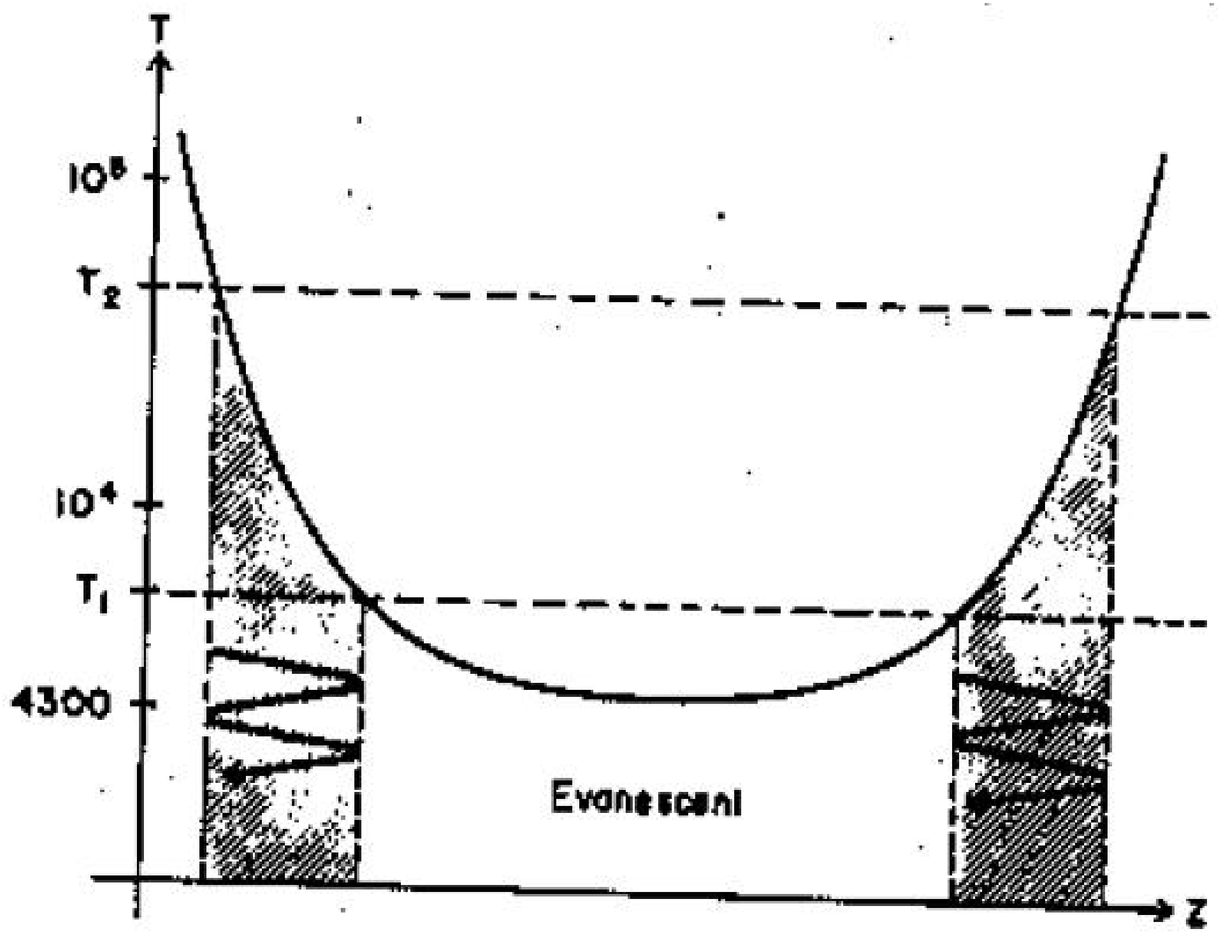


Fig. 4.12. A sketch of the temperature as a function of height in the solar atmosphere, indicating the location of two cavities (shaded), one in the chromosphere and the other in or below the photosphere. Acoustic waves may propagate within a cavity but not below or above it, where they become evanescent (non-propagating). Similar cavities exist which can trap gravity waves.

Shock waves

Uniformly propagating wave: $C = (\frac{rp}{\rho})^{1/2}$ wave profile maintains a fixed shape each Part of the wave travels at the same speed.

If crest of a sound wave moves fast than its leading edge – progressive steepering of wave front.

If a steady wave shape is attained – shock wave is formed.

shock wave speed > C_S strong shock
shock wave speed < C_S weak shock

Shocks can dissipate – convert flow energy to heat

Shock front is a very thin transition region -- a few mean free path wide.

$$\frac{\delta E}{\delta t} = \rho\gamma \left(\frac{\delta v}{\delta x}\right)^2 \qquad \delta t \approx \frac{\delta x}{v_1}$$

$$\delta x = \frac{\rho\gamma \, (v_1 - v_2)^2}{v_1 \delta E} \qquad \delta x = \frac{\gamma}{v_1}$$

Reynold number $\frac{v_1 \delta x}{\gamma} \approx 1$

Two reference frames: Rest frame
shock frame

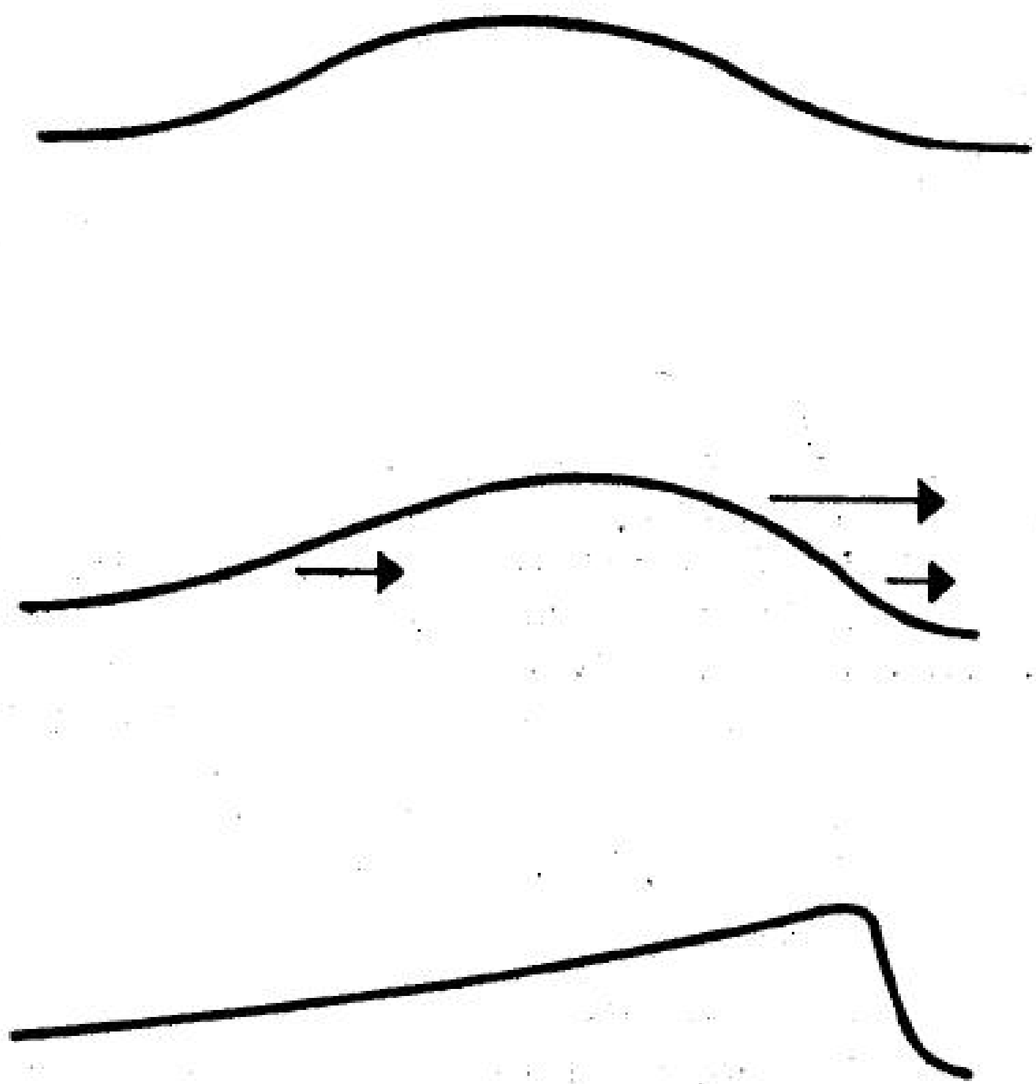


Fig. 5.1. The steepening of a finite-amplitude wave profile to form a shock wave.

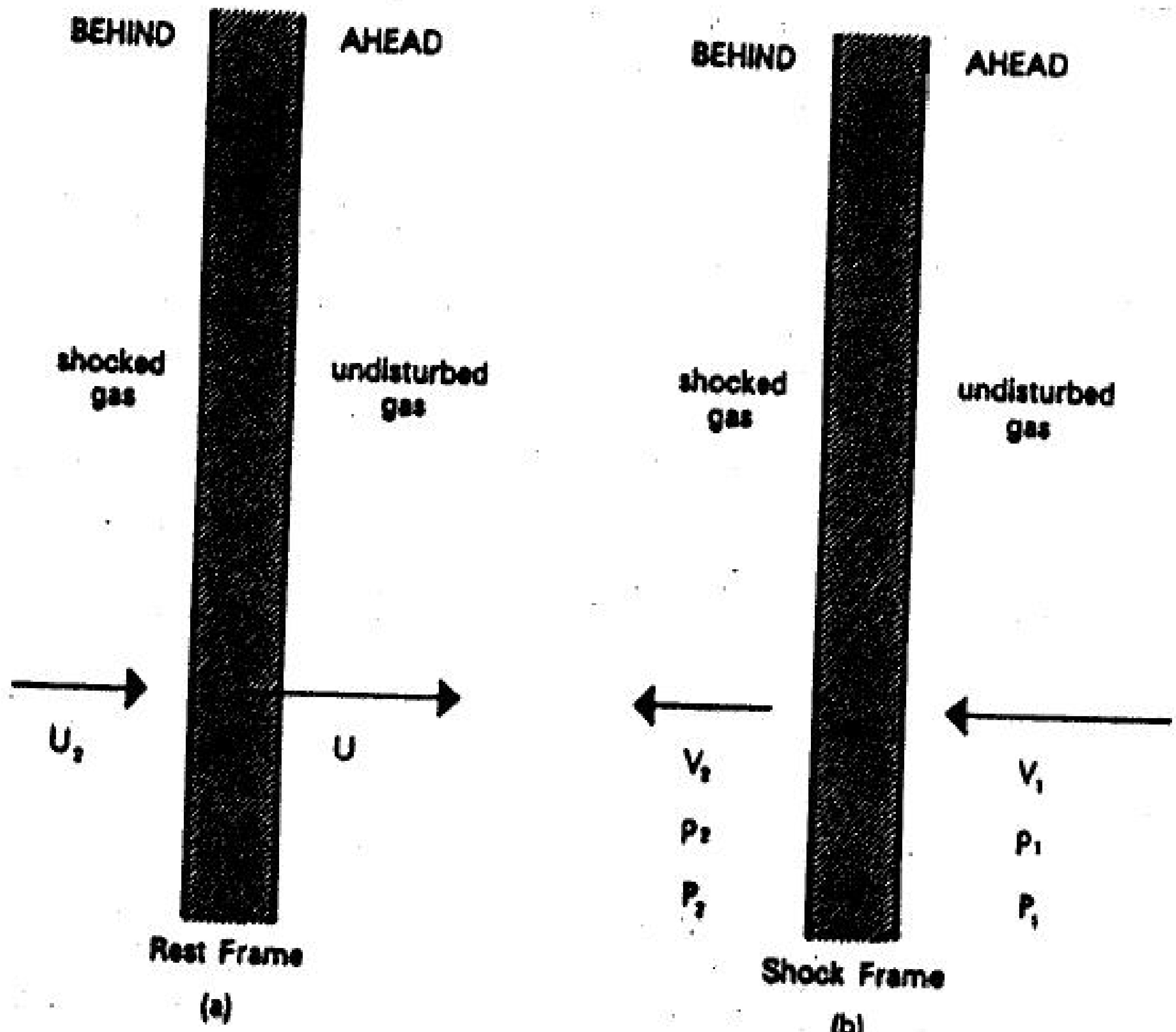


Fig. 5.3. The notation for a plane hydrodynamic shock wave moving to the right with speed u into a gas at rest. Properties ahead of the shock are denoted by 1 and those behind by 2.

Effect of magnetic fields

3 waves

Slow $\longrightarrow$ slow magnetoacoustic shock

Intermediate $\longrightarrow$ Alfven wave (no shock)

Fast $\longrightarrow$ fast magnetoacoustic shock

Only fast shock can propagate directly across magnetic fileds

fast mode: $v = \sqrt{C_s^2 + v_A^2}$

For conducting gas (ionized gas, there is an extra dissipative mechanism: ohmic heating due to finite Electric conductivity. It does not affect gas before & after shock, but affects the thickness of shock.

Energy dissipation: $\dfrac{\delta E}{\delta t} \approx \dfrac{j^2}{\sigma}$

Ampere's law: $j = \dfrac{B_{1y} - B_{2y}}{\mu \delta x}$

$$\delta x = \frac{(B_{1y} - B_{2y})^2}{\mu^2 \sigma v_1 \sigma E}$$

Hydrodynamic shock

Jump relations

Conservation of mas $\rho_2 v_2 = \rho_1 v_1$

Conservation of momentum $p_2 + \rho_2 v_2 = p_1 + \rho_1 v_1$

Conservation of energy $p_2 v_2 + (\rho_2 e_2 + \frac{1}{2}\rho_2 v_2^2) v_2 = p_1 v_1 + (\rho_1 e_1 + \frac{1}{2}\rho_1 v_1^2) v_1$

e : internal energy $= \frac{p}{(r-1)\rho}$ for perfect gas

Then: $\frac{rp_2}{(r-1)\rho_2} + \frac{1}{2} v_2^2 = \frac{rp_1}{(r-1)\rho_1} + \frac{1}{2} v_1^2$

Solution (prove as home work) $\frac{\rho_2}{\rho_1} = \frac{(r+1)M_1^2}{2+(r-1)M_1^2}$ $M_1 = \frac{v_1}{C_{S1}}$

$\frac{v_2}{v_1} = \frac{2+(r-1)M_1^2}{(r+1)M_1^2}$ $C_{S1} = (\frac{rp_1}{\rho_1})^{1/2}$

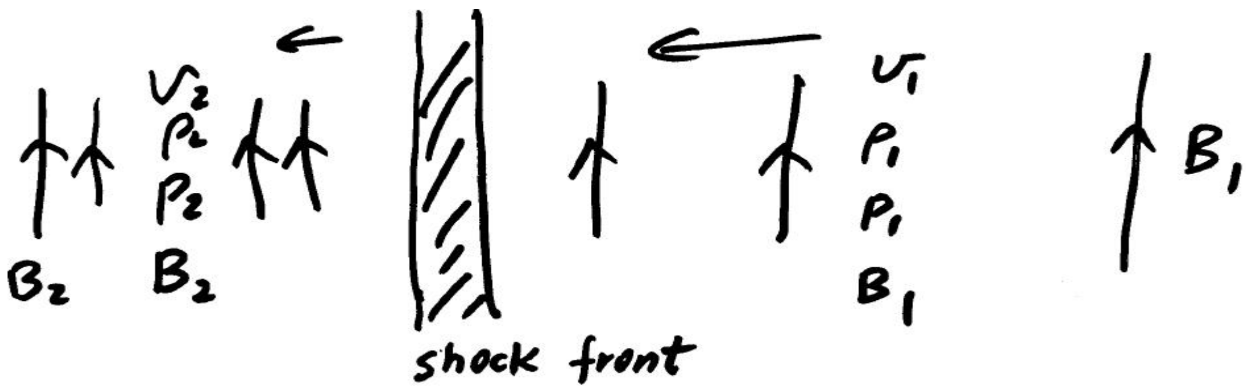
$\frac{p_2}{p_1} = \frac{2rM_1^2 - (r-1)}{r+1}$

Entropy $S = C_v \log \frac{p}{\rho r}$ $S_2 >> S_1$ entropy increase

We find $M_1 \geq 1$ shock spec $\geq$ sound speed ahead

$v_2 \leq C_{S2}$ $p_2 \geq p_1$ $\rho_2 \geq \rho_1$ $T_2 \geq T_1$ (heating)

Perpendicular shock **(magnetic shock)**



$$\rho_2 v_2 = \rho_1 v_1 , \qquad B_2 v_2 = B_1 v_1$$

$$p_2 + \frac{B_2^2}{2\mu} + \rho_2 v_2^2 = p_1 + \frac{B_1^2}{2\mu} + \rho_1 v_1^2$$

$$(p_2 + \frac{B_2^2}{2\mu})v_2 + (\rho_2 e_2 + \frac{1}{2}\rho_2 v_2^2 + \frac{B_2^2}{2\mu})v_2 = (p_1 + \frac{B_1^2}{2\mu})v_1 + (\rho_1 e_1 + \frac{1}{2}\rho_1 v_1^2 + \frac{B_1^2}{2\mu})v_1$$

solution $\quad \frac{\rho_2}{\rho_1} = x \qquad \mathrm{M}_1 = \frac{v_1}{C_s} \qquad \beta_1 = \frac{2\mu p_1}{B_1^2} = \frac{2C_{S1}^2}{r v_{A1}^2}$

$$\frac{v_2}{v_1} = x^{-1} \qquad \frac{B_2}{B_1} = x \qquad \frac{p_2}{p_1} = rM_1^2(1-x)^{-1} + \beta_1^2(1-x^2)$$

Solution for x $\quad f(x) = 2(2-r)x^2 + (2\beta_1 + (r-1)\beta_1 M_1^2 + 2)rx - r(r+1)\beta_1 M_1^2 = 0$

$1 < r < 2 \quad \sim \frac{5}{3}$ ideal gas

$\beta_1 >> 1, \quad v_1^2 >> C_{S1} + v_{A1}^2 \qquad 1 < \frac{B_2}{B_1} < \frac{r+1}{r-1} \qquad$ so: $1 < \frac{B_2}{B_1} < 4$

Oblique shock

B field is in x-y plane

Solutions $\frac{v_{2x}}{v_{1x}} = x^{-1}$, $\frac{v_{2y}}{v_{1y}} = \frac{v_1^2 - v_{A1}^2}{v_1^2 - xv_{A1}^2}$, $\frac{B_{2x}}{B_{1x}} = 1$, $\frac{B_{2y}}{B_{1y}} = \frac{(v_1^2 - v_{A1}^2)x}{v_1^2 - xv_{A1}^2}$, $\frac{p_2}{p_1} = x + \frac{(r-1)xv_1^2}{2C_{S1}^2}(1 - \frac{v_2^2}{v_1^2})$

$$(v_1^2 - Xv_{A1}^2)\left\{XC_{S1}^2 + \frac{1}{2}v_1^2\cos^2\theta[X(r-1)-(r+1)]\right\} + \frac{1}{2}v_{A1}^2 v_1 \sin^2\theta X\left\{r + X(2-z)v_1^2 - Xv_{A1}^2[(r+1)-X(r-1)]\right\} = 0$$

There are 3 solutions: fast, Alfven waves, slow

Two special cases:

switch-off shocks: $v_1 = v_{A1}$, $x = \frac{v_1^2}{v_A^2}$

switch-off shocks: $v_{A1} > C_{S1}$, $x = \frac{r + 1 - 2C_{S1}^2 / v_{A1}^2}{r-1}$

Intermediate wave: wave front propagate at Alfven speed

$$\frac{v_{2y}}{v_{1y}} = \frac{B_{2y}}{B_{1y}} \quad p_2 = p_1 \quad B_{2y}^2 = B_{1y}^2 \quad B_{2y} = -B_{1y} \quad B_{2x} = B_{1x} \quad v_{2y} = -v_{1y}$$

It is actually not a shock, as no change in pressure or density

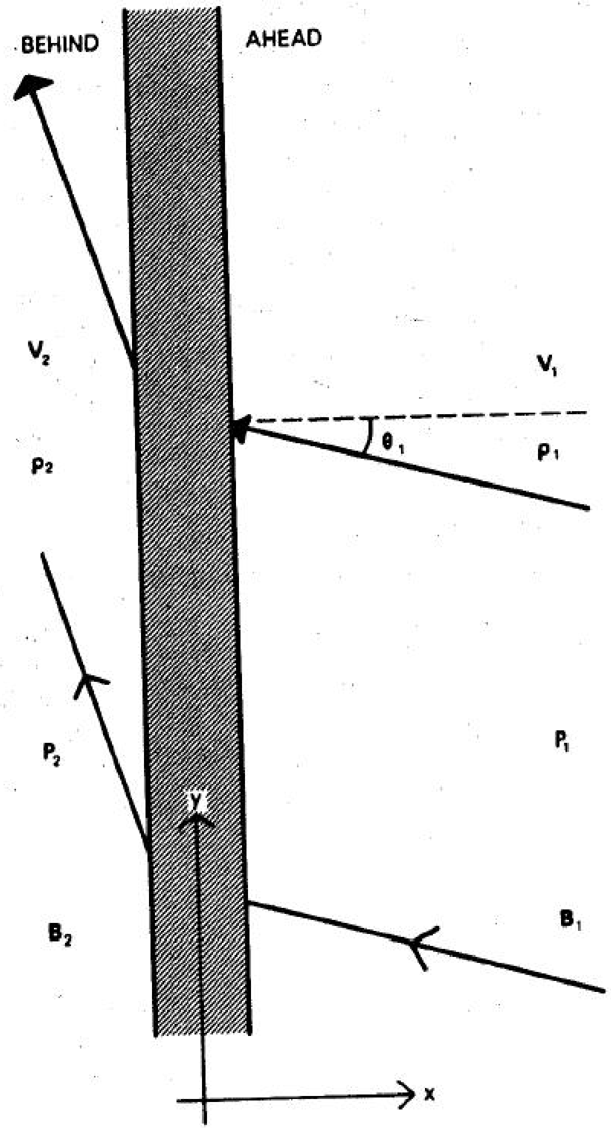


Fig. 5.5. The notation for an oblique shock wave in a frame of reference with two velocity components: the component along the shock normal is the same as the shock front, whereas the speed along the shock front is chosen to make the plasma velocity (v) parallel to the magnetic induction (B).

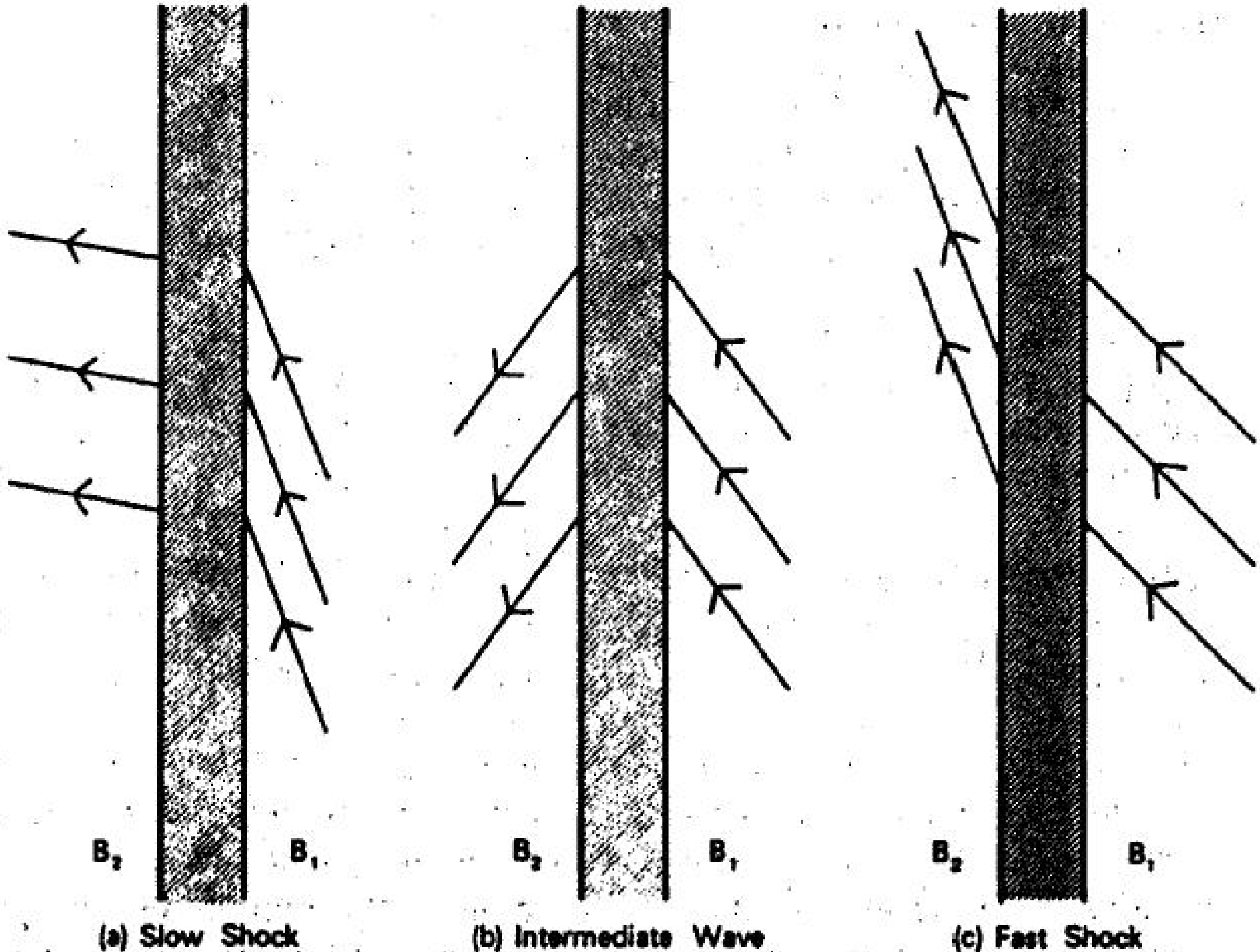
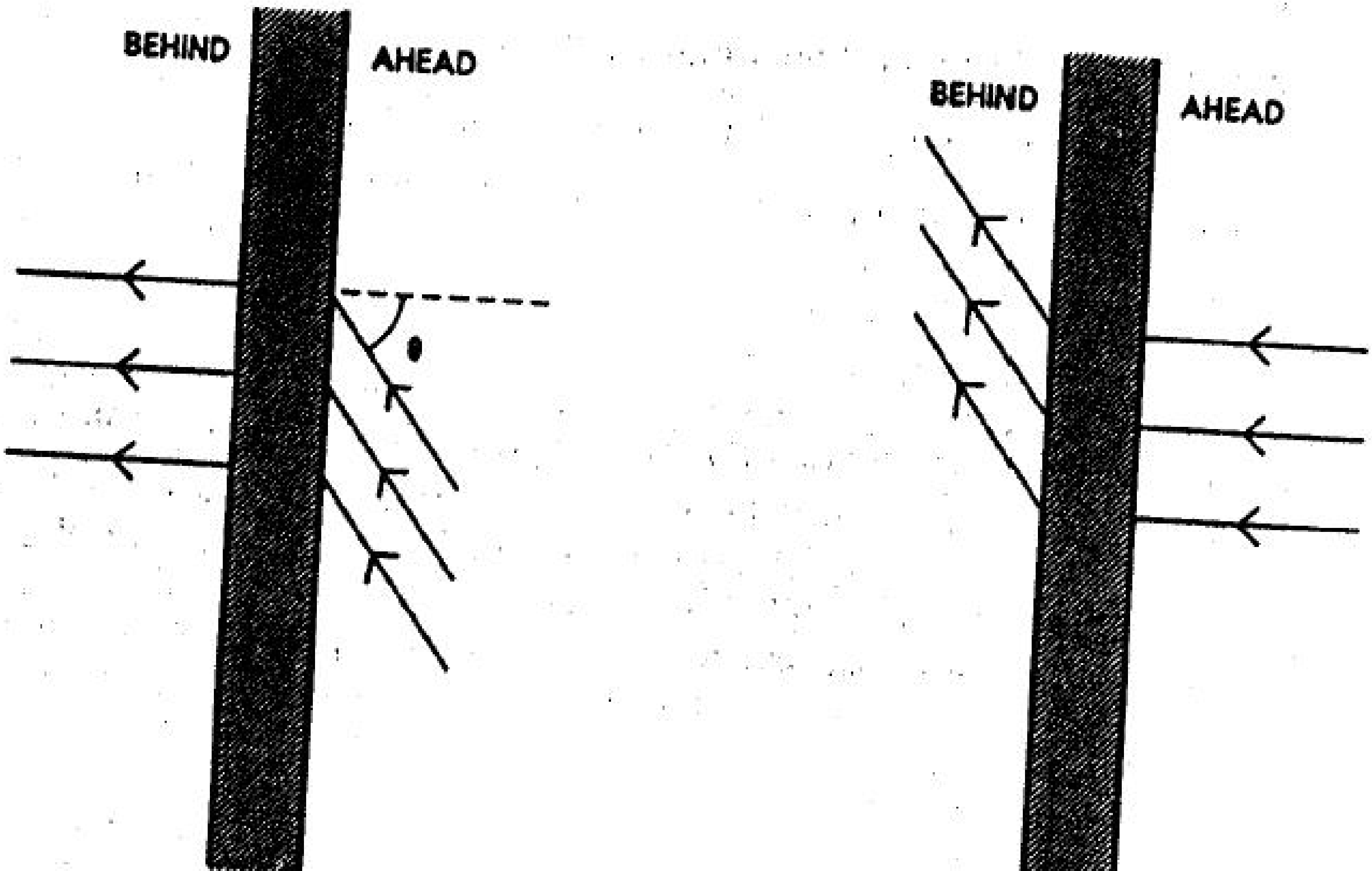


Fig. 5.6. The changes in magnetic field direction that are caused by the three types of oblique wave.



(a) Switch—off

(b) Switch—on

Fig. 5.7. The magnetic field changes for switch-off and switch-on shock waves.

Homework

Use a sequence of CaK images obtained at Big Bear Solar Observatory on 4th of July 2004, derive 2-dimensional κ-ω diagram to show oscillation power spectra. The 3-D IDL save set can be found in a link in the course web page. Array fn1 contains time information. Image pixel size is 1".

(1) What is the peak frequency?

(2) Interpret the diagram.