Lecture 10

Solar Wind

Variations on earth's magnetic fields – geomagnetic storms were found 1–2 days after big solar flares.
Chapman (1929) first suggested that the storms are caused by a stream of plasma ejected from solar flares traveling at 1000 km/s.
Brermann (1951) first proposed that Sun continuously emits "Solar Corpuscles" and making comet tails away from the Sun.

<u>Chapman Solar Corona Theory</u>

① Energy transported by conduction only

$$4\pi r^2 k \frac{dT}{dr} = const \qquad k = k_0 T^{5/2} \qquad \text{Thermal conductivity}$$

boundary condition,

$$T = T_0 \quad at \quad r = R_\Theta \qquad T_0 = 10^6\,\mathrm{K}$$
$$T = 0 \quad at \quad r = \infty$$

solution

$$T = T_0 \left(\frac{R_\Theta}{r} \right)^{2/7} \quad at \quad r = 1\,\mathrm{AU} \quad T = 10^5\,\mathrm{K} \quad \cdots\gg \text{ too Hot !}$$

② Hydrostatic equilibrium

$$\frac{dp}{dr} = \frac{-GM_{\Theta}mn}{r^2} \qquad p = nkT \quad at \;\; r = R_{\Theta} \;\; n = n_0 \;\; p = p_0$$

$$p = p_0 \exp\left\{ \frac{7GM_{\Theta}mn_0}{5p_0R_{\Theta}} \left[\left(\frac{R_{\Theta}}{r} \right)^{5M} - 1 \right] \right\}$$

$$p = \frac{p_0}{e} \qquad as \quad r \to \infty$$ which is a constant

Pressure is too big for interstellar pressure.

Something must be wrong!

<u>Parker's Solution</u>

Corona is not static, instead, it is expanding – Solar wind concept

Isothermal: $T = const, \qquad p = nkT$

Mass Continuity $4\pi r^2 n\upsilon = const$

Equation of motion $nm\upsilon \frac{d\upsilon}{dr} = -\frac{dp}{dr} - \frac{GM_{\Theta}nm}{r^2}$

Eliminating n, p only keep υ

$$\boxed{\left(\upsilon - \frac{\upsilon_c^{\ 2}}{\upsilon}\right)\frac{d\upsilon}{dr} = \frac{2\upsilon_c^{\ 2}}{r} - \frac{GM_\Theta}{r^2}} \tag{12・7}$$

Please derive 12・7 as a home work

$$\upsilon_c = \sqrt{\frac{kT}{m}}$$ Isothermal sound speed

$$< \sqrt{\frac{rp}{\rho}}$$ Adiabatic sound speed

When $\upsilon = \upsilon_c$, $r = r_c = \dfrac{GM_\Theta}{2\upsilon_c^{\ 2}}$ Both sides of equation = 0.

This is called critical point.

Solution of 12・7 is simple:

$$\left(\frac{\upsilon}{\upsilon_c}\right)^2 - \ln\left(\frac{\upsilon}{\upsilon_c}\right) = 4\ln\left(\frac{r}{r_c}\right) + \frac{2GM}{r\upsilon_c^{\ 2}} + c. \tag{12・8}$$

Please derive 12・8 as 2nd part of home work.

This solution is graphically presented in Fig 12.1.
They are divided into 5 classes Ⅰ, Ⅱ, Ⅲ, Ⅳ, and Ⅴ.
A is the critical point.

Solutions Ⅰ & Ⅱ are unacceptable: they have double value and do not connect solar surface.

TypeⅢ: Supersonic at Sun – not observed
so only classes Ⅳ and Ⅴ are acceptable.

TypeⅣ: passes through critical point A, and corresponds to value

$$c = -3, \qquad at \quad r = r_c, \quad \upsilon = \upsilon_c$$

This type is called solar wind solution at large *r*, $\upsilon >> \upsilon_c \quad \upsilon \sim (\ln r)^{1/2}$

$$n \sim r^{-2} (\ln r)^{-1/2}$$

so pressure drops to 0 at $r = \infty$

If $T = 10^6 \mathrm{K}, \qquad \upsilon(1\,\mathrm{AU}) = 100\,\mathrm{km\,s^{-1}}$

TypeⅤ: always subsonic — Solar Breeze
Large r, υ falls off like $r \sim r^{-2}$
p & ρ are constant at large *r*

Many modifications have been added to Parker's solution to fit observations better
constant *T* is a major problem.

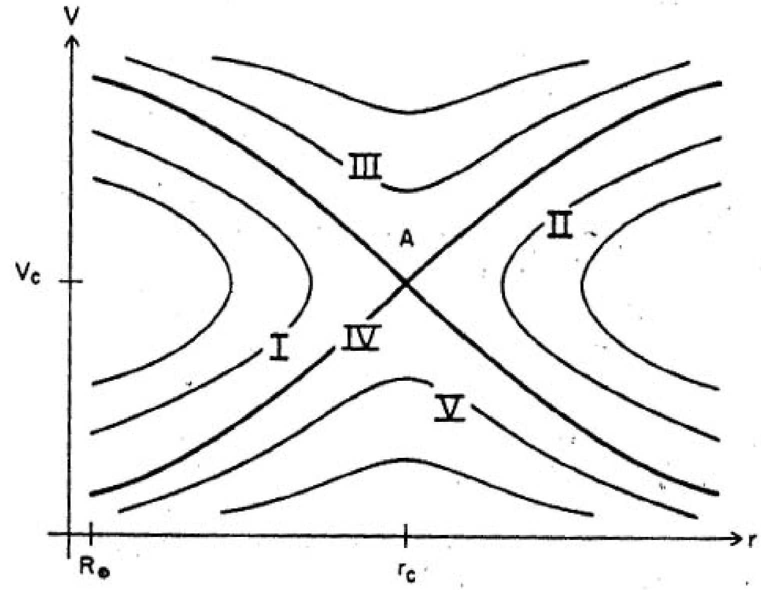


Fig. 12.1. A sketch of Parker's isothermal solutions, showing the different classes I, II, III, IV, V. Type IV (solar wind) passes through the critical point (A), where $v = v_c$ and $r = r_c$. Type V represents the subsonic, solar breeze solutions.

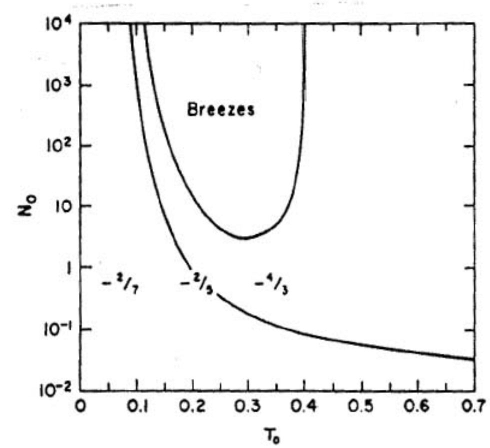


Fig. 12.2. The dependence of the asymptotic temperature behaviour on the surface density (n_0) and temperature (T_0) in units of $\kappa_0(GM_\odot R_0 T_0^5)^{1/2}k_B^{-1}$ and $Gm_pM_\odot/(k_BR_\odot)$, respectively. Wind solutions with $T \sim r^{-2/7}$ are separated from those with $T \sim r^{-4/3}$ by a curve on which $T \sim r^{-2/5}$. The breeze solutions have zero total energy (after Roberts and Soward, 1972).

Energy Equation Considering Variation of T

$$nm\upsilon A\left(\frac{1}{2}\upsilon^2 + \frac{5p}{2nm} - \frac{GM_\Theta}{r}\right) = Ak\frac{dt}{dr} + E_\infty$$

Kinetic enthalpy gravitation conduction

Combined with $$nm\upsilon\frac{d\upsilon}{dr} = -\frac{dp}{dr} - \frac{GM_\Theta nm}{r^2}$$

We get:

$$n\upsilon\left(\frac{3}{2}k\frac{dT}{dr} - \frac{kT}{n}\frac{dn}{dr}\right) = \frac{1}{A}\frac{d}{dr}\left(Ak\frac{dT}{dr}\right)$$

$$k = k_0 T^{5/2}, \qquad A = r^2$$

Topology of the solution is shown in Fig 12.2

Few solutions:

Parker (1964), constant conduction $T \sim r^{-2/7}$

Whang & Chang (1965), kinetic energy dominates, $T \sim r^{-2/5}$

Durney (1971), conduction flux is negligible, $T \sim r^{-4/3}$

Two Flow Model

Protons and electrons are treated separately.
Two temperatures are introduces. $T_e \,\&\, T_p$

Then: $4\pi r^2 n\upsilon = const$

$$nm\upsilon \frac{d\upsilon}{dr} = -\frac{d}{dr}\left(nk\left(T_e + T_p\right)\right) - \frac{GM_\Theta mn}{r^2}$$

Two separate energy equations

$$n\upsilon\left(\frac{3}{2}k\frac{dT_e}{dr} - \frac{kT_e}{n}\frac{dn}{dr}\right) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 k_e \frac{dT_e}{dr}\right) - \frac{3}{2}\nu nk\left(T_e - T_p\right)$$

$$n\upsilon\left(\frac{3}{2}k\frac{dT_p}{dr} - \frac{kT_p}{n}\frac{dn}{dr}\right) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 k_p \frac{dT_p}{dr}\right) - \frac{3}{2}\nu nk\left(T_e - T_p\right)$$

ν is a constant — coupling due to coulomb collision between electrons & protons

$$k_e = \left(\frac{m_i}{m_e}\right)^{1/2} k_p = 43k_p$$

Solution is shown in Fig 12.3 — T_p drops faster.

Magnetic Field Effect

The solution above still has problem. T_e is foolish, because of big conduction. Then magnetic fields have to be introduced.

Major effects: causes difference in some parameters in $\perp$ or $\|$ to field direction; supports waves; produces micro-instabilities.

The plasma close to the Sun co-rotates with Sun, but becomes spiral away from the Sun (Fig 12.4)

B is inclined at an angle φ to the radial plasma velocity.

$\upsilon_{\|}$ is $\upsilon_{\|}$ to *B.* Its speed relative to the solar surface is $\Omega(r - R_{\Theta})$ normal to radius vector.

Thus $\upsilon \sin\psi = \Omega(r - R_{\Theta})\cos\varphi$ *or* $\tan\psi = \Omega(r - R_{\Theta})/\upsilon$

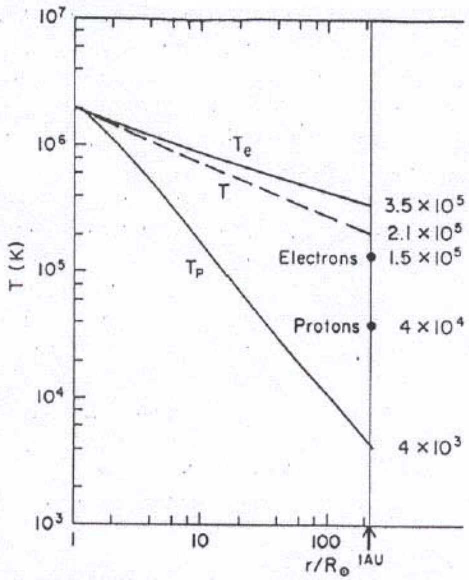
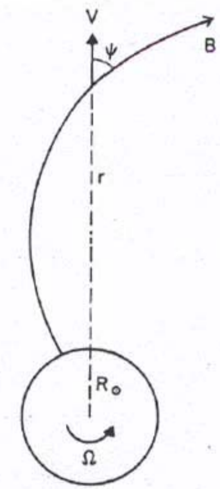


Fig. 12.3. The temperatures of electrons (T_e) and protons (T_p) as functions of distance for the two-fluid solar wind model of Hartle and Sturrock. The single-fluid profile is shown dashed and the observed values of low-speed streams are indicated by dots (after Hundhausen, 1972).



12.4. A spiral magnetic field line attached to the Sun, which is rotating with angular speed Ω and wed from above the north pole. The solar wind is here assumed to dominate the magnetic field energetically, and it moves radially with speed v inclined at ψ to the magnetic field.

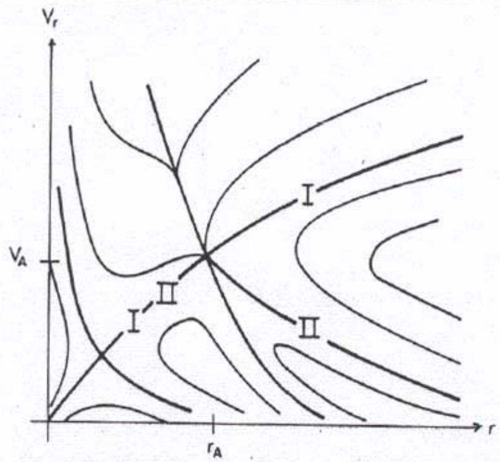


Fig. 12.5. A sketch of the solution topology for a spherical expansion from a rotating magnetised Sun (after Weber and Davis, 1967).

$$\varphi \approx \frac{1}{4}\pi \quad \text{at earth}$$

$$\varphi = 0 \qquad \text{at} \quad r = R_\Theta \quad \text{and} \quad r = \infty$$

There is an important parameter: Alfbeń radius, at which flow speed becomes Alfvenic.

$r < r_a$ Magnetic field is strong that it keeps wind rotating with the Sun, so plasma's angular momentum increases as it moves out.

$r > r_a$ Magnetic field has no effect, and solar wind keeps conserved angular momentum. $L = m\upsilon r$

Weber & Davis's work in spheric polar system:

$$\vec{B} = (B_r,\ 0,\ B_\varphi) \qquad \vec{\upsilon} = \left(\upsilon,\ 0,\ \upsilon_\varphi\right) \qquad \text{depend on } r \text{ only}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{implys :} \qquad B_r = \frac{B_0 R_\Theta{}^2}{r^2}, \qquad B_0 : \text{field strenght at surface.}$$

Equation of motion:

$$nm(\vec{\upsilon}\cdot\nabla)\vec{\upsilon} = -\nabla\left(p + \frac{B^2}{2\mu}\right) + \left(\vec{B}\cdot\nabla\right)\frac{\vec{B}}{\mu} - \frac{GM_\Theta nm}{r^2}\hat{r}$$

Other equations: $n\upsilon_r r^2 = const$

$$\frac{p}{\rho^{\alpha}} = const$$ (polytrophic law, simplicity assumption)

$$\nabla \times \left(\vec{\upsilon} \times \vec{B}\right) = 0 \quad \rightarrow \quad \upsilon_r B_\varphi - \upsilon_\varphi B_r = \frac{c}{r}$$

at $r = R_\Theta \quad B_\varphi = 0, \quad \upsilon_\varphi = \Omega R_\Theta, \quad c = -\Omega R_\Theta{}^2 B_0$

then $B_\varphi = \dfrac{\upsilon_\varphi - r\Omega}{\upsilon_r} B_r$

Next, φ component of equation of motion becomes

$$nm\upsilon_r \frac{d}{dr}\left(r\upsilon_\varphi\right) = B_r \frac{d}{dr}\left(rB_\varphi\right)$$

$n\upsilon_r / B_r$ is constant

After integrating: $r\upsilon_\varphi = \dfrac{B_r}{nm\upsilon_r} rB_\varphi = L$

L is total angular momentum par unit of mass carried by both mass and magnetic field.

Lorentz force changes to torque to solar surface to slow down the Sun.

Then $$\upsilon_\varphi = \Omega_r \frac{M_A L/(r^2\Omega) - 1}{M_A{}^2 - 1}$$

At Alfbeń critical point, $r = r_A \quad (10 \text{ to } 20 \; R_\Theta)$

$M_A = 1, \qquad L = \Omega \upsilon_A{}^2$ ——— Solid body

Radial component of equation of motion.

$$nm\upsilon_r \frac{d\upsilon_r}{dr} - \frac{nm\upsilon_\varphi{}^2}{r} = -\frac{dp}{dr} - \frac{B_\varphi}{r}\frac{d}{dr}\left(rB_\varphi\right) - \frac{GM_\Theta nm}{r^2}$$

υ_r, r diagram is shown in Fig 12.5.

There are 3 critical points, where flow speeds affairs successively the speed for slow, Alfbeń and fast waves. As gas pressure is much smaller than magnetic pressure. Alfbeń and fast wave speeds are very close — υ_A if rotation=0. Alfbeń and fast mode critical points coincide exactly. Only solutions Ⅰ & Ⅱ pass through all 3 critical points. Ⅱ give $\upsilon_r(1\text{AU}) \; 9\,\text{km/s}$ should be excluded. Type Ⅰ gives $\upsilon_r = 425\,\text{km/s}, \;\; \upsilon_\varphi = 1\,\text{km/s}$ at 1AU

Streamers and Coronal Holes

So far, we assume that solar wind expansion is spherically systematic, but in reality, there are two parts of corona:

open field — coronal hales
closed field — coronal loops

Large scale closed loop surrounded by open loop is called streamer (helmet streamer, Fig 12.6 a & b).
In such structure, exist current sheet, heating & field jump.

Pneuman – Kopp Streamer Model

Steady axisymmetric coronal expansion with dipole field; Isothermal with $T = 1.56 \times 10^6 \mathrm{K}$

equations:

$$mn(\vec{\upsilon} \cdot \nabla)\vec{\upsilon} = -\nabla p - \frac{GM_{\Theta}mn}{r^2}\hat{r} + \vec{j} \times \vec{B}$$

$$\nabla \times (\vec{\upsilon} \times \vec{B}) = 0, \quad \nabla \cdot (n\vec{\upsilon}) = 0$$

$$p = nkT, \qquad \vec{j} = {\nabla \times \vec{B}} \big/ {\mu}, \qquad \nabla \cdot B = 0$$

$$r = r_0, \qquad n = n_0, \qquad B_r = B_{0p} \sim \text{polar field} \approx 1\mathrm{G}$$

Solution is shown in Fig 12.6 (b).
Field lines are dragged out by solar wind to form a current sheet; at large distance field lines become radial.

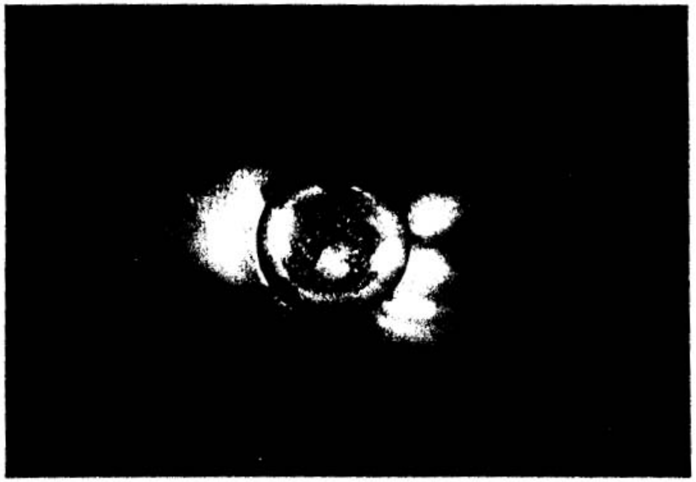


Fig. 12.6(a). A total eclipse (30 June, 1973), showing the outer corona in white light (courtesy High Altitude Observatory, NCAR, Boulder) and the inner corona in soft X-rays (courtesy American Science and Engineering, Cambridge, Mass.).

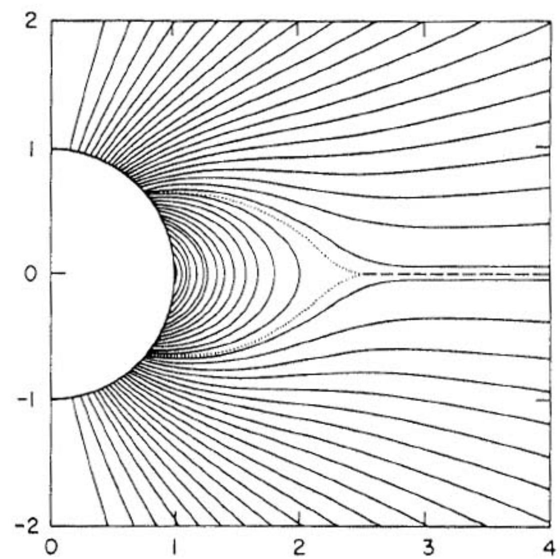


Fig. 12.6(b). Magnetic field lines for the Pneuman–Kopp model of isothermal, axisymmetric expansion from a dipolar surface field. The dotted curve marks the vortex-current sheet separating open and closed regions, while the dashed line indicates the current sheet extending away from the cusp-type neutral point (after Pneuman and Kopp, 1971).

The model is later modified by replacing the isothermal approximation with a full energy equation. This is the problem of solving time dependent MHD equations. (See Fig12.7)

Conclusion: center of coronal hole produces a smooth hot, rare, fast flow at large distance.

In the current sheet, flow speed is slower and plasma is able to leak out by magnetic diffusion.

Coronal Hole Model

In coronal holes, speed of solar wind is higher, the density is lower (1/3 of regular corona density)

e.g. at $r = 2R_\Theta, \qquad n = 3\times 10^{11}\,\mathrm{m}^{-3}, \qquad \upsilon = 120\mathrm{km/s}$

$\text{particl flux} = 3\times 10^{12}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$

Kopp & Holzer polytrophic model

$$n\upsilon A = const$$

$$mn\upsilon\frac{d\upsilon}{dr} = -\frac{dp}{dr} - \frac{GM_\Theta mn}{r^2} \qquad \frac{p}{n^\alpha} = const$$

$$p = nkT, \qquad M = \frac{c}{c_s}, \qquad c_s^{\,2} = \frac{dp}{nm}, \qquad \alpha = 1.1$$

$$\frac{M^2 - 1}{2m^2}\frac{dM^2}{dr} = \frac{1}{2}\left[\frac{\alpha+1}{\alpha-1}\right]\left[1 + \left(\frac{\alpha-1}{2}\right)M^2\right]\frac{1}{g}\frac{dg}{dr}$$

$$g = A(r)^{2(\alpha-1)(\alpha+1)}\left(E + \frac{GM_{\Theta}}{r}\right)$$

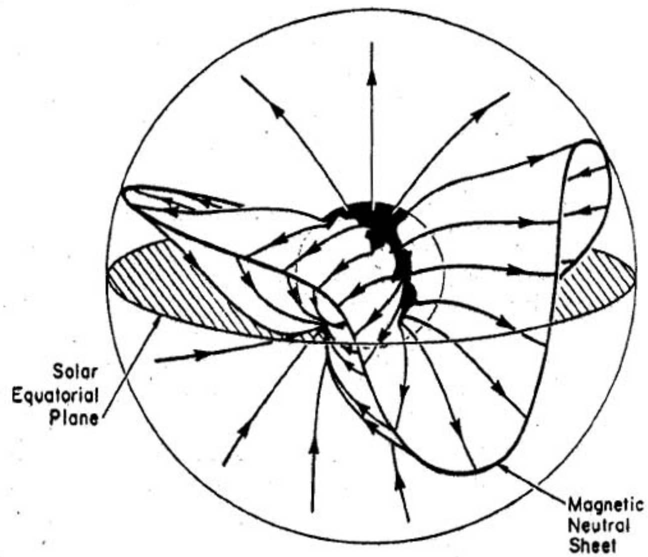


Fig. 12.7(a). A schematic drawing of the warped neutral current sheet that was present during the summer of 1973, when Skylab observed a large coronal hole stretching across the equator (Section 1.3.4C). The neutral sheet probably crosses the solar equatorial plane four times (after Hundhausen, 1981).

Fig. 12.7(b). A soft X-ray picture taken on June 1, 1973, of the coronal hole.

Energy $E = \frac{1}{2}\upsilon^2 + \frac{\alpha}{\alpha - 1}\frac{p}{nm} - \frac{GM_\Theta}{r} \approx 1.8 \times 10^{11}\ \mathrm{J\,kg^{-1}}$

would produce flow speed 600km/s.

The corresponding temperature at corona base = 2×10^6K

In above equations, a new parameter

$A(r) = A_\Theta \left(\frac{r}{R_\Theta}\right)^2 f(r)$ ———— Area function.

$A(r) \sim r^2$ for spherical expansion — $f(r) \equiv 1$

$f(r)$ increases from 1 to f max at a large distance. So at large r, $A(r) \sim r^2$ again.

Solution is shown in Fig.12.8: There are two extra critical points (× and 0) below $r = 2R_\Theta$ For large f max, solar wind becomes supersonic at inner critical point;

$(r < 2R_\Theta)$ For small f max, solar wind becomes supersonic at out critical point at $\sim 4.5R_\Theta$

Solar wind and corona observations: Skylab, Ace, Wind, SOHO,

Two parts: B, υ shape SEPs

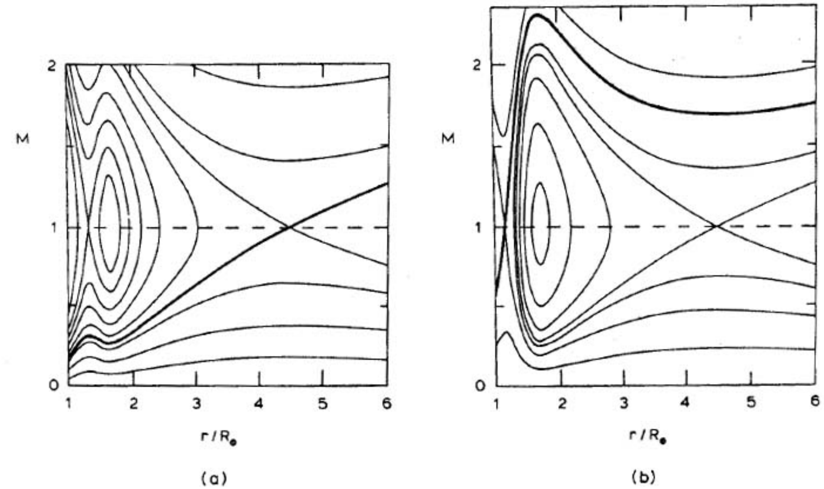


Fig. 12.8. The Mach number ($M = v/c_s$) as a function of radial distance (r) from the solar centre for polytropic flow in a coronal hole whose net non-radial divergence (f_{max}) is (a) 3 and (b) 12 (Kopp and Holzer, 1976).

critical point is always present at 4.5 $R_\odot$, but when f_{max} exceeds 2 an extra pair of critical points appears between $1R_\odot$ and $2R_\odot$, the inner one being X-type and the outer one O-type. When the coronal hole does not diverge too much ($f_{max} < 7.5$), the flow is close to spherical and the only solar wind flow (with a continuous connection of subsonic values at the Sun to supersonic values at large distances) is similar to Parker's solution: it is accelerated to a local maximum at the radius where the inner critical point is located, and then it remains subsonic until it crosses the outer critical point at 4.5 $R_\odot$, as exemplified in Figure 12.8(a). When the coronal hole expands rapidly ($f_{max} > 7.5$), the solar wind flow is rapidly accelerated to supersonic speeds at the inner critical point low down in the corona (1.15 $R_\odot$ for $f_{max} = 12$ in Figure 12.8(b)). The flow speed then attains a local maximum at the middle critical point, and thereafter it remains supersonic. As f_{max} is increased above 7.5, so the base flow becomes faster and the density falls everywhere, especially in the low corona where the rapid acceleration is occurring.

The main point of the Kopp–Holzer paper is that it explains qualitatively the low densities that are observed near the base of coronal holes. However, the rapid field divergence does not account for the high speeds that are observed at Earth, since all of their solar wind solutions, regardless of the value of f_{max}, possess the same asymptotic flow speed at large distances. Any polytropic model, even with a spherical expansion, can provide a high enough speed at 1 AU if the base temperature is taken high enough (2×10^6 K for the Kopp–Holzer model), but when a full energy equation is employed such high speeds are no longer produced.

The other contribution of Kopp and Holzer was to elucidate the topology of the