Ph777, week 10: **Solar Dynamo**

Today's topic is solar dynamo, see Chapter 9 of textbook (Priest). It is necessary to study dynamo, convection, and rotation altogether. We start with convection and then dynamo. Rotation will briefly be discussed in the dynamo section.

A. Convection

Solar Observations

Granulation (1-2″)

- Spectral wiggles: hot gas moving upwards and cooler downwards
- Brighter at center & darker at boundary: $\delta T \approx 300$ K
- High correlation between v and $\delta T (> 80\%)$: "Buoyancy-Driven"
- Cell structure: convective overshoot

Supergranulation (40″)

- Cell size $\gg$ pressure scale height
- Concentration of magnetic fields at boundaries closely coincide with chromospheric network
- $\delta T > 0$ in higher layers and < 0 in deeper layers of cell boundaries. δT is unexpectedly small.

Giant cell (10^8 m)

- Traced with s.g., the chromospheric network (Ca II mottle Schwan & Wohl 1978, AA 70), magnetic field (Bumba 1967) etc.

Mesogranulation (10″)

- Oda (1984 SP 93): a pattern of active granule $\sim 10''$.
- November et al. (1982 ApJ 258): mesogranulation (~50 m/s) when the s.g. and the oscillation (~500 m/s) are removed.

Granule	10^3 km	8-15 min	1.0 km/s	filigree
MesoGranule	10^4 km	2 hr	0.1 km/s	none
SuperGranule	$3 \cdot 10^4$ km	1-2 d	0.5 km/s	mag. network
GiantCell	10^5 km	a few mo.	0.1 km/s	active complex

- *Why convection occurs in the sun?* (Unsöld 1931, Z. Ap. 1, 138)

The Schwarzschield instability criterion is $\nabla_R > \nabla_A$ where the radiative gradient, $\nabla_R = 3\pi F\kappa_R p/16\sigma g\rho T^4$ ("diffusion appr.") and adiabatic gradient $\nabla_A \equiv (d\ln T/d\ln P)_A = (\Gamma-1)/\Gamma$. $\Gamma = 5/3$ for monoatomic perfect gas and $\Gamma \approx 1.1$ when H is highly ionized. Also at the region of H ionization, the opacity κ_R increases and so ∇_R. Therefore H-ionization can lead to the convective instability.

- *(Possible) Hierarchy of convective motions* (Simon & Weiss 1968 ApJ 140)

Granulation	H^+	$\sim 1,000$ km
MesoGranulation	He^+	$\sim 5,000$ km
SuperGranulation	He^{++}	$\sim 15,000$ km
GiantCell	Co.zone	$\sim$2-3$\times 10^5$ km

- *Mixing Length Theory* (see Mihalas 1978)

1. Assume some temperature distribution (e.g. grey atmosphere with κ_R).
2. Carry out a step-by-step integration of $dp/d\tau$ and $d\tau$.
3. At each step calculate ∇_R and ∇_A.
4. If at some point convective instability is found, determine the true gradient ∇ (note $\nabla_R \geq \nabla \geq \nabla_E \geq \nabla_A$) which satisfies the flux conservation:

$$F_{rad} + F_{conv} = L/4\pi r^2 = \sigma T_{eff}^4$$

where

$$F_{rad} = -K\frac{dT}{dz} = K\frac{T}{H_p}\nabla$$

$$F_{conv} = \rho C_p \bar{v}\delta T = \left(\frac{gQH}{32}\right)^{1/2}(\rho C_p T)(\nabla - \nabla_E)^{3/2}\left(\frac{l}{H}\right)^2$$

With the solution ∇, advance from the surface until hitting the base of the convection zone where no more convectively unstable.

- *Other issues on (stellar) convection*

1. Stellar evolution, 2. Li-Be problem, 3. Dynamo (convection + diff. rot.)

 "The Sun" by M. Stix

Table 6.1. Convection zone of a standard solar model ($Z+n$ means $Z\times10^n$)

$r/r_\odot$	P [Pa]	T [K]	η_H	η_{He}	η_{He^+}	$\nabla-\nabla_a$	ΔT [K]	v [m/s]	F_C/F
1.000	1.16+04	5.78+3	.00	.00	.00	-1.0-1	0.	0	.00
1.000	1.84+04	7.74+3	.00	.00	.00	6.3-1	2.1+3	2225	.59
1.000	2.92+04	1.00+4	.04	.00	.00	1.8-1	1.2+3	1983	.98
1.000	4.63+04	1.13+4	.10	.00	.00	8.9-2	6.9+2	1680	1.00
.999	7.34+04	1.23+4	.15	.00	.00	5.2-2	4.4+2	1471	1.00
.999	1.16+05	1.32+4	.20	.00	.00	3.4-2	3.1+2	1302	1.00
.999	1.84+05	1.41+4	.25	.00	.00	2.3-2	2.2+2	1158	1.00
.999	2.92+05	1.51+4	.30	.00	.00	1.6-2	1.7+2	1032	1.00
.998	4.63+05	1.60+4	.34	.00	.00	1.1-2	1.3+2	922	1.00
.998	7.34+05	1.71+4	.39	.00	.00	8.4-3	9.9+1	824	1.00
.998	1.16+06	1.82+4	.43	.00	.00	6.2-3	7.8+1	739	1.00
.997	1.84+06	1.94+4	.48	.01	.00	4.6-3	6.2+1	663	1.00
.997	2.92+06	2.08+4	.52	.01	.00	3.5-3	5.0+1	597	1.00
.996	4.63+06	2.23+4	.57	.02	.00	2.7-3	4.1+1	539	1.00
.996	7.34+06	2.41+4	.62	.03	.00	2.1-3	3.4+1	488	1.00
.995	1.16+07	2.61+4	.67	.06	.00	1.6-3	2.9+1	443	1.00
.994	1.84+07	2.85+4	.71	.10	.00	1.2-3	2.5+1	403	1.00
.994	2.92+07	3.13+4	.76	.17	.00	9.7-4	2.1+1	368	1.00
.993	4.63+07	3.46+4	.80	.28	.00	7.5-4	1.8+1	337	1.00
.992	7.34+07	3.85+4	.83	.42	.00	5.9-4	1.6+1	310	1.00
.991	1.16+08	4.33+4	.86	.57	.00	4.7-4	1.4+1	287	1.00
.989	1.84+08	4.92+4	.89	.72	.00	3.7-4	1.3+1	268	1.00
.988	2.92+08	5.67+4	.92	.83	.00	3.0-4	1.2+1	251	1.00
.986	4.63+08	6.60+4	.94	.89	.01	2.3-4	1.0+1	233	1.00
.984	7.34+08	7.74+4	.95	.90	.04	1.7-4	8.9+0	214	1.00
.982	1.16+09	9.08+4	.96	.85	.12	1.2-4	7.3+0	194	1.00
.979	1.84+09	1.06+5	.97	.72	.26	8.1-5	5.9+0	176	1.00
.975	2.92+09	1.24+5	.97	.56	.43	5.7-5	4.9+0	160	1.00
.972	4.63+09	1.46+5	.98	.39	.60	4.2-5	4.2+0	148	1.00
.967	7.34+09	1.72+5	.98	.26	.74	3.1-5	3.6+0	137	1.00
.961	1.16+10	2.05+5	.98	.17	.83	2.2-5	3.1+0	126	1.00
.955	1.84+10	2.44+5	.98	.11	.89	1.6-5	2.7+0	116	1.00
.947	2.92+10	2.92+5	.98	.08	.92	1.1-5	2.3+0	107	1.00
.938	4.63+10	3.49+5	.98	.06	.94	8.2-6	2.0+0	99	1.00
.927	7.34+10	4.18+5	.99	.04	.96	5.8-6	1.7+0	91	.99
.915	1.16+11	5.02+5	.99	.03	.97	4.1-6	1.4+0	83	.99
.900	1.84+11	6.02+5	.99	.03	.97	2.9-6	1.2+0	77	.98
.883	2.92+11	7.22+5	.99	.02	.98	2.1-6	1.0+0	70	.96
.863	4.63+11	8.66+5	.99	.02	.98	1.4-6	8.6-1	64	.92
.840	7.34+11	1.04+6	.99	.02	.98	9.8-7	7.0-1	58	.86
.815	1.16+12	1.25+6	.99	.02	.98	6.5-7	5.6-1	52	.75
.786	1.84+12	1.50+6	.99	.02	.98	4.0-7	4.1-1	44	.57
.767	2.43+12	1.67+6	.99.	01	.99	2.6-7	3.0-1	38	.40
.753	2.92+12	1.80+6	.99	.01	.99	1.5-7	1.9-1	30	.22
.746	3.21+12	1.87+6	.99	.01	.99	9.2-8	1.2-1	24	.11
.739	3.51+12	1.94+6	.99	.01	.99	-4.0-3	0.	0	.00

$1\ \text{Pa} = 1\ \text{N/m}^2 = 10\ \text{dyne/cm}^2$

B. Solar Dynamo

1. Basic Idea

- Observation -

Solar magnetic fields show some patterns:

(1) the 11-yr cycle in spot number,

(1) the sunspot belt of latitudes,

(1) the spread and drift of sunspots toward the equator,

(1) the inclination of sunspot groups ($\sim 10^\circ$) to the equator,

(1) the laws of polarity,

(1) the reversal of polar fields near solar maximum,

which are commonly thought to be caused by some kind of dynamo operating in deeper convection zone.

- Theory -

A magnetic field is maintained by currents induced in a plasma by its motion across lines of force in a following way:

- A motion ($\mathbf{v}$) across $\mathbf{B}$ leads to an induced electric field ($\mathbf{v} \times \mathbf{B}$)
- ($\mathbf{v} \times \mathbf{B}$) drives an electric current $\mathbf{j}$ which gives a magnetic field $\mathbf{B}'$.
- The magnetic field creates $\mathbf{E}$ and $\mathbf{j} \times \mathbf{B}'$ which can oppose the motion.

To describe this dynamo process, one should, in principle, solve a set of coupled equations:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \qquad \text{(induction equation)}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mathbf{j} \times \mathbf{B} + ... \qquad : \text{(equation of motion)}.$$

However, the problem is too difficult and do instead solve only the induction equation with $\mathbf{v}$ specified *a priori*. This is called **Kinematic Dynamo Theory**.

2. Cowling's theorem

"A steady axisymmetric magnetic field cannot be maintained."

Write a steady, axisymmetric field as sum of a toroidal (azimuthal B_ϕ) and a poloidal fields $\mathbf{B}_p$ (radial and axial components in cylindrical coordinates):

$$\mathbf{B} = \mathbf{B}_\phi + \mathbf{B}_p$$

Because of the axisymmetry, there exists at least one O-type neutral point (N) where $\mathbf{B}_p$ vanishes so that the field is purely azimuthal. Now Ohm's law in the form $\mathbf{j}/\sigma = \mathbf{E} + \mathbf{v} \times \mathbf{B}$ may be integrated around the closed line of force (C) through the neutral points (N) to give

$$\oint_C \frac{j_\phi}{\sigma} ds = \oint_C \mathbf{E} \cdot d\mathbf{S} + \oint_C \mathbf{v} \times \mathbf{B} \cdot d\mathbf{s}.$$

We are looking for field s.t. $\partial \mathbf{B}/\partial t = 0$. Note that the 1st term on RHS is $\oint_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = \oint_S (\partial \mathbf{B}/\partial t) \cdot d\mathbf{S} = 0$, and also $\oint_C \mathbf{v} \times \mathbf{B} \cdot ds = 0$ since $\mathbf{B} \,||\, d\mathbf{s}$. Then RHS vanishes but LHS remains nonzero. Since this equality is not satisfied, a steady state field cannot be symmetric.

Alternatively, we may consider what happens when you assume • Axisymmetry $(\frac{\partial}{\partial \phi} = 0)$ and • Axisymmetric flow $(\mathbf{v} = \mathbf{v}_\phi)$ in the induction equation. You will find:

$$\nabla \times (\mathbf{v} \times \mathbf{B})\Big]_\phi \neq 0, \qquad \nabla \times (\mathbf{v} \times \mathbf{B})\Big]_{r,z} = 0$$

This means that the toroidal component $\mathbf{B}_p$ only decays under ohmic diffusion without re-generation:

$$\frac{\partial \mathbf{B}_\phi}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})\Big]_\phi - \eta \nabla^2 \mathbf{B}_\phi, \qquad \frac{\partial \mathbf{B}_{r,z}}{\partial t} = -\eta \nabla^2 \mathbf{B}_{r,z}.$$

"Axisymmetric magnetic fields cannot be maintained by axisymmetric motion of a conducting fluid."

3. α**-effect** (the electromotive force proportional to magnetic field)

A resolution for the difficulty with an axisymmetric field (Cowling's theorem) was put forward by Parker (1955ApJ122). Let me introduce it in the following way. To re-state the problem, the induction equation from the previous page is:

$$\frac{\partial B_\phi}{\partial t} = B_z \frac{\partial V_\phi}{\partial z} + B_r \Big(\frac{\partial V_\phi}{\partial r} - \frac{V_\phi}{r} \Big) + \eta \nabla^2 B_\phi \qquad (1)$$

$$\frac{\partial \mathbf{B}_p}{\partial t} = \eta \nabla^2 \mathbf{B}_p$$

which means B_ϕ (poloidal) can be generated from $\mathbf{B}_p = (B_r, 0, B_z)$ (toroidal components), but not vice versa.

Now wish there is generation of $\mathbf{B}_p$ in some way and we denote this by a term $(\partial \mathbf{B}_p/\partial t)'$. Then the last equation will change to

$$\frac{\partial \mathbf{B}_p}{\partial t} = \eta \nabla^2 \mathbf{B}_p + \Big(\frac{\partial \mathbf{B}_p}{\partial t} \Big)'$$

By Maxwell's law, we can replace $(\partial \mathbf{B}_p/\partial t)'$ with $-\nabla \times \mathbf{E}$. We can also put $\mathbf{B}_p = \nabla \times \mathbf{A}$ from $\nabla \cdot \mathbf{B}_p = 0$. It can easily be shown that $\mathbf{A}$ in the form $(0, A_\phi, 0)$ generates $\mathbf{B}_p$ only. Thus the above reduces to

$$\frac{\partial A_\phi}{\partial t} = \eta \nabla^2 A_\phi - E_\phi, \qquad (2)$$

which means that A_ϕ and thus $\mathbf{B}_p$ can also be amplified provided there is such electric field E_ϕ.

Parker's idea is that as blobs of plasma rise and expand, they rotate under the Coriolis force and this twist may convert toroidal fields into poloidal ones. Parker modeled the net effect of many convective cells by adding an electric field as

$$-E_\phi = \alpha B_\phi.$$

Here the constant of proportionality, α, in the mean e.m.f (E_ϕ) over many eddies is *a measure of the mean rotational speed of eddies*, and the dynamo action in this way is called α-dynamo. To have an α-effect, we need *asymmetry* between up and down motions (maybe possible from stratification, convection geometry). Other ways of creating α-effect include hydromagnetic inertial waves and magnetic buoyancy.

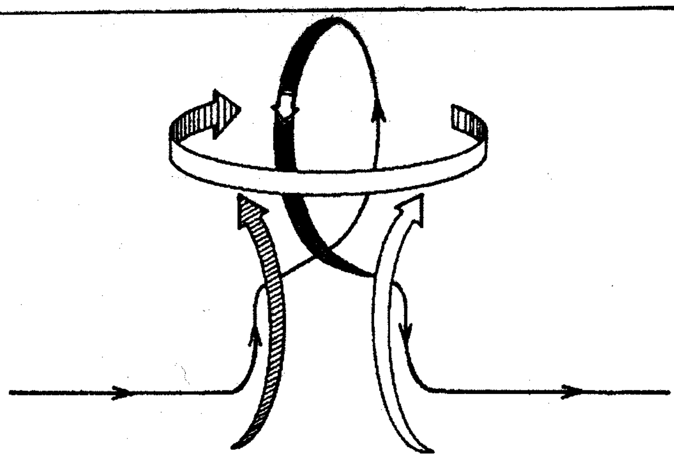


FIG. 1.—Schematic drawing showing how a cyclonic convective cell raises and twists the azimuthal fields B_ϕ into a loop with nonvanishing projection in the meridional plane. Coalescence of many such loops leads to a general meridional field B_m. Solid line represents a magnetic line of force; ribbon arrows represent the cyclonic fluid motions.

Parker '70 ApJ 162, 665

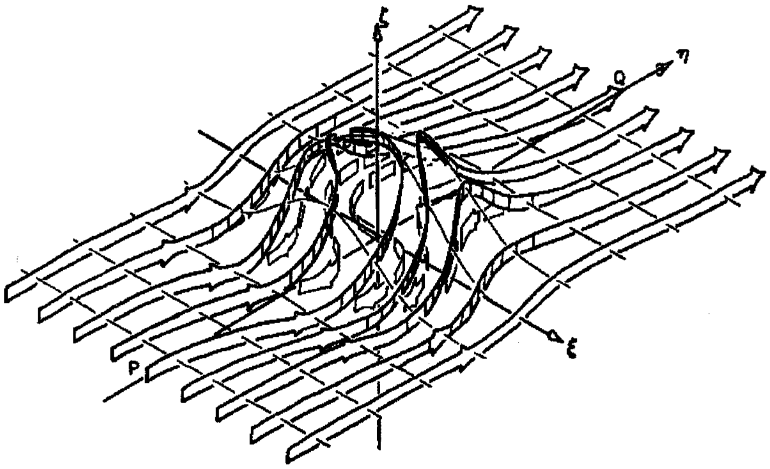
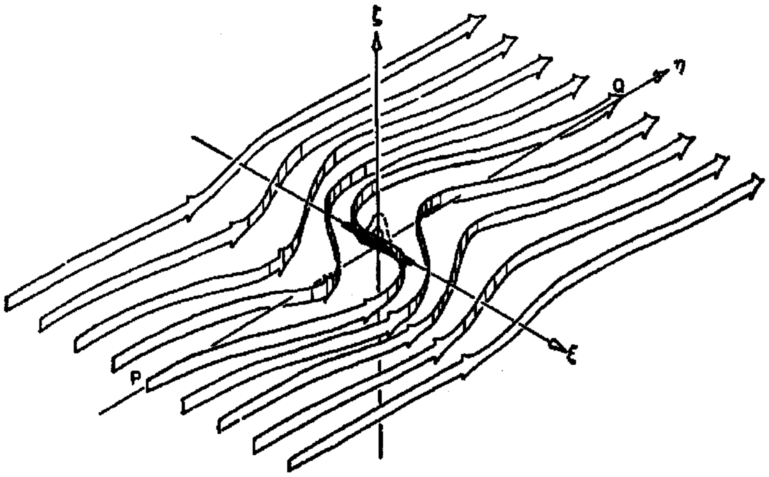


FIG 1.—The deformation of a slab of toroidal field by cyclonic fluid motion. The ribbons underneath represent $-\beta$.



Parker 1955 ApJ 122, 293

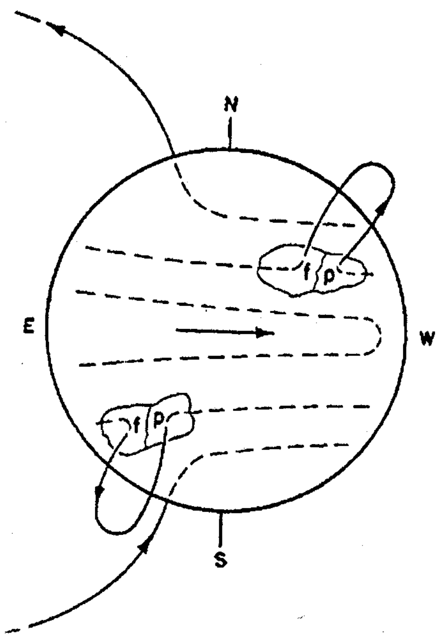
FIG. 2.—The deformation of a slab of toroidal field by a cyclonic fluid motion in the absence of vertical motion. This is the field $+\beta$.

4. ω-effect

ω-effect refers to velociy shear due to differential rotation. This has earlier been incorporated in some phenomenological models for solar cycle (Babcock 1961ApJ133, Leighton 1964ApJ140, 1969ApJ156)

Babcock (1961): a qualitative model for the solar cycle

- At solar solar minimum there exists below the surface a weak poloidal field stretched by differential rotation to form a strong toroidal field.



- Kinks in the toroidal field rise to the surface at low latitudes to form sunspots.
- In the process of rising they are rotated by Coriolis forces, so that a poloidal field is generated in the opposite direction to the original one.
- The reversed poloidal flux is able to diffuse the poles due to a supergranular eddy diffusion in time for the next solar minimum. Note that leading spot's polarity = that of the pole.

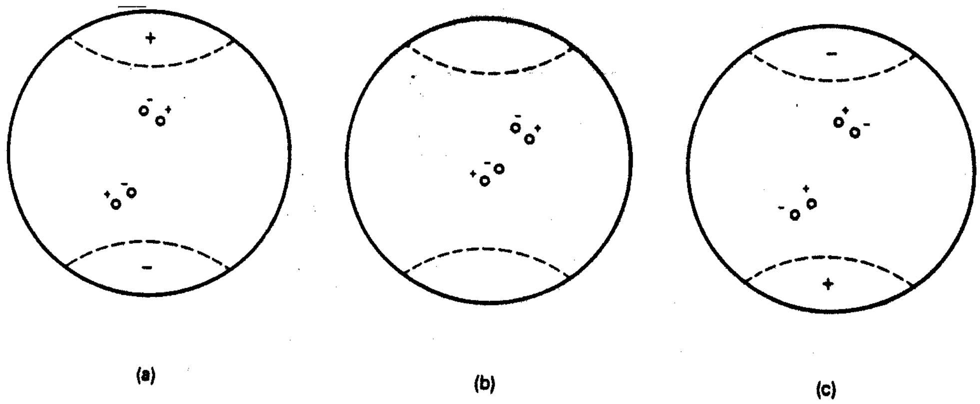


Fig. 1.29. The polarity of sunspots and polar regions for (a) the start of one cycle, (b) the maximum of that cycle and (c) the start of the next cycle.

Leighton (1969): a phenomenological modeling for solar cycle

1. The differential rotation:

$$\Omega = \Omega_s + (a + b \ \sin^n \theta) \frac{R - r}{H}$$

confined to $R - H \le r \le R$ acts on (B_r, B_θ) to change B_ϕ:

$$\left(\frac{\partial B_\phi}{\partial t}\right)_1 = \nabla \times (\mathbf{v} \times \mathbf{B})\Big]_\phi = \sin\theta \Big(B_\theta \frac{\partial \Omega}{\partial \theta} + R B_r \frac{\partial \Omega}{\partial r}\Big) = ...$$

2. When B_ϕ exceeds some B_c, field erupts, which reduces the strength of the remaining B_ϕ:

$$\left(\frac{\partial B_\phi}{\partial t}\right)_2 = -\frac{a B_\phi}{2\pi R \tau} = -a_0 \frac{|B_\phi|}{B_c} \frac{B_\phi}{2\pi R\tau} \qquad |B_\phi| > B_c$$

$$= \ 0 \qquad |B_\phi| \le B_c$$

3. Eruption of B_ϕ produces B_r having a net meridional component:

$$\left(\frac{\partial B_r}{\partial t}\right)_1 = \frac{Ha}{2\pi R^2 \tau} \frac{\partial (B_\phi \sin\gamma)}{\partial \mu} \qquad |B_\phi| > B_c$$

$$= \ 0 \qquad |B_\phi| \le B_c$$

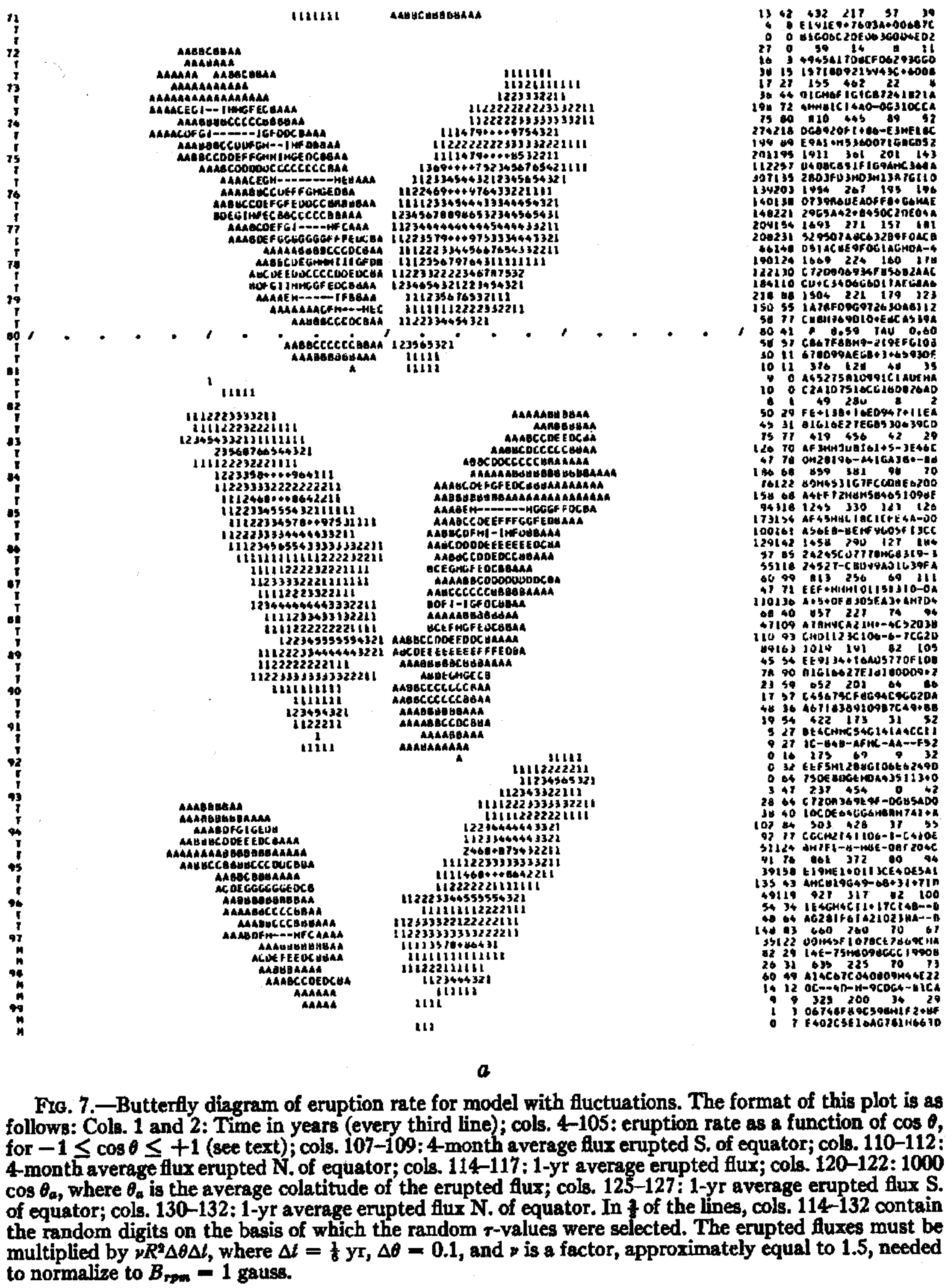
4. B_r is also dispersed by a random walk (Leighton 1964):

$$\left(\frac{\partial B_r}{\partial t}\right)_2 = \frac{1}{\tau_D} \frac{\partial}{\partial \mu} \Big[(1 - \mu^2) \frac{\partial B_r}{\partial \mu}\Big]$$

Pro: The result of numerical simulation is very successful in reproducing the main features of the solar cycle. (see the next figure.)

Con: 1. $d\omega/dr < 0$ produces dynamo action most easily, but is it?

2. The lack of an $r-$dependence of $\mathbf{B}$ means that the generation of poloidal field is not incorporated adequately.



a

FIG. 7.—Butterfly diagram of eruption rate for model with fluctuations. The format of this plot is as follows: Cols. 1 and 2: Time in years (every third line); cols. 4–105: eruption rate as a function of cos θ, for $-1 \leq \cos\theta \leq +1$ (see text); cols. 107–109: 4-month average flux erupted S. of equator; cols. 110–112: 4-month average flux erupted N. of equator; cols. 114–117: 1-yr average erupted flux; cols. 120–122: 1000 cos θ_a, where θ_a is the average colatitude of the erupted flux; cols. 125–127: 1-yr average erupted flux S. of equator; cols. 130–132: 1-yr average erupted flux N. of equator. In $\frac{2}{3}$ of the lines, cols. 114–132 contain the random digits on the basis of which the random τ-values were selected. The erupted fluxes must be multiplied by $\nu R^2 \Delta\theta \Delta t$, where $\Delta t = \frac{1}{3}$ yr, $\Delta\theta = 0.1$, and ν is a factor, approximately equal to 1.5, needed to normalize to $B_{rpm} = 1$ gauss.

5. Migratory Dynamo and Dynamo number

Parker (1955, 1979) obtained a simple solution to the dynamo equation under plane geometry and a constant velocity shear, which leads to a useful parameter, N_D, the dynamo number. Consider the magnetic field and a toroidal velocity velocity in the form:

$$\mathbf{B} = \Big[-\frac{\partial A}{\partial z}, B, \frac{\partial A}{\partial x} \Big], \qquad \mathbf{v} = \mathbf{v}_y(z)$$

where z is locally normal to the solar surface and y in the east. Assume a uniform vertical shear $(dv_y/dz) =$ constant.

$$\Big[\frac{\partial}{\partial t} - \eta\nabla^2\Big] B = \Big(\frac{dv_y}{dz}\Big)\frac{\partial A}{\partial x},$$

$$\Big[\frac{\partial}{\partial t} - \eta\nabla^2\Big] A = \alpha B.$$

These linear dynamo equations possess plane-wave solutions of the form

$$B = B_0 \exp[pt + i(k_x x + k_y y)]$$

where

$$p = -\eta k^2 \pm (1+i)\Big[\frac{\alpha k_x}{2}\frac{dv_y}{dz}\Big]^{1/2}$$

The first term gives simple ohmic decay and the second term gives field generation by α-effect. Define dynamo number by the ratio of the square of these terms:

$$N_D = \frac{\alpha k_x}{2\eta^2 k^4}\frac{dv_y}{dz}$$

$N_D > 1$ gives growing mode traveling northward and $N_D < -1$, southward. $N_D = 1$ gives steady field generation.

Migratory dynamo:

In the sun α changes its sign at the equator so the waves approach or leave the equator depending on the sign of $\alpha(dv_y/dz)$. The waves migrate equatorwards like sunspots if the toroidal velocity increases with depth $(dv_y/dz) < 0$. The wave period agrees with the solar-cycle duration if an eddy diffusion 10^9 m^2 s^{-1} is adopted for η.

Steady dynamo: maybe terrestrial or galactic fields.

Solutions (113) of the dynamo equations become the migratory *dynamo waves*,

$$B = B_0 \exp\left[\left(\Omega - \frac{k^2}{\mu\sigma}\right)t\right] \exp\left[i\left(k\xi \pm \Omega t\right)\right], \tag{120}$$

$$A = -B_0 \frac{\Omega}{kH}(1 \mp i) \exp\left[\left(\Omega - \frac{k^2}{\mu\sigma}\right)t\right] \exp\left[i\left(k\xi \pm \Omega t\right)\right]. \tag{121}$$

If $kH\Gamma > 0$, we have a dynamo wave traveling in the negative ξ-direction; if $kH\Gamma < 0$, the wave travels in the positive ξ-direction. The vector potential is $\pi/4$ out of phase with B. Assuming that $\Omega > k^2/\mu\sigma$, the amplitude increases exponentially with time. If the conductivity is large enough that $\Omega \gg k^2/\mu\sigma$, the amplitude increases by a factor of exp 2π, or about a factor of 500, for every $2\pi/k$ (one wave length) the wave propagates. The velocity of propagation Ω/k varies as $k^{-1/2}$ and hence increases as the square root of the wave length.

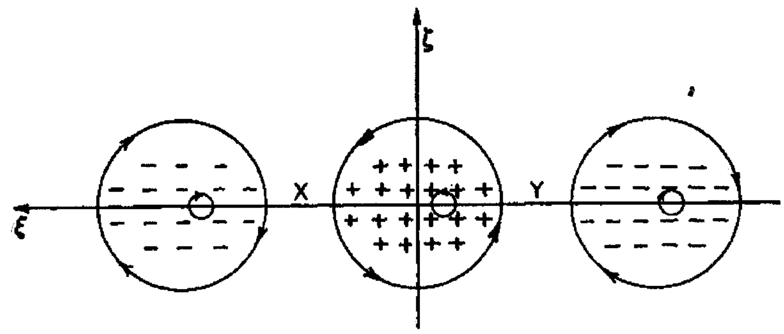
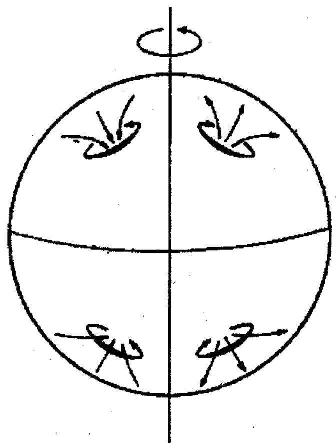


FIG. 5.—Schematic drawing of a section along a train of migratory dynamo waves The + signs represent flux coming out of the paper, and − signs into the paper.



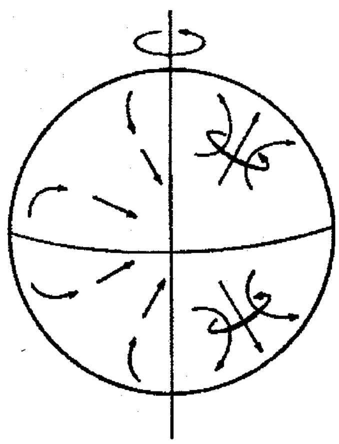


FIG. 1 (*left*).—Sketch illustrating the divergence (or convergence) and the resulting sense of rotation of rising and sinking fluid in the solar convection zone.

FIG. 2 (*right*).—Sketch illustrating the suggested sense of rotation in concentrated rising cells of fluid in the core of Earth. The broad subsidence without much rotation between cells is also indicated.

6. α-ω dynamo

Roberts (1972) solved the dynamo equation in a sphere including:

- an α-effect : $\alpha = \alpha_0 \cos\theta$ which reverses sign at the equator ($\theta = \pi/2$) and is due to small-scale turbulent velocity $\mathbf{v}$,
- an ω-effect: $\omega = v_\phi/R \equiv r\omega_0$, differential rotation due to a shear in angular velocity,
- a large scale flow v_0 and turbulent diffusivity $\tilde{\eta} \gg \eta$.

This leads to the two scalar dynamo equations:

$$\frac{\partial B_\phi}{\partial t} + R[\mathbf{v}_p \cdot \nabla]\frac{B_\phi}{R} = R(\mathbf{B}_p \cdot \nabla)\omega + \nabla \times (\alpha \mathbf{B}_p) + \tilde{\eta}(\nabla^2 - R^{-2})B_\phi \qquad (1')$$

$$\frac{\partial A_p}{\partial t} + \frac{\mathbf{v}_p}{R} \cdot \nabla(RA_p) = \alpha B_\phi + \tilde{\eta}(\nabla^2 - R^{-2})A_p \qquad (2')$$

which are similar to (1) and (2) except $\eta \to \tilde{\eta}$ and α effect is now included in ($1'$).

- When rotation is weak, the α effect dominates and both B_ϕ and B_p are generated by α effect $\to$ α^2-dynamo. This is a steady dynamo and seems more relevant to the terrestrial dynamo.
- When rotation is strong, an α-ω dynamo results in which ω-effect generates toroidal fields and α-effect, the poloidal fields–probably the solar dynamo.

7. Further theoretical considerations on α and ω

- Stix (1972): includes a feedback of the Lorentz force on the motion

$$\alpha = \frac{\alpha_0}{2}\left[1 - erf\frac{B - B_c}{c}\right]$$

- Yoshimura (1975, 1978, 1979): -assumes various functional forms for α and ω to describe flux eruption by magnetic buoyancy and a time-delay for the feedback of the magnetic field on the dynamo process. -reproduces a realistic butterfly

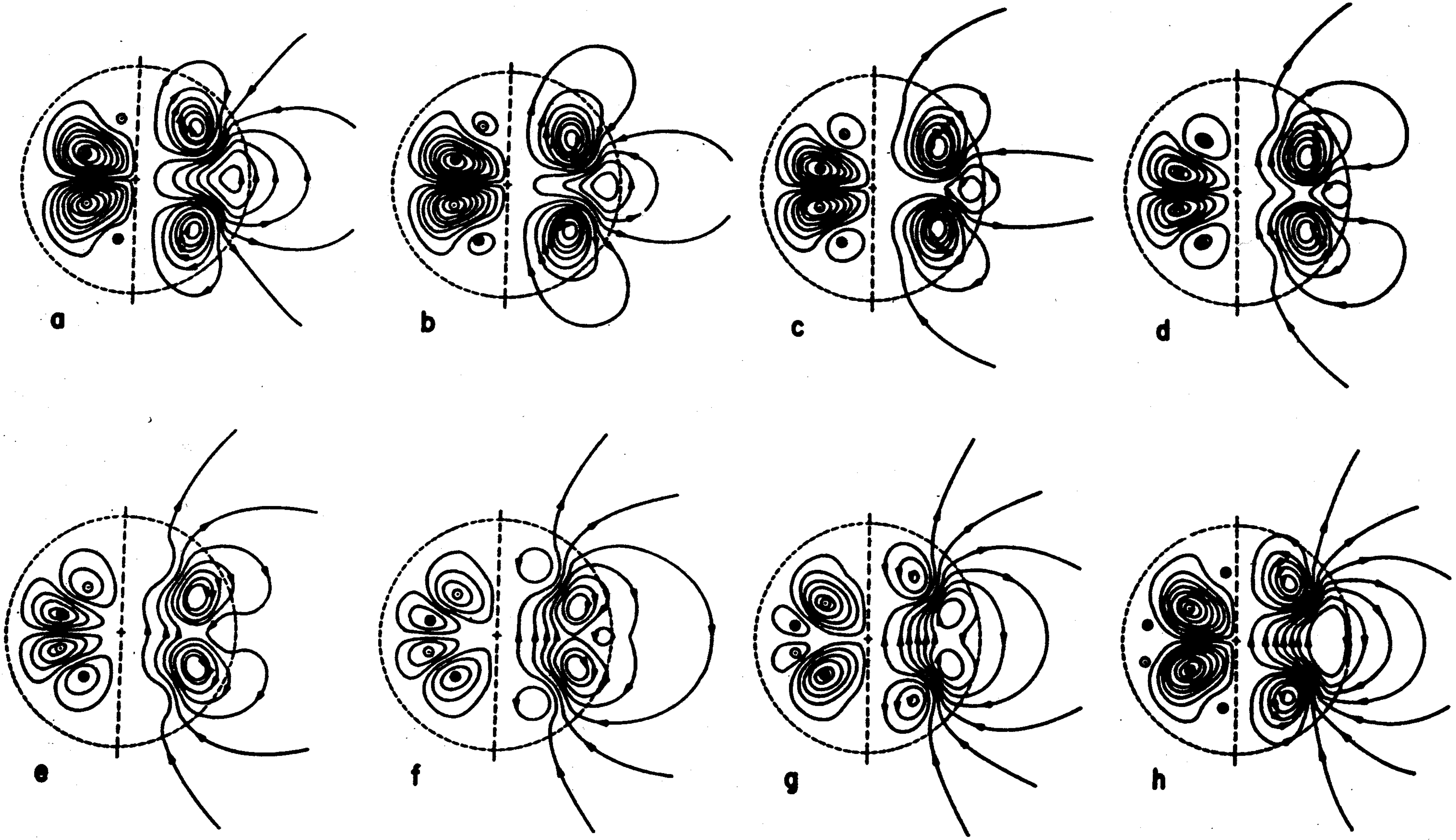


Fig. 9.4 A half-cycle for an α–ω dynamo with $\alpha = \alpha(R, \theta)$ and $\omega = \omega(R)$. The dynamo number for this marginal oscillation is $X = -206$, and $\Omega = (a^2/\eta)\mathbb{I}(p)$ is 47.4. $\mathbf{B}_p$-lines are shown in each right-hand hemisphere and lines of constant B_φ on the left at intervals of $\Omega t = \pi/8$. (From Roberts, 1972.)

diagram, the poleward migration of mid-latitude poloidal flux in a secondary dynamo wave, a 55-year modulation of the 11-yr cycle, & the Maunder Minimum.

- Malkus & Proctor (1975): possible nonlinear effect to limit the growth of a linear α-ω dynamo
- Leighton (1969), Yoshimura (1975): the possible removal of flux by magnetic buoyancy .
- Stix (1975), Jepps (1975): the reduction of α due to the inhibition of convection when $\mathbf{B}$ increases.
- Gilman (1969): the limitation of differential rotation by the Lorentz force
- Zeldovich & Ruzmaikin (1980): third order nonlinear system
- Jones (1982), Ito (1980), Cook & Roberts (1970)......

"Any success in reproducing solar behavior does not prove the model."

8. Convective dynamo

→ Dynamo models in which *buoyancy* is the driving force in the Boussinesq approximation.

- Soward (1974): 3-D motion in the shape of square or hexagonal cells, nonlinear oscillatory solution.

- Busse (1973, 1975): shear flow parallel to the axis of a horizontal 2D roll.

- Gilman (1977, 1978): Boussinesq convection in a rotating spherical shell. The resulting differential rotation is driven by Reynolds stress rather than meridional circulation, and possesses equatorial acceleration. However too large magnitude of helicity was required.

- Gilman and Miller (1981): extended Gilman's work 1977 by numerically solving the coupled equations of induction, motion, energy and continuity for a convectively-driven dynamo. Many physics are incorporated but the solution does not behave like solar magnetic fields.

9. Difficulties with solar dynamo theory

(1) The filamentary nature of solar magnetic fields should be incorporated.

(2) α is estimated under the first-order smoothing approximation ($\langle \mathbf{v} \times \mathbf{B} \rangle = 0$), which assumes either that small-scale turbulence is weak or that life time of an eddy much shorter than its circulation time.

(3) Dynamo models need $\alpha \approx 1 - 10$ cm s^{-1}, but α calculated from mixing-length theory is much too high ≈ 100 m s^{-1}. The theory give too large an effect of turbulence on the magnetic field. (Maybe this effect can be reduced by incorporating a feedback from the Lorentz force and a filamentation.)

(4) Observation gives $\eta \approx 10^9$ m^{-2} s^{-1} or larger, but this is ten times higher than required in some dynamo models.

(5) The sign of $d\omega/dr$. $d\omega/dr < 0$ is needed to have drift toward equator, but observation .... (cf. solar seismology, torsional oscillation, meridional flow.)

(6) The whole concept of the turbulent diffusion of a magnetic field.

(7) The large amount of magnetic fluxes emerges as ephemeral active regions, rather than sunspots.

(8) Observations of changes in solar radius, luminosity and surface temperatures may help.

(9) The apparent rigid-body rotation of coronal holes.