## CHAPTER 3

# HYDRAULICS OF CULVERTS 

The hydraulic design procedure for culverts requires:

1. Determination of Design Flow
2. Selection of Culvert Size
3. Determination of Outlet Velocity

## DETERMINATION OF DESIGN FLOW

The United States Geological Survey has developed a nationwide series of water-supply papers titled the "Magnitude and Frequency of Floods in the United States." These reports contain tables of maximum known floods and charts for estimating the probable magnitude of floods of frequencies ranging from 1. 1 to 50 years. Table 11 indicates the Geological Survey regions, USGS district and principal field offices and the applicable water-supply paper numbers. Most states have adapted and consolidated those parts of the water-supply papers which pertain to specific hydrologic areas within the particular state. The hydrologic design procedures developed by the various states enable quick and accurate determination of design flow. It is recommended that the culvert design flow be determined by methods based on USGS data.

If USGS data are not available for a particular culvert location, flow quantities may be determined by the Rational Method or by statistical methods using records of flow and runoff. An example of the latter method is a nomograph developed by California and shown in Figure 27.

## FACTORS AFFECTING CULVERT DISCHARGE

Factors affecting culvert discharge are depicted on the culvert cross section shown in Illustration 3.1 and are used in determining the type of discharge control.

Inlet Control. The control section is located at or near the culvert entrance, and, for any given shape and size of culvert, the discharge is dependent only on the inlet geometry and headwater depth. Inlet control will exist as long as water can flow through the barrel of the culvert at a greater rate than water can enter the inlet. Since the control section is at the inlet, the capacity is not affected by any hydraulic factors beyond the culvert entrance such as slope, length or surface roughness. Culverts operating under inlet control will always flow partially full.

## Illustration 3.1-Factors Affecting Culvert Discharge

$$
\begin{aligned}
\text { D } & =\text { Inside diameter for circular pipe } \\
H W & =\text { Headwater depth at culvert entrance } \\
\mathrm{L} & =\text { Length of culvert } \\
\mathrm{n} & =\text { Surface roughness of the pipe wall, usually expressed in terms of } \\
& \text { Manning's } \mathrm{n} \\
\mathrm{~S}_{\mathrm{o}} & =\text { Slope of the culvert pipe } \\
\mathrm{TW} & =\text { Tailwater depth at culvert outlet }
\end{aligned}
$$



Outlet Control. The control section is located at or near the culvert outlet and for any given shape and size of culvert, the discharge is dependent on all of the hydraulic factors upstream from the outlet such as shape, slope, length, surface roughness, tailwater depth, headwater depth and inlet geometry. Outlet control will exist as long as water can enter the culvert at a greater rate than water can flow through it. Culverts operating under outlet control can flow either full or partially full.

Critical Depth. Critical flow occurs when the sum of the kinetic energy (velocity head) plus the potential energy (static or depth head equal to the depth of the flow) for a given discharge is at a minimum. Conversely, the discharge through a pipe with a given total energy head will be maximum at critical flow. The depth of the flow at this point is defined as critical depth, and the slope required to produce the flow is defined as critical slope. Capacity of a culvert with an unsubmerged outlet will be established at the point where critical flow occurs. Since under inlet control, the discharge of the culvert is not reduced by as many hydraulic factors as under outlet control, for a given energy head, a culvert will have maximum possible discharge if it is operating at critical flow with inlet control. The energy head at the inlet control section is approximately equal to the head at the inlet minus entrance losses. Discharge is not limited by culvert roughness or outlet conditions but is dependent only on the shape and size of the culvert entrance. Although the discharge of a culvert operating with inlet control is not related to the pipe roughness, the roughness does determine the minimum slope (critical slope) at which inlet control will occur. Pipe with a smooth interior can be installed on a very flat slope and still have inlet control. Pipe with a rough interior must be installed on a much steeper slope to have inlet control. Charts of critical depth for various pipe and box section sizes and flows are shown in Figures 28 through 32.

## SELECTION OF CULVERT SIZE

The many hydraulic design procedures available for determining the required size of a culvert vary from empirical formulas to a comprehensive mathematical analysis. Most empirical formulas, while easy to use, do not lend themselves to proper evaluation of all the factors that affect the flow of water through a culvert. The mathematical solution, while giving precise results, is time consuming. A systematic and simple design procedure for the proper selection of a culvert size is provided by Hydraulic Engineering Circular No. 5, "Hydraulic Charts for the Selection of Highway Culverts" and No. 10, "Capacity Charts for the Hydraulic Design of Highway Culverts," developed by the Bureau of Public Roads. The procedure when selecting a culvert is to determine the headwater depth from the charts for both assumed inlet and outlet controls. The solution which yields the higher headwater depth indicates the governing control. When this procedure is followed, Inlet Control Nomographs, Figures 33 through 37, and Outlet Control Nomographs, Figures 38 through 41, are used.

An alternative and simpler method is to use the Culvert Capacity Charts, Figures 42 through 145. These charts are based on the data given in Circular No. 5 and enable the hydraulic solution to be obtained directly without using the double solution for both inlet and outlet control required when the nomographs are used.

Culvert Capacity Chart Procedure. The Culvert Capacity Charts are a convenient tool for selection of pipe sizes when the culvert is installed with conditions as indicated on the charts. The nomographs must be used for other shapes, roughness coefficients, inlet conditions or submerged outlets.

## List Design Data

A. Design discharge $Q$, in cubic feet per second, with average return period (i.e., $Q_{25}$ or $Q_{50}$, etc.).
B. Approximate length $L$ of culvert, in feet.
C. Slope of culvert.
D. Allowable headwater depth, in feet, which is the vertical distance from the culvert invert (flow line) at the entrance to the water surface elevation permissible in the headwater pool or approach channel upstream from the culvert.
E. Mean and maximum flood velocities in natural stream.
F. Type of culvert for first trial selection, including barrel cross sectional shape and entrance type.

## Select Culvert Size

A. Select the appropriate capacity chart, Figures 42 to 145, for the culvert size approximately equal to the allowable headwater depth divided by 2.0.
B. Project a vertical line from the design discharge $Q$ to the inlet control curve. From this intersection project a line horizontally and read the headwater depth on the vertical scale. If this headwater depth is more
than the allowable, try the next larger size pipe. If the headwater depth is less than the allowable, check the outlet control curves.
C. Extend the vertical line from the design discharge to the outlet control curve representing the length of the culvert. From this intersection project a line horizontally and read the headwater depth plus $\mathrm{S}_{0} \mathrm{~L}$ on the vertical scale. Subtract $S_{0} L$ from the outlet control value to obtain the headwater depth. If the headwater depth is more than the allowable, try the next larger size pipe. If the headwater depth is less than the allowable, check the next smaller pipe size following the same procedure for both inlet control and outlet control.
D. Compare the headwater depths for inlet and outlet control. The higher headwater depth indicates the governing control.

## Determine Outlet Velocity

A. If outlet control governs, the outlet velocity equals the flow quantity divided by the flow cross sectional area at the outlet. Depending upon the tailwater conditions, this flow area will be between that corresponding to critical depth and the full area of the pipe. If the outlet is not submerged, it is usually sufficiently accurate to calculate the flow area based on a depth of flow equal to the average of the critical depth and the vertical height of the pipe.
B. If inlet control governs, the outlet velocity may be approximated by Manning's formula using Figures 2 through 19 for full flow values and Figures 20 through 24 for partial flow values.

## Record Selection

Record final selection of culvert with size, type, required headwater and outlet velocity.

Nomograph Procedure. The nomograph procedure is used for selection of culverts with entrance conditions other than projecting or for submerged outlets.

## List Design Data

A. Design discharge $Q$, in cubic feet per second, with average return period (i.e., $Q_{25}$ or $Q_{50}$, etc.).
B. Approximate length $L$ of culvert, in feet.
C. Slope of culvert.
D. Allowable headwater depth, in feet, which is the vertical distance from the culvert invert (flow line) at the entrance to the water surface elevation permissible in the headwater pool or approach channel upstream from the culvert.
E. Mean and maximum flood velocities in natural stream.
F. Type of culvert for first trial selection, including barrel cross sectional shape and entrance type.

## Select Trial Culvert Size

Select a trial culvert with a rise or diameter equal to the allowable headwater divided by 2.0.

## Find Headwater Depth for Trial Culvert

A. Inlet Control
(1) Given Q, size and type of culvert, use appropriate inlet control nomograph Figures 33 through 37 to find headwater depth:
(a) Connect with a straightedge the given culvert diameter or height (D) and the discharge Q; mark intersection of straightedge on HW/D scale marked (1).
(b) HW/D scale marked (1) represents entrance type used, read HW/D on scale (1). If another of the three entrance types listed on the nomograph is used, extend the point of intersection in (a) horizontally to scale (2) or (3) and read HW/D.
(c) Compute HW by multiplying HW/D by D.
(2) If HW is greater or less than allowable, try another trial size until HW is acceptable for inlet control.
B. Outlet Control
(1) Given Q, size and type of culvert and estimated depth of tailwater TW, in feet, above the invert at the outlet for the design flood condition in the outlet channel:
(a) Locate appropriate outlet control nomograph (Figures 38 through 41) for type of culvert selected. Find $\mathrm{k}_{\mathrm{e}}$, for entrance type from Table 12.
(b) Begin nomograph solution by locating starting point on length scale for proper $\mathrm{k}_{\mathrm{e}}$.
(c) Using a straightedge, connect point on length scale to size of culvert barrel and mark the point of crossing on the "turning line."
(d) Pivot the straightedge on this point on the turning line and connect given discharge rate. Read head in feet on the head (H) scale.
(2) For tailwater TW elevation equal to or greater than the top of the culvert at the outlet set $h_{0}$ equal to TW and find HW by the following equation:

$$
\begin{equation*}
H W=H+h_{0}-S_{0} L \tag{3}
\end{equation*}
$$

(3) For tailwater TW elevations less than the top of the culvert at the outlet, use $h_{o}=\frac{d_{c}+D}{2}$ or TW, whichever is the greater, where $d_{c}$, the critical depth in feet is determined from the appropriate critical depth chart (Figures 28 through 32).
C. Compare the headwaters found in paragraphs A (Inlet Control) and B (Outlet Control). The higher headwater governs and indicates the flow control existing under the given conditions for the trial size selected.
D. If outlet control governs and the HW is higher than acceptable, select a larger trial size and find HW as instructed under paragraph B. Inlet control need not be checked, if the smaller size was satisfactory for this control as determined under paragraph A.

## Try Another Culvert

Try a culvert of another size or shape and repeat the above procedure.

## Determine Outlet Velocity

A. If outlet control governs, the outlet velocity equals the flow quantity divided by the flow cross sectional area at the outlet. Depending upon the tailwater conditions, this flow area will be between that corresponding to critical depth and the full area of the pipe. If the outlet is not submerged, it is sufficiently accurate to calculate flow area based on a depth of flow equal to the average of the critical depth and vertical height of the pipe.
B. If inlet control governs, the outlet velocity may be approximated by Manning's formula using Figures 2 through 19 for full flow values and Figures 20 through 24 for partial flow values.

## Record Selection

Record final selection of culvert with size, type, required headwater and outlet velocity.

## EXAMPLE PROBLEMS

EXAMPLE 3-I CULVERT CAPACITY CHART PROCEDURE

## List Design Data

A. $Q_{25}=180$ cubic feet per second $Q_{50}=225$ cubic feet per second
B. $L=200$ feet
C. $S_{0}=0.01$ feet per foot
D. Allowable HW = 10 feet for 25 and 50 -year storms
E. $T W=3.5$ feet for 25 -year storm

TW = 4.0 feet for 50-year storm
F. Circular concrete culvert with a projecting entrance, $n=0.012$

Select Culvert Size
A. Try $D=\frac{H W}{2.0}=\frac{10}{2.0}=5$ feet or 60 inch diameter as first trial size.
B. In Figure 54, project a vertical line from $Q=180$ cubic feet per second
to the inlet control curve and read horizontally $\mathrm{HW}=6.2$. Since $\mathrm{HW}=$ 6.2 is considerably less than the allowable try a 54 inch diameter.

In Figure 53, project a vertical line from $Q=180$ cubic feet per second to the inlet control curve and read horizontally $\mathrm{HW}=7.2$ feet.
In Figure 53, project a vertical line from $\mathrm{Q}=225$ cubic feet per second to the inlet control curve and read horizontally $\mathrm{HW}=9.6$ feet.
C. In Figure 53, extend the vertical line from Q = 180 cubic feet per second to the $L=200$ feet outlet control curve and read horizontally $H W+S_{0} L=8.0$ feet.
In Figure 53, extend the vertical line from $Q=225$ cubic feet per second to the $\mathrm{L}=200$ feet outlet control curve and read horizontally $H W+S_{0} L=10.2$ feet. $S_{0} L=0.01 \times 200=2.0$ feet.
Therefore HW = 8.0-2.0 = 6.0 feet for 25-year storm $H W=10.2-2.0=8.2$ feet for 50 -year storm
D. Since the calculated HW for inlet control exceeds the calculated HW for outlet control in both cases, inlet control governs for both the 25 and 50 -year storm flows.

## Determine Outlet Velocity

B. Enter Figure 4 on the horizontal scale at a pipe slope of 0.01 feet per foot ( 1.0 feet per 100 feet). Project a vertical line to the line representing 54 -inch pipe diameter. Read a full flow value of 210 cubic feet per second on the vertical scale and a full flow velocity of 13.5 feet
per second. Calculate $\frac{Q_{50}}{Q_{\text {Full }}}=\frac{225}{210}=1.07$.
Enter Figure 20 at 1.07 on the horizontal scale and project a vertical line to the "flow" curve. At this intersection project a horizontal line to the "velocity" curve. Directly beneath this intersection read
$\mathrm{V}_{50}=1.12$ on the horizontal scale. Calculate $\mathrm{V}_{50}=1.12 \mathrm{~V}_{\text {Full }}=1.12 \mathrm{X}$
$13.5=15.1$ feet per second.

## Record Selection

Use a 54-inch diameter concrete pipe with allowable HW $=10.0$ feet and actual HW $=7.2$ and 9.6 feet respectively for the 25 and 50 year storm flows, and a maximum outlet velocity of 15.1 feet per second.

## EXAMPLE 3-2

## NOMOGRAPH PROCEDURE

## List Design Data

A. $Q_{25}=180$ cubic feet per second

$$
Q_{50}=225 \text { cubic feet per second }
$$

B. $L=200$ feet
C. $S_{0}=0.01$ feet per foot
D. Allowable HW = 10 feet for 25 and 50 -year storms
E. TW $=3.5$ feet for 25 -year storm $T W=4.0$ feet for 50 -year storm
F. Circular concrete culvert with a projecting entrance, $n=0.012$

## Select Trial Culvert Size

$$
D=\frac{H W}{2.0}=\frac{10}{2.0}=5 \text { feet }
$$

## Determine Trial Culvert Headwater Depth

A. Inlet Control
(1) For $Q=180$ cubic feet per second and $D=60$ inches, Figure 33 indicates HW/D $=1.25$. Therefore HW $=1.25 \times 5=6.2$ feet.
(2) Since HW $=6.2$ feet is considerably less than allowable try a 54inch pipe.
For $Q=180$ cubic feet per second and $D=54$ inches, Figure 33 indicates HW/D $=1.6$. Therefore $\mathrm{HW}=1.6 \times 4.5=7.2$ feet.
For $\mathrm{Q}=225$ cubic feet per second and $\mathrm{D}=54$ inches, Figure 33 indicates HW/D $=2.14$. Therefore HW 2.14 X $4.5=9.6$ feet.

## B. Outlet Control

(I) $\mathrm{TW}=3.5$ and 4.0 feet is less than $\mathrm{D}=4.5$ feet.
(3) Table 12, $\mathrm{k}_{\mathrm{e}}=0.2$.

For D = 54 inches, $Q=180$ cubic feet per second, Figure 28 indicates $d_{\mathrm{C}}, 3.9$ feet which is less than $\mathrm{D}=4.5$ feet. Calculate

$$
h_{0}=\frac{d_{c+} D}{2}=\frac{3.9+4.5}{2}=4.2 \text { feet. }
$$

For $\mathrm{D}=54$ inches, $\mathrm{Q}=180$ cubic feet per second, $\mathrm{k}_{\mathrm{e}} .=0.2$ and $\mathrm{L}=$ 200 feet.
Figure 38 indicates $\mathrm{H}=3.8$ feet.
Therefore HW $=3.8+4.2-(0.01 \times 200)=6.0$ feet (Equation 3).
For D $=54$ inches, $\mathrm{Q}=225$ cubic feet per second, Figure 28 indicates $d_{c}$, $=4.2$ feet which is less than $D=4.5$ feet. Calculate

$$
\mathrm{h}_{\mathrm{o}}=\frac{\mathrm{d}_{\mathrm{c}+} \mathrm{D}}{2}=\frac{4.2+4.5}{2}=4.3 \text { feet. }
$$

For $\mathrm{D}=54$ inches, $\mathrm{Q}=225$ cubic feet per second, $\mathrm{k}_{\mathrm{e}},=0.2$ and $\mathrm{L}=$ 200 feet.
Figure 38 indicates $\mathrm{H}=5.9$ feet.
Therefore HW $=5.9+4.3-(0.01 \times 200)=8.2$ feet (Equation 3).
C. Inlet control governs for both the 25 and 50 -year design flows.

## Try Another Culvert

A 48 -inch culvert would be sufficient for the 25 -year storm flow but for the 50 -year storm flow the HW would be greater than the allowable.

## Determine Outlet Velocity

B. Enter Figure 4 on the horizontal scale at a pipe slope of 0.01 feet per foot ( 1.0 feet per 100 feet). Project a vertical line to the line representing 54 -inch pipe diameter. Read a full flow value of 210 cubic feet per second on the vertical scale and a full flow velocity of 13.5 feet per second. Calculate
$\frac{Q_{50}}{Q_{\text {Full }}}=\frac{225}{210}=1.07$.
Enter Figure 20 at 1.07 on the horizontal scale and project a vertical line to the "flow" curve. At this intersection project a horizontal line to the "velocity" curve. Directly beneath this intersection read
$\frac{V_{50}}{V_{\text {Ful }}}=1.12$ on the horizontal scale. Calculate $\mathrm{V}_{50}=1.12 \mathrm{~V}_{\text {Full }}=1.12 \mathrm{X}$ $13.5=15.1$ feet per second.

## Record Selection

Use a 54 -inch diameter concrete pipe with allowable HW $=10.0$ feet and actual HW $=7.2$ and 9.6 feet respectively for the 25 and 50 -year storm flows, and a maximum outlet velocity of 15.1 feet per second.

## EXAMPLE 3-3

CULVERT DESIGN
General: A highway is to be constructed on embankment over a creek draining 400 acres. The embankment will be 41 -feet high with 2:1 side slopes and a top width of 80 feet. Hydraulic design criteria requires a circular concrete pipe, $n=0.012$, with the inlet projecting from the fill. To prevent flooding of upstream properties, the allowable headwater is 10.0 feet, and the design storm frequency is 25 years.
Given: Drainage Area Roughness Coefficient $\quad \mathrm{n}=0.012$ (See discussion of Manning's " $n$ " Value)
Headwater

$$
\text { HW = } 10 \text { feet (allowable) }
$$

Find: The required culvert size.

## Solution: 1. Design Flow

The design flow for 400 acres should be obtained using USGS data. Rather than present an analysis for a specific area, the design flow will be assumed as 250 cubic feet per second for a 25-year storm.
2. Selection of Culvert Size

The culvert will be set on the natural creek bed which has a one percent slope. A cross sectional sketch of the culvert and embankment indicates a culvert length of about 250 feet. No flooding of the outlet is expected.
Trial diameter HW/D $=2.0$ feet $\quad D=\frac{10}{2}=5$ feet.
Enter Figure 54, from Q = 250 cubic feet per second project a line vertically to the inlet control curve, read HW $=8.8$ feet on the vertical scale. Extend the vertical line to the outlet control curve for $\mathrm{L}=250$ feet, read $H+\mathrm{S}_{0} \mathrm{~L}=9.6$ on the vertical scale. $\mathrm{S}_{0} \mathrm{~L}=$ $250 \times 0.01=2.5$ feet. Therefore, outlet control HW = 9.6-2.5 = 7.1 feet and inlet control governs.

Enter Figure 53, from Q = 250 cubic feet per second project a line vertically to the inlet control curve, read $\mathrm{HW}=10.8$ feet which is greater than the allowable.

## 3. Determine Outlet Velocity

For inlet control, the outlet velocity is determined from Manning's formula. Entering Figure 4, a 60 -inch diameter pipe with $\mathrm{S}_{0}=1.0$ foot per 100 feet will have a velocity $=14.1$ feet per second flowing full and a capacity of 280 cubic feet per second. Enter Figure 20 with a Proportion of Value for Full Flow = $\frac{250}{280}$ or 0.9, read Depth of Flow $=0.74$ and
Velocity Proportion $=1.13$. Therefore, outlet velocity $=1.13 \mathrm{X}$ $14.1=15.9$ feet per second.

Answer: A 60-inch diameter circular pipe would be required.

## EXAMPLE 3-4

CULVERT DESIGN
General: An 800 -foot long box culvert with an $n=0.012$ is to be installed on a $0.5 \%$ slope. Because utility lines are to be installed in the embankment above the box culvert, the maximum rise is limited to

8 feet. The box section is required to carry a maximum flow of 1,000 cubic feet per second with an allowable headwater depth of 15 feet.

## List Design Data

A. $Q=1,000$ cubic feet per second
B. $L=800$ feet
C. $S_{0}=0.5 \%=0.005$ feet per foot
D. Allowable HW = 15 feet
E. Box culvert with projecting entrance and $n=0.012$

## Select Culvert Size

Inspecting the box section culvert capacity charts for boxes with rise equal to or less than 8 feet, it is found that a $8 \times 8$ foot and a $9 \times 7$ foot box section will all discharge 1,000 cubic feet per second with a headwater depth equal to or less than 15 feet under inlet control. Therefore, each of the two sizes will be investigated.

## Determine Headwater Depth

8 X 8 foot Box Section
A. Inlet Control

Enter Figure 124, from Q $=1,000$ project a vertical line to the inlet control curve. Project horizontally to the vertical scale and read a headwater depth of 14.8 feet for inlet control.
B. Outlet Control

Continue vertical projection from $\mathrm{Q}=1,000$ to the outlet control curve for $L=800$ feet. Project horizontally to vertical scale and read a value for $\left(H W+S_{0} L\right)=17.5$ feet. Then HW = $17.5-S_{0} L=17.5-(0.005 \mathrm{X}$ $800)=13.5$ feet for outlet control.

Therefore inlet control governs.

## $9 \times 7$ - foot Box Section

Entering Figure 127, and proceeding in a similar manner, find a headwater depth of 14.7 for inlet control and 13.1 feet for outlet control with inlet control governing.

## Determine Outlet Velocity

Entering Table 6, find area and $\mathrm{C}_{1}$, value for each size box section and Table A-1 find value of $\mathrm{S}^{1 / 2}$ for $\mathrm{S}_{0}=0.005$, then Qutull $=\mathrm{C}_{1} \mathrm{~S}^{1 / 2}$.

For 8 X 8 - foot Box Section
$Q_{\text {full }}=12700 \times 0.07071=898$ cubic feet per second
$\mathrm{V}_{\text {full }}=\mathrm{Q} / \mathrm{A}=899 \div 63.11=14.2$ feet per second.

Then

$$
\frac{\mathrm{Q}_{\text {partial }}}{\mathrm{Q}_{\text {tull }}}=\frac{1000}{899}=1.11 .
$$

Entering Figure 24.9 on the horizontal scale at 1.11, project a vertical line to intersect the flow curve. From this point, proceed horizontally to the right and intersect the velocity curve. From this point drop vertically to the horizontal scale and read a value of 1.18 for $\mathrm{V}_{\text {partial }} / V_{\text {full }}$ ratio.

Then
$V_{\text {partial }}=1.18 \times 14.2=16.8$ feet per second
Proceeding in a similar manner for the 9 X 7 foot box section, Figure 24.7 , find a $\bigvee_{\text {partial }}=16.9$ feet per second.

## Record Selection

Use either a 8 X 8 foot box section with an actual HW of 14.8 feet and an outlet velocity of 16.8 feet per second or a $9 \times 7$ foot box section with an actual HW of 14.7 feet and an outlet velocity of 16.9 feet per second.

