Math 654, Spring 2011
Homework 2 Due: Friday, 2/18/2010

1. Under the structure imposed for the 5-FU data presented on page 14 of Lecture Slides 3, do the following:

   (a) Show that
   \[ E[\pi_i(1 - \pi_i)] = E\left[\frac{n_ip_i(1 - p_i)}{n_i - 1}\right]. \]

   (b) Find an estimate of \( E[\pi_i(1 - \pi_i)] \) using the data and decide when using these historical data is more efficient (Hint: use the results on page 27 of Lecture Slides 3).

2. In a small hypothetical randomized clinical trial, 5 patients were randomized to 2 treatments: 3 to treatment A and 2 to treatment B. The trial resulted in the following data on response:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>A</th>
<th>A</th>
<th>A</th>
<th>B</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

The test statistic used to evaluate whether treatment A is better than treatment B is based on \( \overline{Y}_A - \overline{Y}_B \), where \( \overline{Y}_A \) and \( \overline{Y}_B \) are the average responses among patients receiving treatments A and B respectively. We want to test the sharp null hypothesis that treatments A and B will produce exactly the same response for each subject against the alternative that treatment A is better than treatment B (so it implies one-sided test).

   (a) Find the permutational distribution of the test statistic under the sharp null hypothesis (Find out all possible values the test statistic may take and the probability associated with each value).

   (b) What is the p-value of the test?

3. We are to conduct a randomized clinical trial to compare treatment A to treatment B. In order for other people not able to break the randomization
code, we decided to use varying block sizes 2, 4 with probabilities 0.5, 0.5. Then within each block half of patients will be assigned to treatment A and half to treatment B through permutation. Now suppose 20 patients are available for randomization. Using the following uniform numbers, assign these 20 patients to either treatment A or treatment B (you may not need all the numbers; but you are required to use the numbers sequentially):

- Uniform numbers for block sizes: 0.65, 0.06, 0.43, 0.93, 0.94, 0.41, 0.55, 0.77, 0.16, 0.08, 0.78, 0.41, 0.87, 0.85
- Uniform numbers for blocks: 0.49, 0.87, 0.55, 0.01, 0.64, 0.52, 0.02, 0.38, 0.03, 0.49, 0.22, 0.09, 0.61, 0.51, 0.32, 0.84, 0.99, 0.73, 0.99, 0.30, 0.73, 0.38, 0.22, 0.36, 0.95, 0.75, 0.78, 0.41, 0.87, 0.85, 0.14, 0.03, 0.15, 0.56, 0.31, 0.84, 0.97, 0.52, 0.11, 0.91, 0.53, 0.22, 0.34, 0.44, 0.82, 0.92, 0.28.