1. Consider the simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ with all the assumptions (L-I-N-E). Based on 12 observations with

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 = 5.2193, \quad \bar{X} = -0.4356,$$

someone obtained the following estimated model

$\hat{Y}$	=	0.75	-	2.22 X,
(SE)		(0.185)		(0.234)

$$R^2 = 89.96\%, \quad \hat{\sigma} = 0.5347, \quad F - Statistic = 89.58.$$

(a) If the previous experience claimed that the intercept is zero, test the claim with significance level $\alpha = 0.05$.

(b) If the previous experience suggested that when X increases by 1 unit, the expected response Y will increase by 2 unit. Give a statistical test for the suggestion with $\alpha = 0.05$.

(c) Suppose two individuals have 2 units difference in X. What is the expected difference in their response? give a 95% CI for the difference.

(d) Suppose a new individual has $X = 1.5$, find the 95% confidence prediction interval for the possible response from this individual.

(a) $H_0 : \beta_0 = 0$,

$$|t^*| = |\frac{0.75 - 0}{0.185}| = 4.054 > t(0.975, 10) = 2.228$$

We reject H_0.

(b) $H_0 : \beta_1 = -2$,

$$|t^*| = |\frac{-2.22 + 2}{0.234}| = 0.94 < t(0.975, 10) = 2.228$$

We accept H_0.

(c) we need to find the CI for $2\beta_1$, which is

$$2(b_1 \pm t(0.975, 10) * s(b_1)) = 2 * (-2.22 \pm 2.228 * 0.234) = [-5.4827, -3.3973]$$

(d) $\hat{Y} = 0.75 - 2.22 * 1.5 = -2.5800$ and

$$s(pred) = \hat{\sigma}\{1 + 1/12 + (-0.4356 - 1.5)^2/5.2193\}^{1/2} = 0.7176$$

Thus the PI is

$$-2.5800 \pm 0.7176 * 2.228 = [-4.1788, -0.9812]$$

2. Someone fits a regression model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ and get the following ANOVA table

Response: Y

	Df	Sum Sq	Mean Sq	F-value
X	1	18.4873	18.4873	11.355
Residuals	13	21.1660	1.6282	

(a) Find the value for the least estimation squares

$$\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \{Y_i - \beta_0 - \beta_1 X_i\}^2$$

(b) Test $H_0 : \beta_1 = 0$ with $\alpha = 0.05$ under the assumptions (L,I, N, E).

(a)

$$\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \{Y_i - \beta_0 - \beta_1 X_i\}^2 = SSE = 21.1660$$

(b) $H_0 : \beta_1 = 0$

$$F = 11.355 > F(0.95, 1, 13) = 4.67$$

Thus, we reject H_0.

3. For data $(X_i, Y_i), i = 1, ..., 10$ and its model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, some parts, marked by ???, of the calculation are missing.

Coefficients:

	Estimate	Std. Error	t value	p-value
(Intercept)	0.4409	0.1352	**3.2611**	0.0115
x	-1.0344	**0.1161**	-8.913	1.99e-05

Residual standard error: 0.4258 on 8 degrees of freedom

Multiple R-squared: **0.9085**

F-statistic: **79.4416** on 1 and 8 DF, p-value: 1.99e-05

(a) Fill in the missed values.

(b) Set up the ANOVA table.

(a) See the bold values

(b)

$$SSE = MSE * 8 = 0.4258^2 * 8 = 1.4504 \quad \text{with d.f } 8$$

and

$$(t^*)^2 = 79.4416 = F^* = \frac{SSR/1}{MSE}$$

Thus

$$SSR = 79.4416 * 0.4258^2 = 14.4032 \quad \text{with d.f } 1$$

$$SST = 14.4032 + 1.4504 = 15.8536 \quad \text{with d.f } 9$$