**Math 644, Fall 2012**

**Homework 3 Due: Friday, 10/19/2012**

1. Suppose we have $n = 10$ observations $(X_i, Y_i)$ and fit the data with model

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

with $\varepsilon_i, i = 1, ..., 10$ are IID $N(0, \sigma^2)$. We have the following calculations.

$$\bar{X} = 0.5669, \quad \bar{Y} = 0.9624, \quad \sum_{i=1}^{n} Y_i^2 = 10.2695,$$
$$\sum_{i=1}^{n} X_i^2 = 4.0169, \quad \sum_{i=1}^{n} X_i Y_i = 6.2841.$$

   (a) Write down the estimated model.

   (b) Test $H_0 : \beta_1 = 1$ with $\alpha = 0.05$.

   (c) For a new $X = 1$, find the 95% CI for its expected response.

   (d) For a new $X = 0.5$, find the 95% prediction interval for its possible response.

2. For the least square estimators $b_0$ and $b_1$ of simple linear regression model, find $Cov(b_0, b_1)$.

3. Suppose $A : m \times n$ is a constant matrix and $Y : n \times 1$ is a random vector. Then $\text{Var}(AY) = A\text{Var}(Y)A'$. Please give your proof for $m = 2, n = 3$.

4. For multiple linear regression, the normal equations are

$$\sum_{i=1}^{n} e_i = 0$$
$$\sum_{i=1}^{n} e_i X_{i1} = 0$$
$$\vdots$$
$$\sum_{i=1}^{n} e_i X_{ip} = 0.$$

Prove that

$$\sum_{i=1}^{n} \hat{Y}_i e_i = 0.$$

5. For each of the following regression models, indicate whether it is a general linear regression model. If not, state whether it can be expressed in the form of a linear regression model after some suitable transformation.

   (a) $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 \log X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$.

   (b) $Y_i = \varepsilon_i \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}^2)$, with $\varepsilon_i > 0$.

   (c) $Y_i = \beta_0 \log(\beta_1 X_{i1}) + \varepsilon_i$.

   (d) $Y_i = \log(\beta_1 X_{i1}) + \beta_2 \log(X_{i2}) + \varepsilon_i$.

   (e) $Y_i = [1 + \exp(\beta_0 + \beta_1 X_{i1} + \varepsilon_i)]^{-1}$.

6. Consider the multiple linear regression models

$$Y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i, i = 1, \ldots, n,$$

   where $\varepsilon_i$ are uncorrelated with $E\varepsilon_i = 0$ and $E\varepsilon_i^2 = \sigma^2$. State the least square criterion and derive the least squares estimators for $\beta_1$ and $\beta_2$.