

Familywise Error Rate Controlling Procedures for Discrete Data- Supplementary Materials

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November 16, 2017

S1 Results from Independent Simulation Settings

The simulation results under independent setting for stepwise procedures comparisons are shown in this section. Table S1 and S2 are single-step procedures comparisons using Fisher Exact Test, Table S3 and S4 are single-step procedures comparisons Binomial Exact Test.

Table S5 and S6 are step-down procedures comparisons using Fisher Exact Test, they can be plotted as Figure S1 and S2. Table S7 and S8 are step-up procedures comparisons using Fisher Exact Test, they can be plotted as Figure S3 and S4.

S2 Results from Dependent Settings

Step 1: Generate dependent Poisson observed counts for each group

In order to generate m dependent BET statistics T_i , we use the following algorithm to generate m dependent Poisson random variables within each group, note the Poisson random variables between two groups are independent.

1. Generate m independent Poisson random variable $Y_{1i} \sim Poi((1 - \rho)\lambda_{1i}) = Poi(2(1 - \rho))$ and one $Y_{01} \sim Poi(\rho\lambda_{1i}) = Poi(2\rho)$.

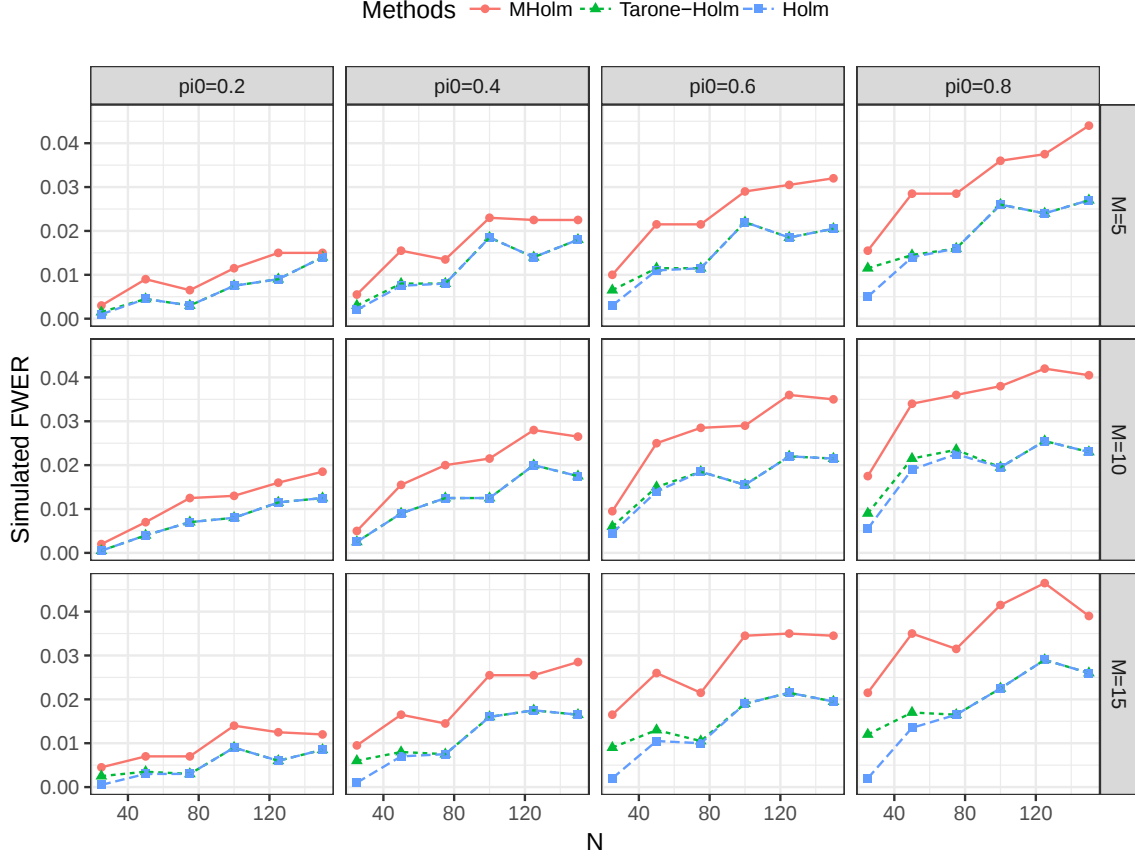


Figure S1: Simulated FWER comparisons for different step-down procedures based on FET.

2. Let $X_{1i} = Y_{1i} + Y_{10}$, then $X_{1i} \sim Poi(2)$ are dependent for $i = 1, \dots, m$, and the correlation between X_{1i} and X_{1j} is $\frac{Cov(X_{1i}, X_{1j})}{\sqrt{Var(X_{1i})}\sqrt{Var(X_{1j})}} = \frac{Var(Y_{10})}{\sqrt{2}\sqrt{2}} = \frac{2\rho}{2} = \rho$ for $i, j = 1, \dots, m$ and $i \neq j$.
3. For $i = 1, \dots, m_0$, generate m_0 independent Poisson random variable $Y_{2i} \sim Poi((1 - \rho)\lambda_{2i}) = Poi(2(1 - \rho))$ and one $Y_{20} \sim Poi(\rho\lambda_{2i}) = Poi(2\rho)$. For $i = m_0 + 1, \dots, m$, generate $m - m_0$ independent Poisson random variable $Y_{2i} \sim Poi((1 - \rho)\lambda_{2i}) = Poi(10(1 - \rho))$ and one $Y'_{20} \sim Poi(\rho\lambda_{2i}) = Poi(10\rho)$.
4. Let $X_{2i} = Y_{2i} + Y_{20}$ for $i = 1, \dots, m_0$, then $X_{2i} \sim Poi(2)$ are dependent for $i = 1, \dots, m_0$, and the correlation between X_{2i} and X_{2j} is $\frac{Cov(X_{2i}, X_{2j})}{\sqrt{Var(X_{2i})}\sqrt{Var(X_{2j})}} = \frac{Var(Y_{20})}{\sqrt{2}\sqrt{2}} = \frac{2\rho}{2} = \rho$ for $i, j = 1, \dots, m_0$ and $i \neq j$; let $X_{2i} = Y_{2i} + Y'_{20}$ for $i = m_0 + 1, \dots, m$, then $X_{2i} \sim Poi(10)$ are dependent for $i = m_0 + 1, \dots, m$, and the

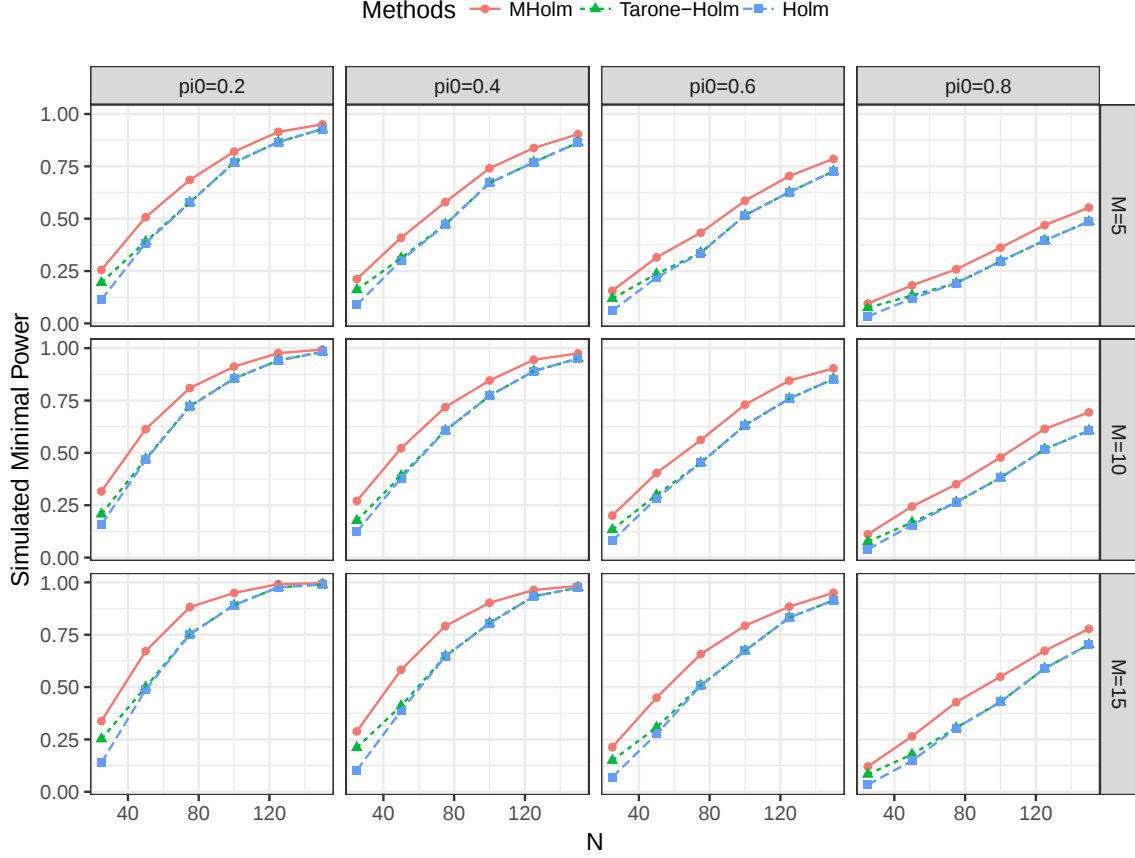


Figure S2: Simulated minimal power comparisons for different step-down procedures based on FET.

correlation between X_{2i} and X_{2j} is $\frac{Cov(X_{2i}, X_{2j})}{\sqrt{Var(X_{2i})}\sqrt{Var(X_{2j})}} = \frac{Var(Y'_{20})}{\sqrt{10}\sqrt{10}} = \frac{10\rho}{10} = \rho$
for $i, j = m_0 + 1, \dots, m$ and $i \neq j$.

Step 2: Obtain the conditional test statistics

Since the generated Poisson random variables between two groups are independent, we can directly conduct BET for each hypothesis. after generating Poisson observed counts x_{1i} and x_{2i} , let $c_i = x_{1i} + x_{2i}$ be the total observed count for two groups. Then the test statistics T_i is conditional test statistics X_{1i} given $X_{1i} + X_{2i} = c_i$. Then the critical value is observed count x_{1i} .

Step 3: Conditional distribution of the test statistics

Based on the conditional inference in Lehman and Romano (2005). which is the BET in our paper, the conditional distribution of X_{1i} given $X_{1i} + X_{2i} = c_i$ is binomial $Bin(c_i, p_i)$, where $p_i = \frac{\lambda_{1i}}{\lambda_{1i} + \lambda_{2i}}$.

Step 4: Calculate available p -value P_i and attainable p -values When H_i is true,

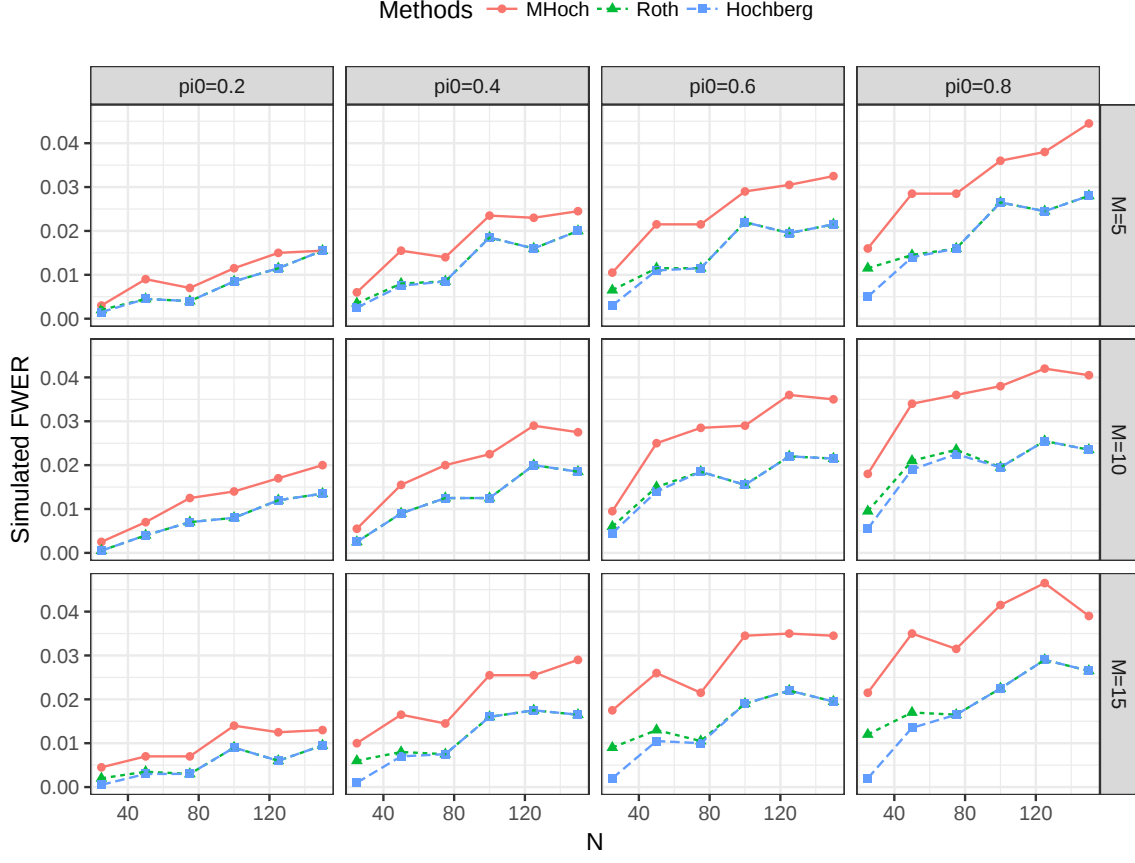


Figure S3: Simulated FWER comparisons for different step-up procedures based on FET.

$\lambda_{1i} = \lambda_{2i}$, then $p_i = 0.5$. That is, $X_{1i}|X_{1i}+X_{2i} = c_i \sim \text{Bin}(c_i, 0.5)$ under null H_i . Therefore, the available conditional p -value for H_i can be calculated by

$$\begin{aligned}
 P_i &= \Pr_{H_i} \{X_{1i} \geq x_{1i} | X_{1i} + X_{2i} = c_i\} \\
 &= \sum_{j=x_{1i}}^{c_i} \binom{c_i}{j} 0.5^i (1 - 0.5)^{c_i-j} \\
 &= \sum_{j=x_{1i}}^{c_i} \binom{c_i}{j} 0.5^{c_i}.
 \end{aligned} \tag{1}$$

The corresponding attainable p -values can be calculated by

$$\Pr_{H_i} \{X_{1i} \geq x | X_{1i} + X_{2i} = c_i\} = \sum_{j=x}^{c_i} \binom{c_i}{j} 0.5^{c_i} \quad \text{for } x = 0, 1, \dots, c_i. \tag{2}$$

Then the simulation results under the above dependent setting for stepwise procedures comparisons are shown in Table S9-S14 and Figure S5-S8.

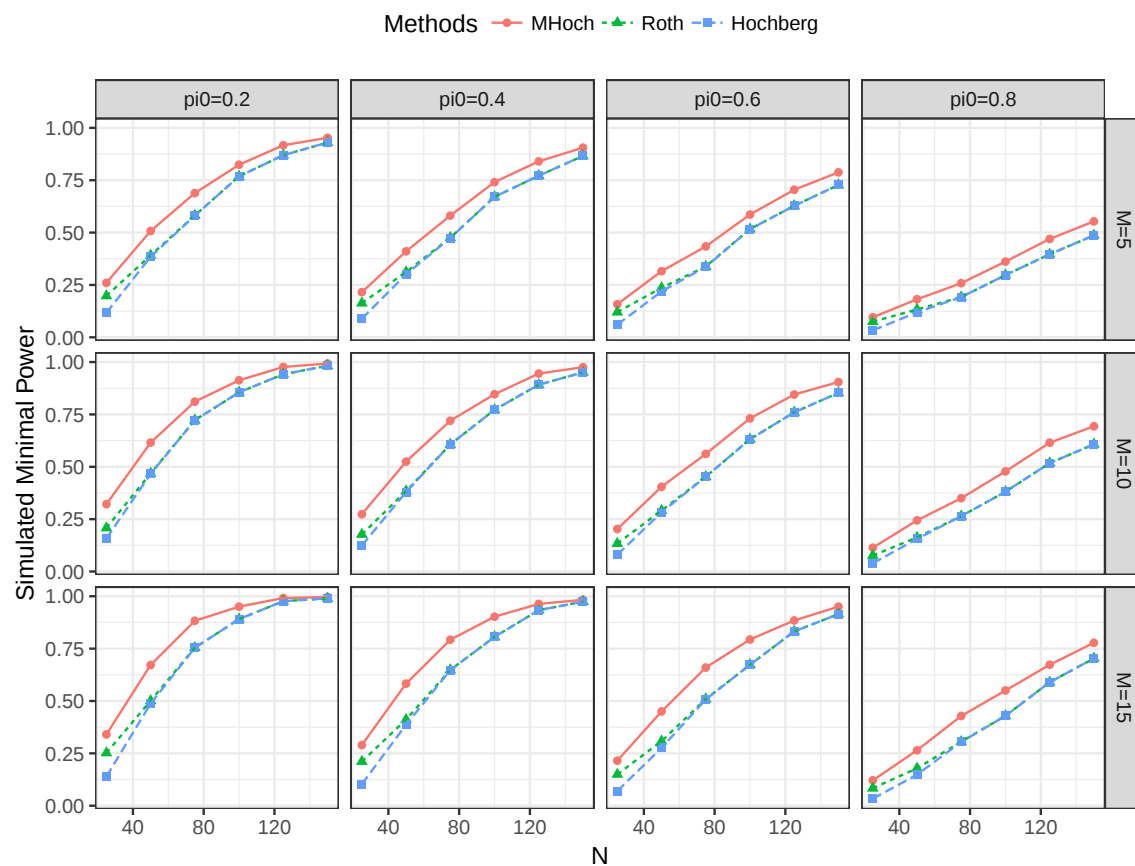


Figure S4: Simulated minimal power comparisons for different step-up procedures based on FET.

Table S1: Simulated FWER comparisons for single-step procedures with independent p -values generated from Fisher's Exact Test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MBonf	0.0025	0.0060	0.0035	0.0075	0.0075	0.0095
	Tarone	0.0015	0.0030	0.0015	0.0055	0.0045	0.0085
	Bonf	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
	Sidak	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
$m = 5$ $\pi_0 = 0.4$	MBonf	0.0045	0.0130	0.0120	0.0170	0.0135	0.0145
	Tarone	0.0030	0.0060	0.0065	0.0140	0.0090	0.0100
	Bonf	0.0015	0.0060	0.0065	0.0140	0.0090	0.0100
	Sidak	0.0015	0.0060	0.0065	0.0140	0.0090	0.0100
$m = 5$ $\pi_0 = 0.6$	MBonf	0.0085	0.0200	0.0195	0.0235	0.0225	0.0245
	Tarone	0.0060	0.0105	0.0105	0.0180	0.0155	0.0170
	Bonf	0.0025	0.0100	0.0105	0.0180	0.0155	0.0170
	Sidak	0.0025	0.0100	0.0105	0.0180	0.0160	0.0175
$m = 5$ $\pi_0 = 0.8$	MBonf	0.0140	0.0265	0.0270	0.0340	0.0315	0.0370
	Tarone	0.0110	0.0140	0.0155	0.0245	0.0215	0.0220
	Bonf	0.0045	0.0135	0.0155	0.0245	0.0215	0.0220
	Sidak	0.0045	0.0135	0.0155	0.0245	0.0220	0.0230
$m = 10$ $\pi_0 = 0.2$	MBonf	0.0020	0.0060	0.0100	0.0115	0.0095	0.0110
	Tarone	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
	Bonf	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
	Sidak	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
$m = 10$ $\pi_0 = 0.4$	MBonf	0.0050	0.0145	0.0165	0.0190	0.0215	0.0190
	Tarone	0.0025	0.0090	0.0120	0.0100	0.0140	0.0125
	Bonf	0.0025	0.0090	0.0120	0.0100	0.0140	0.0125
	Sidak	0.0025	0.0090	0.0120	0.0110	0.0145	0.0130
$m = 10$ $\pi_0 = 0.6$	MBonf	0.0090	0.0245	0.0260	0.0265	0.0300	0.0255
	Tarone	0.0055	0.0150	0.0185	0.0150	0.0180	0.0155
	Bonf	0.0045	0.0140	0.0185	0.0150	0.0180	0.0155
	Sidak	0.0045	0.0140	0.0185	0.0160	0.0195	0.0155
$m = 10$ $\pi_0 = 0.8$	MBonf	0.0175	0.0335	0.0345	0.0370	0.0390	0.0360
	Tarone	0.0090	0.0215	0.0225	0.0190	0.0220	0.0200
	Bonf	0.0055	0.0190	0.0225	0.0190	0.0220	0.0200
	Sidak	0.0055	0.0190	0.0225	0.0210	0.0240	0.0200
$m = 15$ $\pi_0 = 0.2$	MBonf	0.0040	0.0060	0.0065	0.0120	0.0080	0.0100
	Tarone	0.0020	0.0030	0.0030	0.0065	0.0045	0.0070
	Bonf	0.0005	0.0030	0.0030	0.0065	0.0045	0.0070
	Sidak	0.0005	0.0030	0.0030	0.0075	0.0045	0.0070
$m = 15$ $\pi_0 = 0.4$	MBonf	0.0090	0.0150	0.0140	0.0240	0.0210	0.0200
	Tarone	0.0060	0.0075	0.0065	0.0125	0.0150	0.0105
	Bonf	0.0010	0.0070	0.0065	0.0125	0.0150	0.0105
	Sidak	0.0010	0.0070	0.0065	0.0145	0.0150	0.0105
$m = 15$ $\pi_0 = 0.6$	MBonf	0.0165	0.0250	0.0210	0.0325	0.0320	0.0280
	Tarone	0.0090	0.0130	0.0095	0.0170	0.0205	0.0180
	Bonf	0.0020	0.0105	0.0095	0.0170	0.0205	0.0180
	Sidak	0.0020	0.0105	0.0095	0.0190	0.0205	0.0180
$m = 15$ $\pi_0 = 0.8$	MBonf	0.0210	0.0345	0.0315	0.0400	0.0460	0.0360
	Tarone	0.0115	0.0170	0.0155	0.0215	0.0285	0.0240
	Bonf	0.0020	0.0135	0.0155	0.0215	0.0285	0.0240
	Sidak	0.0020	0.0135	0.0155	0.0240	0.0285	0.0240

Table S2: Simulated minimal power comparisons for single-step procedures with independent p -values generated from Fisher's exact test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MBonf	0.2550	0.5060	0.6855	0.8195	0.9145	0.9505
	Tarone	0.1945	0.3900	0.5775	0.7680	0.8655	0.9275
	Bonf	0.1125	0.3825	0.5765	0.7680	0.8655	0.9275
	Sidak	0.1125	0.3825	0.5850	0.7680	0.8710	0.9340
$m = 5$ $\pi_0 = 0.4$	MBonf	0.2110	0.4085	0.5785	0.7405	0.8375	0.9025
	Tarone	0.1605	0.3110	0.4715	0.6705	0.7695	0.8625
	Bonf	0.0880	0.3000	0.4700	0.6705	0.7695	0.8625
	Sidak	0.0880	0.3000	0.4770	0.6705	0.7765	0.8680
$m = 5$ $\pi_0 = 0.6$	MBonf	0.1550	0.3130	0.4320	0.5835	0.7025	0.7845
	Tarone	0.1180	0.2365	0.3370	0.5145	0.6255	0.7245
	Bonf	0.0605	0.2190	0.3355	0.5145	0.6255	0.7245
	Sidak	0.0605	0.2190	0.3420	0.5145	0.6330	0.7345
$m = 5$ $\pi_0 = 0.8$	MBonf	0.0945	0.1800	0.2570	0.3595	0.4660	0.5505
	Tarone	0.0740	0.1330	0.1920	0.2955	0.3950	0.4850
	Bonf	0.0330	0.1190	0.1920	0.2955	0.3950	0.4850
	Sidak	0.0330	0.1190	0.1955	0.2955	0.4025	0.5005
$m = 10$ $\pi_0 = 0.2$	MBonf	0.3155	0.6130	0.8090	0.9110	0.9765	0.9930
	Tarone	0.2075	0.4695	0.7220	0.8550	0.9415	0.9820
	Bonf	0.1575	0.4660	0.7220	0.8550	0.9415	0.9820
	Sidak	0.1575	0.4660	0.7220	0.8595	0.9425	0.9830
$m = 10$ $\pi_0 = 0.4$	MBonf	0.2700	0.5220	0.7180	0.8455	0.9440	0.9750
	Tarone	0.1770	0.3905	0.6065	0.7720	0.8905	0.9505
	Bonf	0.1235	0.3795	0.6065	0.7720	0.8905	0.9505
	Sidak	0.1235	0.3795	0.6065	0.7775	0.8920	0.9575
$m = 10$ $\pi_0 = 0.6$	MBonf	0.2005	0.4030	0.5615	0.7300	0.8450	0.9035
	Tarone	0.1330	0.2990	0.4525	0.6315	0.7590	0.8525
	Bonf	0.0800	0.2825	0.4525	0.6315	0.7590	0.8525
	Sidak	0.0800	0.2825	0.4525	0.6375	0.7615	0.8585
$m = 10$ $\pi_0 = 0.8$	MBonf	0.1115	0.2440	0.3500	0.4775	0.6140	0.6935
	Tarone	0.0760	0.1680	0.2645	0.3810	0.5165	0.6060
	Bonf	0.0390	0.1555	0.2645	0.3810	0.5165	0.6060
	Sidak	0.0390	0.1555	0.2645	0.3880	0.5170	0.6185
$m = 15$ $\pi_0 = 0.2$	MBonf	0.3370	0.6715	0.8820	0.9495	0.9915	0.9965
	Tarone	0.2520	0.4995	0.7530	0.8910	0.9765	0.9895
	Bonf	0.1390	0.4870	0.7515	0.8910	0.9765	0.9895
	Sidak	0.1390	0.4870	0.7515	0.8960	0.9765	0.9895
$m = 15$ $\pi_0 = 0.4$	MBonf	0.2880	0.5815	0.7910	0.9025	0.9635	0.9830
	Tarone	0.2110	0.4105	0.6475	0.8050	0.9335	0.9745
	Bonf	0.1030	0.3870	0.6460	0.8050	0.9335	0.9745
	Sidak	0.1030	0.3870	0.6460	0.8125	0.9335	0.9745
$m = 15$ $\pi_0 = 0.6$	MBonf	0.2135	0.4485	0.6570	0.7925	0.8840	0.9500
	Tarone	0.1495	0.3070	0.5085	0.6730	0.8315	0.9140
	Bonf	0.0700	0.2760	0.5065	0.6730	0.8315	0.9140
	Sidak	0.0700	0.2760	0.5065	0.6790	0.8315	0.9140
$m = 15$ $\pi_0 = 0.8$	MBonf	0.1205	0.2635	0.4270	0.5490	0.6710	0.7780
	Tarone	0.0830	0.1785	0.3050	0.4295	0.5890	0.7020
	Bonf	0.0335	0.1480	0.3040	0.4290	0.5890	0.7020
	Sidak	0.0335	0.1480	0.3040	0.4345	0.5895	0.7020

Table S3: Simulated FWER comparisons for single-step procedures with independent p -values generated from Binomial Exact Test statistics.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
$m = 5$ $\alpha = 0.05$	MBonf	0.0020	0.0060	0.0075	0.0165
	Tarone	0.0010	0.0030	0.0055	0.0105
	Bonf	0.0010	0.0020	0.0025	0.0030
	Sidak	0.0010	0.0020	0.0025	0.0030
$m = 10$ $\alpha = 0.05$	MBonf	0.0010	0.0045	0.0130	0.0160
	Tarone	0.0000	0.0010	0.0050	0.0115
	Bonf	0.0000	0.0005	0.0025	0.0025
	Sidak	0.0000	0.0005	0.0025	0.0025
$m = 15$ $\alpha = 0.05$	MBonf	0.0010	0.0065	0.0045	0.0150
	Tarone	0.0000	0.0010	0.0020	0.0070
	Bonf	0.0000	0.0005	0.0000	0.0000
	Sidak	0.0000	0.0005	0.0000	0.0000
$m = 5$ $\alpha = 0.1$	MBonf	0.0070	0.0125	0.0200	0.0365
	Tarone	0.0020	0.0065	0.0110	0.0285
	Bonf	0.0020	0.0055	0.0065	0.0130
	Sidak	0.0020	0.0055	0.0065	0.0130
$m = 10$ $\alpha = 0.1$	MBonf	0.0040	0.0080	0.0275	0.0350
	Tarone	0.0000	0.0030	0.0165	0.0195
	Bonf	0.0000	0.0015	0.0055	0.0060
	Sidak	0.0000	0.0015	0.0055	0.0060
$m = 15$ $\alpha = 0.1$	MBonf	0.0060	0.0155	0.0185	0.0315
	Tarone	0.0005	0.0060	0.0045	0.0200
	Bonf	0.0000	0.0010	0.0020	0.0025
	Sidak	0.0000	0.0010	0.0020	0.0025

Table S4: Simulated minimal power comparisons for single-step procedures with independent p -values generated from Binomial Exact Test statistics.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
$m = 5$ $\alpha = 0.05$	MBonf	0.9205	0.8805	0.7845	0.5565
	Tarone	0.8815	0.8240	0.7395	0.5235
	Bonf	0.8735	0.8055	0.6610	0.4045
	Sidak	0.8735	0.8055	0.6610	0.4045
$m = 10$ $\alpha = 0.05$	MBonf	0.9850	0.9635	0.9035	0.7390
	Tarone	0.9470	0.9240	0.8630	0.6855
	Bonf	0.9315	0.8635	0.7050	0.4775
	Sidak	0.9315	0.8635	0.7050	0.4775
$m = 15$ $\alpha = 0.05$	MBonf	0.9925	0.9810	0.9555	0.8210
	Tarone	0.9825	0.9500	0.9095	0.7845
	Bonf	0.9820	0.9475	0.8560	0.6135
	Sidak	0.9820	0.9475	0.8560	0.6135
$m = 5$ $\alpha = 0.1$	MBonf	0.9680	0.9415	0.8615	0.6330
	Tarone	0.9410	0.9140	0.8240	0.5920
	Bonf	0.9050	0.8375	0.7040	0.4520
	Sidak	0.9050	0.8375	0.7040	0.4520
$m = 10$ $\alpha = 0.1$	MBonf	0.9965	0.9875	0.9620	0.8315
	Tarone	0.9885	0.9660	0.9170	0.7835
	Bonf	0.9870	0.9565	0.8690	0.6600
	Sidak	0.9870	0.9565	0.8690	0.6600
$m = 15$ $\alpha = 0.1$	MBonf	0.9995	0.9970	0.9830	0.9030
	Tarone	0.9960	0.9930	0.9605	0.8400
	Bonf	0.9880	0.9615	0.8830	0.6515
	Sidak	0.9895	0.9635	0.8880	0.6590

Table S5: Simulated FWER comparisons for step-down procedures with independent p -values generated from Fisher's Exact Test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MHolm	0.0030	0.0090	0.0065	0.0115	0.0150	0.0150
	TH^*	0.0015	0.0045	0.0030	0.0075	0.0090	0.0140
	Holm	0.0010	0.0045	0.0030	0.0075	0.0090	0.0140
$m = 5$ $\pi_0 = 0.4$	MHolm	0.0055	0.0155	0.0135	0.0230	0.0225	0.0225
	TH^*	0.0030	0.0080	0.0080	0.0185	0.0140	0.0180
	Holm	0.0020	0.0075	0.0080	0.0185	0.0140	0.0180
$m = 5$ $\pi_0 = 0.6$	MHolm	0.0100	0.0215	0.0215	0.0290	0.0305	0.0320
	TH^*	0.0065	0.0115	0.0115	0.0220	0.0185	0.0205
	Holm	0.0030	0.0110	0.0115	0.0220	0.0185	0.0205
$m = 5$ $\pi_0 = 0.8$	MHolm	0.0155	0.0285	0.0285	0.0360	0.0375	0.0440
	TH^*	0.0115	0.0145	0.0160	0.0260	0.0240	0.0270
	Holm	0.0050	0.0140	0.0160	0.0260	0.0240	0.0270
$m = 10$ $\pi_0 = 0.2$	MHolm	0.0020	0.0070	0.0125	0.0130	0.0160	0.0185
	TH^*	0.0005	0.0040	0.0070	0.0080	0.0115	0.0125
	Holm	0.0005	0.0040	0.0070	0.0080	0.0115	0.0125
$m = 10$ $\pi_0 = 0.4$	MHolm	0.0050	0.0155	0.0200	0.0215	0.0280	0.0265
	TH^*	0.0025	0.0090	0.0125	0.0125	0.0200	0.0175
	Holm	0.0025	0.0090	0.0125	0.0125	0.0200	0.0175
$m = 10$ $\pi_0 = 0.6$	MHolm	0.0095	0.0250	0.0285	0.0290	0.0360	0.0350
	TH^*	0.0060	0.0150	0.0185	0.0155	0.0220	0.0215
	Holm	0.0045	0.0140	0.0185	0.0155	0.0220	0.0215
$m = 10$ $\pi_0 = 0.8$	MHolm	0.0175	0.0340	0.0360	0.0380	0.0420	0.0405
	TH^*	0.0090	0.0215	0.0235	0.0195	0.0255	0.0230
	Holm	0.0055	0.0190	0.0225	0.0195	0.0255	0.0230
$m = 15$ $\pi_0 = 0.2$	MHolm	0.0045	0.0070	0.0070	0.0140	0.0125	0.0120
	TH^*	0.0025	0.0035	0.0030	0.0090	0.0060	0.0085
	Holm	0.0005	0.0030	0.0030	0.0090	0.0060	0.0085
$m = 15$ $\pi_0 = 0.4$	MHolm	0.0095	0.0165	0.0145	0.0255	0.0255	0.0285
	TH^*	0.0060	0.0080	0.0075	0.0160	0.0175	0.0165
	Holm	0.0010	0.0070	0.0075	0.0160	0.0175	0.0165
$m = 15$ $\pi_0 = 0.6$	MHolm	0.0165	0.0260	0.0215	0.0345	0.0350	0.0345
	TH^*	0.0090	0.0130	0.0105	0.0190	0.0215	0.0195
	Holm	0.0020	0.0105	0.0100	0.0190	0.0215	0.0195
$m = 15$ $\pi_0 = 0.8$	MHolm	0.0215	0.0350	0.0315	0.0415	0.0465	0.0390
	TH^*	0.0120	0.0170	0.0165	0.0225	0.0290	0.0260
	Holm	0.0020	0.0135	0.0165	0.0225	0.0290	0.0260

Table S6: Simulated minimal power comparisons for step-down procedures with independent p -values generated from Fisher's Exact Test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MHolm	0.2555	0.5070	0.6855	0.8200	0.9145	0.9505
	TH^*	0.1945	0.3905	0.5780	0.7680	0.8660	0.9280
	Holm	0.1130	0.3830	0.5770	0.7680	0.8660	0.9280
$m = 5$ $\pi_0 = 0.4$	MHolm	0.2120	0.4090	0.5790	0.7405	0.8375	0.9030
	TH^*	0.1605	0.3115	0.4725	0.6705	0.7695	0.8630
	Holm	0.0880	0.3005	0.4710	0.6705	0.7695	0.8630
$m = 5$ $\pi_0 = 0.6$	MHolm	0.1555	0.3150	0.4330	0.5855	0.7035	0.7855
	TH^*	0.1185	0.2365	0.3375	0.5160	0.6265	0.7260
	Holm	0.0605	0.2190	0.3360	0.5160	0.6265	0.7260
$m = 5$ $\pi_0 = 0.8$	MHolm	0.0950	0.1815	0.2585	0.3615	0.4690	0.5530
	TH^*	0.0745	0.1330	0.1920	0.2965	0.3960	0.4860
	Holm	0.0330	0.1190	0.1920	0.2965	0.3960	0.4860
$m = 10$ $\pi_0 = 0.2$	MHolm	0.3160	0.6130	0.8095	0.9120	0.9765	0.9930
	TH^*	0.2075	0.4700	0.7220	0.8550	0.9415	0.9820
	Holm	0.1575	0.4660	0.7220	0.8550	0.9415	0.9820
$m = 10$ $\pi_0 = 0.4$	MHolm	0.2705	0.5220	0.7185	0.8455	0.9445	0.9750
	TH^*	0.1770	0.3905	0.6065	0.7720	0.8905	0.9505
	Holm	0.1235	0.3795	0.6065	0.7720	0.8905	0.9505
$m = 10$ $\pi_0 = 0.6$	MHolm	0.2010	0.4035	0.5615	0.7300	0.8450	0.9035
	TH^*	0.1330	0.2990	0.4525	0.6315	0.7590	0.8525
	Holm	0.0800	0.2825	0.4525	0.6315	0.7590	0.8525
$m = 10$ $\pi_0 = 0.8$	MHolm	0.1115	0.2440	0.3500	0.4780	0.6145	0.6935
	TH^*	0.0760	0.1680	0.2645	0.3810	0.5175	0.6065
	Holm	0.0390	0.1555	0.2645	0.3810	0.5175	0.6065
$m = 15$ $\pi_0 = 0.2$	MHolm	0.3375	0.6715	0.8820	0.9495	0.9915	0.9965
	TH^*	0.2520	0.4995	0.7530	0.8910	0.9765	0.9895
	Holm	0.1390	0.4870	0.7515	0.8910	0.9765	0.9895
$m = 15$ $\pi_0 = 0.4$	MHolm	0.2885	0.5825	0.7915	0.9025	0.9635	0.9830
	TH^*	0.2110	0.4105	0.6475	0.8055	0.9335	0.9745
	Holm	0.1030	0.3870	0.6460	0.8055	0.9335	0.9745
$m = 15$ $\pi_0 = 0.6$	MHolm	0.2135	0.4495	0.6575	0.7930	0.8840	0.9500
	TH^*	0.1495	0.3070	0.5085	0.6730	0.8315	0.9140
	Holm	0.0700	0.2760	0.5065	0.6730	0.8315	0.9140
$m = 15$ $\pi_0 = 0.8$	MHolm	0.1205	0.2645	0.4280	0.5495	0.6730	0.7780
	TH^*	0.0835	0.1785	0.3055	0.4295	0.5890	0.7030
	Holm	0.0335	0.1480	0.3045	0.4290	0.5890	0.7030

Table S7: Simulated FWER comparisons for step-up procedures with independent p -values generated from Fisher's Exact Test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MHoch	0.0030	0.0090	0.0070	0.0115	0.0150	0.0155
	Roth	0.0020	0.0045	0.0040	0.0085	0.0115	0.0155
	Hoch	0.0015	0.0045	0.0040	0.0085	0.0115	0.0155
$m = 5$ $\pi_0 = 0.4$	MHoch	0.0060	0.0155	0.0140	0.0235	0.0230	0.0245
	Roth	0.0035	0.0080	0.0085	0.0185	0.0160	0.0200
	Hoch	0.0025	0.0075	0.0085	0.0185	0.0160	0.0200
$m = 5$ $\pi_0 = 0.6$	MHoch	0.0105	0.0215	0.0215	0.0290	0.0305	0.0325
	Roth	0.0065	0.0115	0.0115	0.0220	0.0195	0.0215
	Hoch	0.0030	0.0110	0.0115	0.0220	0.0195	0.0215
$m = 5$ $\pi_0 = 0.8$	MHoch	0.0160	0.0285	0.0285	0.0360	0.0380	0.0445
	Roth	0.0115	0.0145	0.0160	0.0265	0.0245	0.0280
	Hoch	0.0050	0.0140	0.0160	0.0265	0.0245	0.0280
$m = 10$ $\pi_0 = 0.2$	MHoch	0.0025	0.0070	0.0125	0.0140	0.0170	0.0200
	Roth	0.0005	0.0040	0.0070	0.0080	0.0120	0.0135
	Hoch	0.0005	0.0040	0.0070	0.0080	0.0120	0.0135
$m = 10$ $\pi_0 = 0.4$	MHoch	0.0055	0.0155	0.0200	0.0225	0.0290	0.0275
	Roth	0.0025	0.0090	0.0125	0.0125	0.0200	0.0185
	Hoch	0.0025	0.0090	0.0125	0.0125	0.0200	0.0185
$m = 10$ $\pi_0 = 0.6$	MHoch	0.0095	0.0250	0.0285	0.0290	0.0360	0.0350
	Roth	0.0060	0.0150	0.0185	0.0155	0.0220	0.0215
	Hoch	0.0045	0.0140	0.0185	0.0155	0.0220	0.0215
$m = 10$ $\pi_0 = 0.8$	MHoch	0.0180	0.0340	0.0360	0.0380	0.0420	0.0405
	Roth	0.0095	0.0210	0.0235	0.0195	0.0255	0.0235
	Hoch	0.0055	0.0190	0.0225	0.0195	0.0255	0.0235
$m = 15$ $\pi_0 = 0.2$	MHoch	0.0045	0.0070	0.0070	0.0140	0.0125	0.0130
	Roth	0.0020	0.0035	0.0030	0.0090	0.0060	0.0095
	Hoch	0.0005	0.0030	0.0030	0.0090	0.0060	0.0095
$m = 15$ $\pi_0 = 0.4$	MHoch	0.0100	0.0165	0.0145	0.0255	0.0255	0.0290
	Roth	0.0060	0.0080	0.0075	0.0160	0.0175	0.0165
	Hoch	0.0010	0.0070	0.0075	0.0160	0.0175	0.0165
$m = 15$ $\pi_0 = 0.6$	MHoch	0.0175	0.0260	0.0215	0.0345	0.0350	0.0345
	Roth	0.0090	0.0130	0.0105	0.0190	0.0220	0.0195
	Hoch	0.0020	0.0105	0.0100	0.0190	0.0220	0.0195
$m = 15$ $\pi_0 = 0.8$	MHoch	0.0215	0.0350	0.0315	0.0415	0.0465	0.0390
	Roth	0.0120	0.0170	0.0165	0.0225	0.0290	0.0265
	Hoch	0.0020	0.0135	0.0165	0.0225	0.0290	0.0265

Table S8: Simulated minimal power comparisons for step-up procedures with independent p -values generated from Fisher's Exact Test statistics.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$ $\pi_0 = 0.2$	MHoch	0.2600	0.5075	0.6885	0.8240	0.9170	0.9525
	Roth	0.1975	0.3915	0.5820	0.7685	0.8695	0.9300
	Hoch	0.1170	0.3845	0.5810	0.7685	0.8695	0.9300
$m = 5$ $\pi_0 = 0.4$	MHoch	0.2155	0.4105	0.5810	0.7410	0.8400	0.9055
	Roth	0.1630	0.3115	0.4755	0.6705	0.7715	0.8660
	Hoch	0.0885	0.3010	0.4740	0.6705	0.7715	0.8660
$m = 5$ $\pi_0 = 0.6$	MHoch	0.1580	0.3155	0.4340	0.5860	0.7045	0.7875
	Roth	0.1200	0.2365	0.3380	0.5165	0.6280	0.7275
	Hoch	0.0605	0.2190	0.3365	0.5165	0.6280	0.7275
$m = 5$ $\pi_0 = 0.8$	MHoch	0.0955	0.1815	0.2585	0.3615	0.4695	0.5535
	Roth	0.0745	0.1330	0.1920	0.2970	0.3965	0.4870
	Hoch	0.0330	0.1190	0.1920	0.2970	0.3965	0.4870
$m = 10$ $\pi_0 = 0.2$	MHoch	0.3215	0.6155	0.8110	0.9130	0.9765	0.9930
	Roth	0.2080	0.4685	0.7225	0.8555	0.9420	0.9820
	Hoch	0.1580	0.4660	0.7225	0.8555	0.9420	0.9820
$m = 10$ $\pi_0 = 0.4$	MHoch	0.2735	0.5245	0.7200	0.8465	0.9450	0.9755
	Roth	0.1770	0.3840	0.6070	0.7720	0.8920	0.9510
	Hoch	0.1240	0.3795	0.6065	0.7720	0.8920	0.9510
$m = 10$ $\pi_0 = 0.6$	MHoch	0.2030	0.4045	0.5615	0.7310	0.8450	0.9045
	Roth	0.1335	0.2910	0.4525	0.6315	0.7600	0.8530
	Hoch	0.0800	0.2825	0.4525	0.6315	0.7600	0.8530
$m = 10$ $\pi_0 = 0.8$	MHoch	0.1135	0.2440	0.3500	0.4780	0.6150	0.6935
	Roth	0.0765	0.1625	0.2645	0.3810	0.5175	0.6075
	Hoch	0.0390	0.1555	0.2645	0.3810	0.5175	0.6075
$m = 15$ $\pi_0 = 0.2$	MHoch	0.3405	0.6720	0.8830	0.9505	0.9915	0.9965
	Roth	0.2520	0.5010	0.7545	0.8910	0.9765	0.9900
	Hoch	0.1390	0.4875	0.7535	0.8910	0.9765	0.9900
$m = 15$ $\pi_0 = 0.4$	MHoch	0.2895	0.5830	0.7925	0.9025	0.9635	0.9830
	Roth	0.2110	0.4115	0.6485	0.8060	0.9335	0.9745
	Hoch	0.1030	0.3870	0.6470	0.8060	0.9335	0.9745
$m = 15$ $\pi_0 = 0.6$	MHoch	0.2150	0.4500	0.6595	0.7935	0.8845	0.9505
	Roth	0.1495	0.3080	0.5095	0.6730	0.8320	0.9150
	Hoch	0.0700	0.2760	0.5075	0.6730	0.8320	0.9150
$m = 15$ $\pi_0 = 0.8$	MHoch	0.1210	0.2645	0.4285	0.5500	0.6730	0.7780
	Roth	0.0835	0.1785	0.3055	0.4295	0.5895	0.7035
	Hoch	0.0335	0.1480	0.3045	0.4290	0.5895	0.7035

Table S9: Simulated FWER comparisons for single-step procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MBonf	0.0045	0.0050	0.0035	0.0045	0.0025	0.0055	0.0025	0.0080	0.0045	0.0040
	Tarone	0.0015	0.0010	0.0005	0.0020	0.0010	0.0030	0.0005	0.0040	0.0010	0.0005
	Bonf	0.0015	0.0010	0.0005	0.0015	0.0010	0.0025	0.0005	0.0035	0.0000	0.0005
	Sidak	0.0015	0.0010	0.0005	0.0015	0.0010	0.0025	0.0005	0.0035	0.0000	0.0005
$m = 5$ $\pi_0 = 0.6$	MBonf	0.0100	0.0055	0.0080	0.0085	0.0080	0.0050	0.0095	0.0070	0.0045	0.0040
	Tarone	0.0060	0.0025	0.0060	0.0050	0.0050	0.0045	0.0045	0.0055	0.0040	0.0025
	Bonf	0.0025	0.0015	0.0020	0.0035	0.0020	0.0010	0.0010	0.0000	0.0020	0.0010
	Sidak	0.0025	0.0015	0.0020	0.0035	0.0020	0.0010	0.0010	0.0000	0.0020	0.0010
$m = 5$ $\pi_0 = 0.8$	MBonf	0.0125	0.0145	0.0155	0.0160	0.0115	0.0135	0.0110	0.0060	0.0105	0.0045
	Tarone	0.0090	0.0105	0.0065	0.0095	0.0080	0.0085	0.0070	0.0045	0.0065	0.0030
	Bonf	0.0035	0.0005	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
	Sidak	0.0035	0.0005	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
$m = 10$ $\pi_0 = 0.4$	MBonf	0.0035	0.0030	0.0025	0.0035	0.0030	0.0025	0.0025	0.0030	0.0025	0.0035
	Tarone	0.0005	0.0010	0.0010	0.0020	0.0010	0.0010	0.0010	0.0005	0.0010	0.0010
	Bonf	0.0005	0.0005	0.0010	0.0005	0.0010	0.0010	0.0000	0.0005	0.0005	0.0005
	Sidak	0.0005	0.0005	0.0010	0.0005	0.0010	0.0010	0.0000	0.0005	0.0005	0.0005
$m = 10$ $\pi_0 = 0.6$	MBonf	0.0080	0.0065	0.0095	0.0095	0.0065	0.0035	0.0025	0.0055	0.0070	0.0030
	Tarone	0.0020	0.0030	0.0060	0.0050	0.0020	0.0020	0.0010	0.0030	0.0030	0.0025
	Bonf	0.0000	0.0015	0.0020	0.0015	0.0000	0.0010	0.0005	0.0010	0.0010	0.0005
	Sidak	0.0000	0.0015	0.0020	0.0015	0.0000	0.0010	0.0005	0.0010	0.0010	0.0005
$m = 10$ $\pi_0 = 0.8$	MBonf	0.0185	0.0105	0.0115	0.0135	0.0135	0.0150	0.0100	0.0090	0.0050	0.0085
	Tarone	0.0120	0.0080	0.0075	0.0095	0.0075	0.0100	0.0065	0.0030	0.0040	0.0045
	Bonf	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010
	Sidak	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010

Table S10: Simulated minimal power comparisons for single-step procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MBonf	0.9095	0.8880	0.8580	0.8455	0.8135	0.7745	0.7545	0.7295	0.6840	0.6305
	Tarone	0.8500	0.8350	0.8000	0.7945	0.7585	0.7070	0.6815	0.6630	0.6130	0.5605
	Bonf	0.7530	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4685
	Sidak	0.7530	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4685
$m = 5$ $\pi_0 = 0.6$	MBonf	0.8150	0.8020	0.7755	0.7740	0.7655	0.7195	0.7255	0.6790	0.6750	0.6260
	Tarone	0.7635	0.7435	0.7210	0.7075	0.7185	0.6775	0.6815	0.6425	0.6255	0.5865
	Bonf	0.6135	0.5985	0.5740	0.5615	0.5725	0.5410	0.5270	0.5070	0.4715	0.4365
	Sidak	0.6135	0.5985	0.5740	0.5615	0.5725	0.5410	0.5270	0.5070	0.4715	0.4365
$m = 5$ $\pi_0 = 0.8$	MBonf	0.5965	0.6055	0.5955	0.5925	0.6075	0.6095	0.5960	0.5960	0.6075	0.6120
	Tarone	0.5635	0.5730	0.5675	0.5600	0.5755	0.5880	0.5730	0.5730	0.5935	0.5985
	Bonf	0.3825	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
	Sidak	0.3825	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
$m = 10$ $\pi_0 = 0.4$	MBonf	0.9760	0.9460	0.9175	0.8925	0.8570	0.8250	0.7885	0.7270	0.6895	0.6090
	Tarone	0.9470	0.8940	0.8535	0.8295	0.7875	0.7585	0.7120	0.6525	0.6040	0.5260
	Bonf	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
	Sidak	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
$m = 10$ $\pi_0 = 0.6$	MBonf	0.9250	0.9125	0.8845	0.8425	0.8300	0.7920	0.7470	0.7160	0.6590	0.6180
	Tarone	0.8820	0.8630	0.8370	0.7705	0.7680	0.7285	0.6925	0.6645	0.6090	0.5745
	Bonf	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
	Sidak	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
$m = 10$ $\pi_0 = 0.8$	MBonf	0.7675	0.7595	0.7390	0.7330	0.7320	0.6975	0.6865	0.6665	0.6340	0.6160
	Tarone	0.7145	0.7055	0.6910	0.6860	0.6885	0.6540	0.6445	0.6310	0.5925	0.5875
	Bonf	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495
	Sidak	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

Table S11: Simulated FWER comparisons for step-down procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MHolm	0.0085	0.0090	0.0070	0.0090	0.0105	0.0120	0.0095	0.0150	0.0115	0.0055
	TH	0.0055	0.0060	0.0040	0.0050	0.0065	0.0100	0.0040	0.0130	0.0095	0.0045
	Holm	0.0025	0.0040	0.0035	0.0035	0.0035	0.0055	0.0015	0.0075	0.0040	0.0035
$m = 5$ $\pi_0 = 0.6$	MHolm	0.0170	0.0095	0.0125	0.0140	0.0125	0.0095	0.0160	0.0150	0.0090	0.0065
	TH	0.0110	0.0065	0.0095	0.0095	0.0090	0.0070	0.0115	0.0125	0.0075	0.0055
	Holm	0.0030	0.0020	0.0035	0.0055	0.0040	0.0030	0.0045	0.0025	0.0040	0.0030
$m = 5$ $\pi_0 = 0.8$	MHolm	0.0160	0.0205	0.0200	0.0190	0.0170	0.0175	0.0135	0.0075	0.0120	0.0070
	TH	0.0110	0.0145	0.0120	0.0150	0.0120	0.0120	0.0115	0.0065	0.0100	0.0060
	Holm	0.0035	0.0015	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
$m = 10$ $\pi_0 = 0.4$	MHolm	0.0130	0.0150	0.0115	0.0095	0.0090	0.0100	0.0115	0.0130	0.0095	0.0090
	TH	0.0060	0.0030	0.0075	0.0040	0.0040	0.0075	0.0080	0.0105	0.0070	0.0075
	Holm	0.0005	0.0005	0.0015	0.0005	0.0015	0.0015	0.0010	0.0030	0.0010	0.0010
$m = 10$ $\pi_0 = 0.6$	MHolm	0.0160	0.0150	0.0185	0.0165	0.0175	0.0105	0.0125	0.0130	0.0140	0.0055
	TH	0.0055	0.0085	0.0115	0.0100	0.0125	0.0075	0.0065	0.0100	0.0125	0.0045
	Holm	0.0000	0.0020	0.0020	0.0025	0.0000	0.0015	0.0005	0.0015	0.0025	0.0005
$m = 10$ $\pi_0 = 0.8$	MHolm	0.0230	0.0160	0.0195	0.0195	0.0200	0.0215	0.0145	0.0120	0.0090	0.0130
	TH	0.0160	0.0130	0.0145	0.0130	0.0145	0.0165	0.0130	0.0100	0.0075	0.0120
	Holm	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010

Table S12: Simulated minimal power comparisons for step-down procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MHolm	0.9095	0.8880	0.8580	0.8455	0.8135	0.7745	0.7545	0.7300	0.6840	0.6305
	TH	0.8500	0.8350	0.8000	0.7945	0.7585	0.7070	0.6815	0.6630	0.6130	0.5605
	Holm	0.7535	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4690
$m = 5$ $\pi_0 = 0.6$	MHolm	0.8155	0.8020	0.7755	0.7740	0.7655	0.7195	0.7255	0.6795	0.6750	0.6260
	TH	0.7640	0.7440	0.7210	0.7080	0.7185	0.6775	0.6815	0.6425	0.6255	0.5865
	Holm	0.6135	0.5985	0.5750	0.5615	0.5725	0.5415	0.5270	0.5070	0.4715	0.4365
$m = 5$ $\pi_0 = 0.8$	MHolm	0.5980	0.6060	0.5975	0.5930	0.6080	0.6095	0.5965	0.5960	0.6080	0.6125
	TH	0.5640	0.5740	0.5685	0.5600	0.5755	0.5880	0.5735	0.5730	0.5940	0.5990
	Holm	0.3830	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
$m = 10$ $\pi_0 = 0.4$	MHolm	0.9760	0.9460	0.9175	0.8925	0.8570	0.8250	0.7885	0.7270	0.6895	0.6090
	TH	0.9470	0.8940	0.8535	0.8295	0.7875	0.7585	0.7120	0.6525	0.6040	0.5260
	Holm	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
$m = 10$ $\pi_0 = 0.6$	MHolm	0.9265	0.9125	0.8845	0.8425	0.8300	0.7920	0.7470	0.7160	0.6590	0.6180
	TH	0.8820	0.8630	0.8380	0.7705	0.7680	0.7285	0.6925	0.6645	0.6090	0.5745
	Holm	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
$m = 10$ $\pi_0 = 0.8$	MHolm	0.7680	0.7600	0.7390	0.7330	0.7325	0.6975	0.6870	0.6665	0.6340	0.6160
	TH	0.7165	0.7060	0.6925	0.6865	0.6895	0.6540	0.6450	0.6310	0.5925	0.5875
	Holm	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

Table S13: Simulated FWER comparisons for step-up procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MHoch	0.0085	0.0090	0.0070	0.0090	0.0105	0.0120	0.0110	0.0175	0.0130	0.0080
	Roth	0.0060	0.0070	0.0050	0.0065	0.0075	0.0095	0.0060	0.0140	0.0105	0.0070
	Hoch	0.0025	0.0040	0.0035	0.0035	0.0035	0.0060	0.0020	0.0090	0.0055	0.0060
$m = 5$ $\pi_0 = 0.6$	MHoch	0.0170	0.0100	0.0125	0.0160	0.0130	0.0105	0.0165	0.0170	0.0120	0.0120
	Roth	0.0120	0.0075	0.0095	0.0110	0.0085	0.0070	0.0125	0.0140	0.0095	0.0105
	Hoch	0.0030	0.0020	0.0035	0.0055	0.0040	0.0030	0.0050	0.0030	0.0050	0.0075
$m = 5$ $\pi_0 = 0.8$	MHoch	0.0165	0.0210	0.0200	0.0200	0.0170	0.0180	0.0135	0.0095	0.0140	0.0115
	Roth	0.0110	0.0145	0.0130	0.0150	0.0115	0.0130	0.0100	0.0075	0.0120	0.0085
	Hoch	0.0035	0.0015	0.0020	0.0015	0.0025	0.0025	0.0030	0.0010	0.0030	0.0035
$m = 10$ $\pi_0 = 0.4$	MHoch	0.0145	0.0165	0.0115	0.0100	0.0105	0.0105	0.0135	0.0155	0.0110	0.0125
	Roth	0.0075	0.0045	0.0080	0.0035	0.0045	0.0075	0.0095	0.0120	0.0080	0.0115
	Hoch	0.0005	0.0005	0.0015	0.0010	0.0015	0.0015	0.0015	0.0035	0.0025	0.0060
$m = 10$ $\pi_0 = 0.6$	MHoch	0.0165	0.0150	0.0185	0.0165	0.0185	0.0115	0.0150	0.0155	0.0190	0.0085
	Roth	0.0060	0.0070	0.0110	0.0100	0.0110	0.0070	0.0070	0.0105	0.0135	0.0070
	Hoch	0.0000	0.0020	0.0020	0.0025	0.0005	0.0015	0.0010	0.0020	0.0045	0.0015
$m = 10$ $\pi_0 = 0.8$	MHoch	0.0235	0.0170	0.0205	0.0200	0.0210	0.0225	0.0165	0.0140	0.0105	0.0170
	Roth	0.0145	0.0120	0.0125	0.0130	0.0155	0.0155	0.0130	0.0075	0.0080	0.0110
	Hoch	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0035

Table S14: Simulated minimal power comparisons for step-up procedures with dependent p -values generated from Binomial Exact Test statistics.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$ $\pi_0 = 0.4$	MHoch	0.9110	0.8905	0.8595	0.8490	0.8155	0.7780	0.7595	0.7370	0.6985	0.6525
	Roth	0.8565	0.8430	0.8075	0.8020	0.7640	0.7200	0.6945	0.6755	0.6300	0.5955
	Hoch	0.7640	0.7555	0.7105	0.7025	0.6835	0.6310	0.6050	0.5860	0.5435	0.4945
	MHoch	0.8165	0.8065	0.7785	0.7775	0.7690	0.7255	0.7290	0.6880	0.6835	0.6425
	Roth	0.7670	0.7515	0.7245	0.7115	0.7250	0.6845	0.6840	0.6505	0.6325	0.5990
	Hoch	0.6210	0.6045	0.5780	0.5665	0.5800	0.5460	0.5345	0.5140	0.4845	0.4530
	MHoch	0.5985	0.6065	0.5975	0.5940	0.6080	0.6095	0.5965	0.5965	0.6085	0.6130
	Roth	0.5490	0.5590	0.5520	0.5490	0.5610	0.5755	0.5560	0.5570	0.5775	0.5800
	Hoch	0.3830	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3815
$m = 10$ $\pi_0 = 0.4$	MHoch	0.9770	0.9465	0.9205	0.8945	0.8630	0.8300	0.7925	0.7380	0.7055	0.6305
	Roth	0.9495	0.8980	0.8595	0.8325	0.7910	0.7610	0.7190	0.6605	0.6190	0.5625
	Hoch	0.8815	0.8260	0.7630	0.7520	0.6850	0.6705	0.6095	0.5565	0.5075	0.4485
	MHoch	0.9285	0.9155	0.8855	0.8475	0.8340	0.7985	0.7540	0.7235	0.6735	0.6405
	Roth	0.8870	0.8650	0.8425	0.7760	0.7735	0.7335	0.7000	0.6700	0.6145	0.5875
	Hoch	0.7425	0.7260	0.7030	0.6225	0.6290	0.5710	0.5430	0.5000	0.4465	0.4205
	MHoch	0.7725	0.7625	0.7400	0.7380	0.7350	0.6995	0.6890	0.6710	0.6400	0.6285
	Roth	0.7200	0.7055	0.6945	0.6895	0.6905	0.6515	0.6460	0.6365	0.5970	0.5965
	Hoch	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

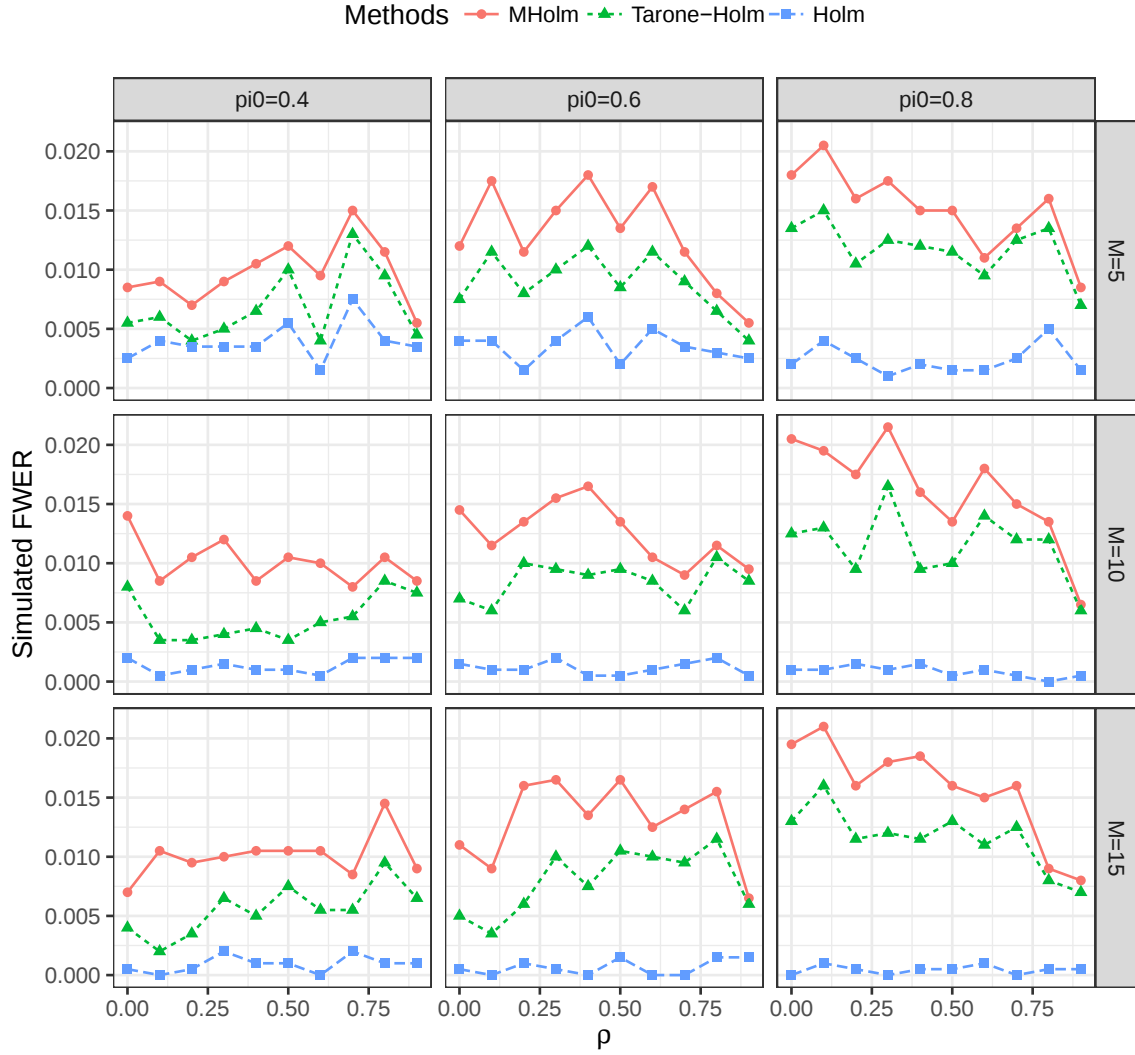


Figure S5: Simulated FWER comparisons for different step-down procedures based on the blocking dependent BET.

R-package for MHTdiscrete: R-package MHTdiscrete containing code to perform the FWER controlling procedures for discrete data described in the article. The package can be downloaded from <https://cran.r-project.org/web/packages/MHTdiscrete>. (GNU zipped tar file)

Web Application for MHTdiscrete: A web application containing the proposed procedures and comparable procedures can be accessed at <https://allen.shinyapps.io/MTPs>.

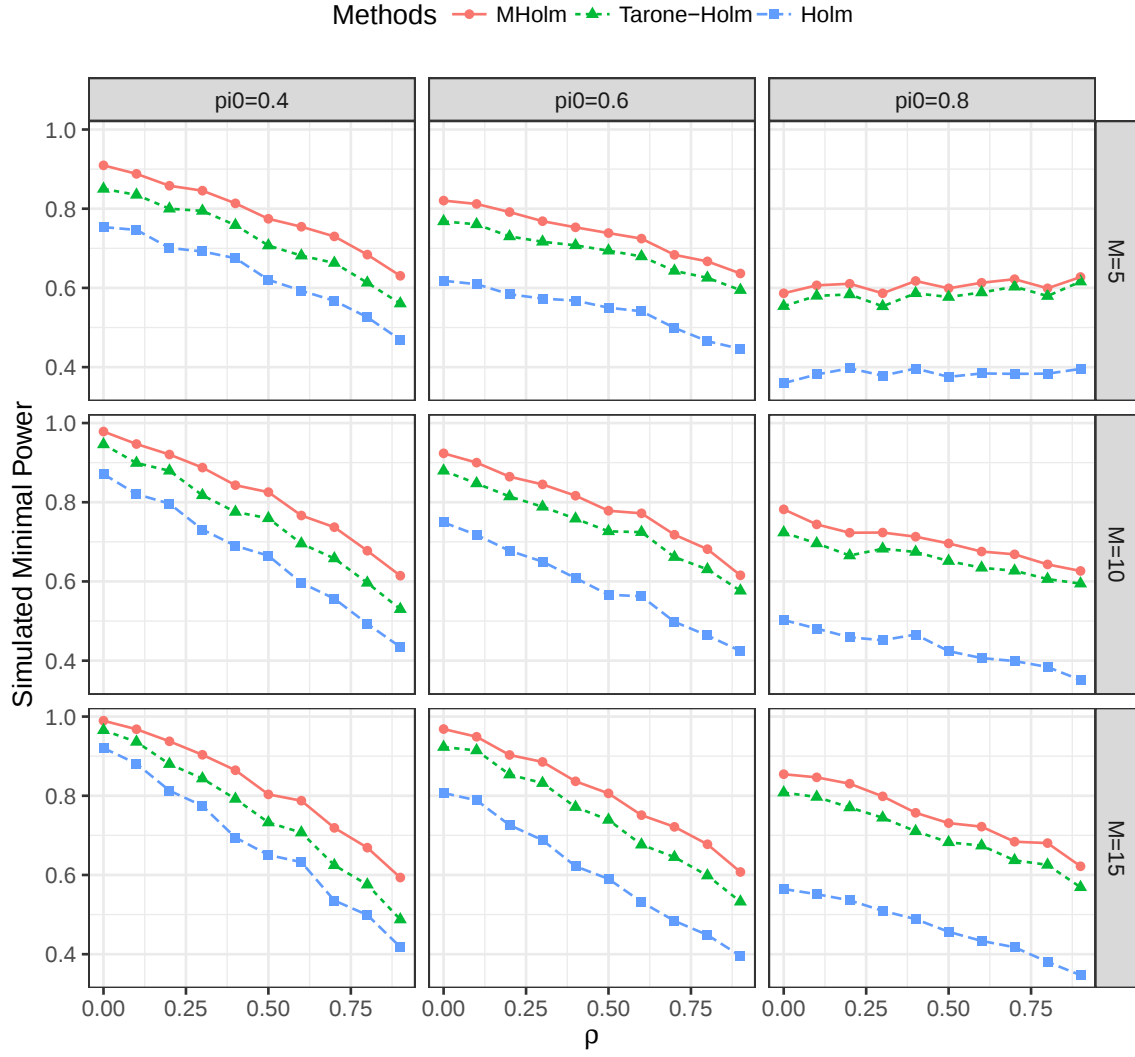


Figure S6: Simulated minimal power comparisons for different step-down procedures based on the blocking dependent BET.

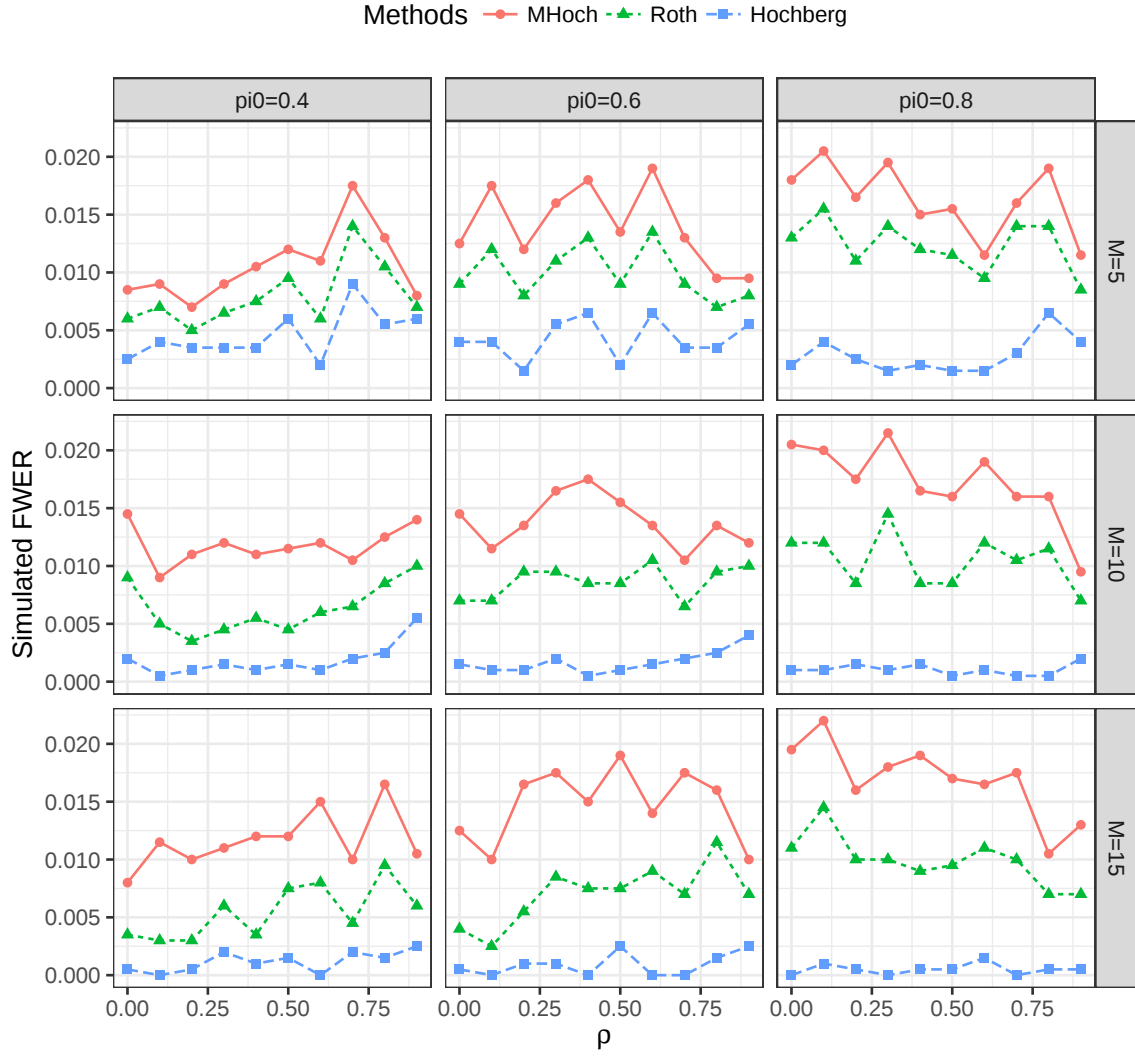


Figure S7: Simulated FWER comparisons for different step-up procedures based on the blocking dependent BET.

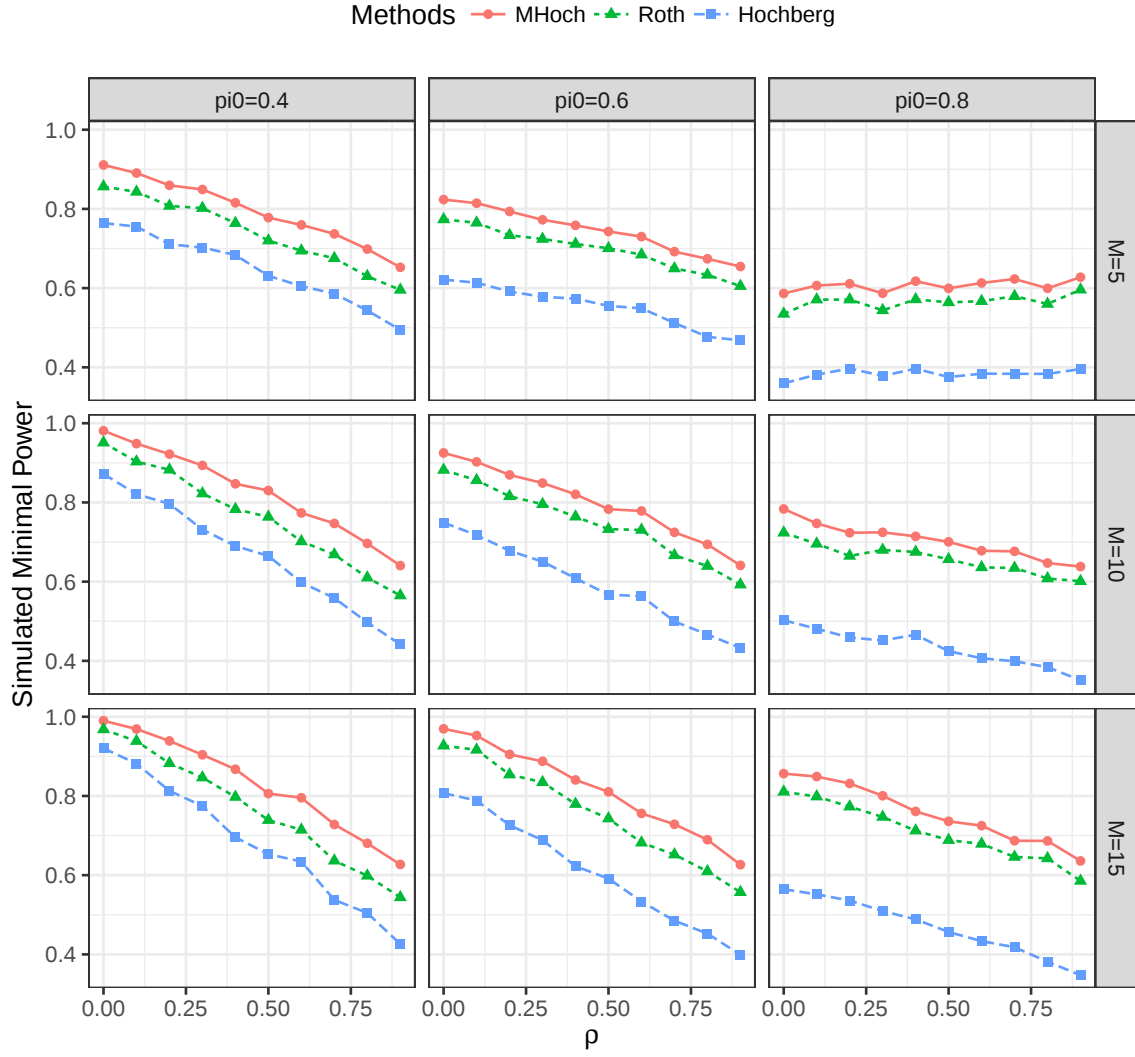


Figure S8: Simulated minimal power comparisons for different step-up procedures based on the blocking dependent BET.