Familywise Error Rate Controlling Procedures for Discrete Data - Supplementary Materials

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S1 Results from Independence Simulation Settings

The simulation results under the independence setting for stepwise procedures comparisons are shown in this section. Tables S1, S2 and Tables S3, S4 respectively provide the results of numerical comparisons of single-step procedures using Fisher and Binomial Exact Tests (as plotted in Figures 1 and 2).

Table S1: Simulated FWER comparisons for single-step procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MBonf	0.0025	0.0060	0.0035	0.0075	0.0075	0.0095
$\pi_0 = 0.2$	Tarone	0.0015	0.0030	0.0015	0.0055	0.0045	0.0085
	Bonf	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
	Sidak	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
$m = 5$	MBonf	0.0045	0.0130	0.0120	0.0170	0.0135	0.0145
$\pi_0 = 0.4$	Tarone	0.0030	0.0060	0.0065	0.0140	0.0090	0.0100
	Bonf	0.0015	0.0060	0.0065	0.0140	0.0090	0.0100
	Sidak	0.0015	0.0060	0.0065	0.0140	0.0090	0.0100
$m = 5$	MBonf	0.0085	0.0200	0.0195	0.0235	0.0225	0.0245
$\pi_0 = 0.6$	Tarone	0.0060	0.0105	0.0105	0.0180	0.0155	0.0170
	Bonf	0.0025	0.0100	0.0105	0.0180	0.0155	0.0170
	Sidak	0.0025	0.0100	0.0105	0.0180	0.0160	0.0175
$m = 5$	MBonf	0.0140	0.0265	0.0270	0.0340	0.0315	0.0370
$\pi_0 = 0.8$	Tarone	0.0110	0.0140	0.0155	0.0245	0.0215	0.0220
	Bonf	0.0045	0.0135	0.0155	0.0245	0.0215	0.0220
	Sidak	0.0045	0.0135	0.0155	0.0245	0.0220	0.0230
$m = 10$	MBonf	0.0020	0.0060	0.0100	0.0115	0.0095	0.0110
$\pi_0 = 0.2$	Tarone	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
	Bonf	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
	Sidak	0.0005	0.0040	0.0065	0.0060	0.0070	0.0060
$m = 10$	MBonf	0.0050	0.0145	0.0165	0.0190	0.0215	0.0190
$\pi_0 = 0.4$	Tarone	0.0025	0.0090	0.0120	0.0100	0.0140	0.0125
	Bonf	0.0025	0.0090	0.0120	0.0100	0.0140	0.0125
	Sidak	0.0025	0.0090	0.0120	0.0110	0.0145	0.0130
$m = 10$	MBonf	0.0090	0.0245	0.0260	0.0265	0.0300	0.0255
$\pi_0 = 0.6$	Tarone	0.0055	0.0150	0.0185	0.0150	0.0180	0.0155
	Bonf	0.0045	0.0140	0.0185	0.0150	0.0180	0.0155
	Sidak	0.0045	0.0140	0.0185	0.0160	0.0195	0.0155
$m = 10$	MBonf	0.0175	0.0335	0.0345	0.0370	0.0390	0.0360
$\pi_0 = 0.8$	Tarone	0.0090	0.0215	0.0225	0.0190	0.0220	0.0200
	Bonf	0.0055	0.0190	0.0225	0.0190	0.0220	0.0200
	Sidak	0.0055	0.0190	0.0225	0.0210	0.0240	0.0200
$m = 15$	MBonf	0.0040	0.0060	0.0065	0.0120	0.0080	0.0100
$\pi_0 = 0.2$	Tarone	0.0020	0.0030	0.0030	0.0065	0.0045	0.0070
	Bonf	0.0005	0.0030	0.0030	0.0065	0.0045	0.0070
	Sidak	0.0005	0.0030	0.0030	0.0075	0.0045	0.0070
$m = 15$	MBonf	0.0090	0.0150	0.0140	0.0240	0.0210	0.0200
$\pi_0 = 0.4$	Tarone	0.0060	0.0075	0.0065	0.0125	0.0150	0.0105
	Bonf	0.0010	0.0070	0.0065	0.0125	0.0150	0.0105
	Sidak	0.0010	0.0070	0.0065	0.0145	0.0150	0.0105
$m = 15$	MBonf	0.0165	0.0250	0.0210	0.0325	0.0320	0.0280
$\pi_0 = 0.6$	Tarone	0.0090	0.0130	0.0095	0.0170	0.0205	0.0180
	Bonf	0.0020	0.0105	0.0095	0.0170	0.0205	0.0180
	Sidak	0.0020	0.0105	0.0095	0.0190	0.0205	0.0180
$m = 15$	MBonf	0.0210	0.0345	0.0315	0.0400	0.0460	0.0360
$\pi_0 = 0.8$	Tarone	0.0115	0.0170	0.0155	0.0215	0.0285	0.0240
	Bonf	0.0020	0.0135	0.0155	0.0215	0.0285	0.0240
	Sidak	0.0020	0.0135	0.0155	0.0240	0.0285	0.0240

Table S2: Simulated minimal power comparisons for single-step procedures with independent p-values generated from Fisher's exact test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MBonf	0.2550	0.5060	0.6855	0.8195	0.9145	0.9505
	Tarone	0.1945	0.3900	0.5775	0.7680	0.8655	0.9275
$\pi_0 = 0.2$	Bonf	0.1125	0.3825	0.5765	0.7680	0.8655	0.9275
	Sidak	0.1125	0.3825	0.5850	0.7680	0.8710	0.9340
$m = 5$	MBonf	0.2110	0.4085	0.5785	0.7405	0.8375	0.9025
	Tarone	0.1605	0.3110	0.4715	0.6705	0.7695	0.8625
$\pi_0 = 0.4$	Bonf	0.0880	0.3000	0.4700	0.6705	0.7695	0.8625
	Sidak	0.0880	0.3000	0.4770	0.6705	0.7765	0.8680
$m = 5$	MBonf	0.1550	0.3130	0.4320	0.5835	0.7025	0.7845
	Tarone	0.1180	0.2365	0.3370	0.5145	0.6255	0.7245
$\pi_0 = 0.6$	Bonf	0.0605	0.2190	0.3355	0.5145	0.6255	0.7245
	Sidak	0.0605	0.2190	0.3420	0.5145	0.6330	0.7345
$m = 5$	MBonf	0.0945	0.1800	0.2570	0.3595	0.4660	0.5505
	Tarone	0.0740	0.1330	0.1920	0.2955	0.3950	0.4850
$\pi_0 = 0.8$	Bonf	0.0330	0.1190	0.1920	0.2955	0.3950	0.4850
	Sidak	0.0330	0.1190	0.1955	0.2955	0.4025	0.5005
$m = 10$	MBonf	0.3155	0.6130	0.8090	0.9110	0.9765	0.9930
	Tarone	0.2075	0.4695	0.7220	0.8550	0.9415	0.9820
$\pi_0 = 0.2$	Bonf	0.1575	0.4660	0.7220	0.8550	0.9415	0.9820
	Sidak	0.1575	0.4660	0.7220	0.8595	0.9425	0.9830
$m = 10$	MBonf	0.2700	0.5220	0.7180	0.8455	0.9440	0.9750
	Tarone	0.1770	0.3905	0.6065	0.7720	0.8905	0.9505
$\pi_0 = 0.4$	Bonf	0.1235	0.3795	0.6065	0.7720	0.8905	0.9505
	Sidak	0.1235	0.3795	0.6065	0.7775	0.8920	0.9575
$m = 10$	MBonf	0.2005	0.4030	0.5615	0.7300	0.8450	0.9035
	Tarone	0.1330	0.2990	0.4525	0.6315	0.7590	0.8525
$\pi_0 = 0.6$	Bonf	0.0800	0.2825	0.4525	0.6315	0.7590	0.8525
	Sidak	0.0800	0.2825	0.4525	0.6375	0.7615	0.8585
$m = 10$	MBonf	0.1115	0.2440	0.3500	0.4775	0.6140	0.6935
	Tarone	0.0760	0.1680	0.2645	0.3810	0.5165	0.6060
$\pi_0 = 0.8$	Bonf	0.0390	0.1555	0.2645	0.3810	0.5165	0.6060
	Sidak	0.0390	0.1555	0.2645	0.3880	0.5170	0.6185
$m = 15$	MBonf	0.3370	0.6715	0.8820	0.9495	0.9915	0.9965
	Tarone	0.2520	0.4995	0.7530	0.8910	0.9765	0.9895
$\pi_0 = 0.2$	Bonf	0.1390	0.4870	0.7515	0.8910	0.9765	0.9895
	Sidak	0.1390	0.4870	0.7515	0.8960	0.9765	0.9895
$m = 15$	MBonf	0.2880	0.5815	0.7910	0.9025	0.9635	0.9830
	Tarone	0.2110	0.4105	0.6475	0.8050	0.9335	0.9745
$\pi_0 = 0.4$	Bonf	0.1030	0.3870	0.6460	0.8050	0.9335	0.9745
	Sidak	0.1030	0.3870	0.6460	0.8125	0.9335	0.9745
$m = 15$	MBonf	0.2135	0.4485	0.6570	0.7925	0.8840	0.9500
	Tarone	0.1495	0.3070	0.5085	0.6730	0.8315	0.9140
$\pi_0 = 0.6$	Bonf	0.0700	0.2760	0.5065	0.6730	0.8315	0.9140
	Sidak	0.0700	0.2760	0.5065	0.6790	0.8315	0.9140
$m = 15$	MBonf	0.1205	0.2635	0.4270	0.5490	0.6710	0.7780
	Tarone	0.0830	0.1785	0.3050	0.4295	0.5890	0.7020
$\pi_0 = 0.8$	Bonf	0.0335	0.1480	0.3040	0.4290	0.5890	0.7020
	Sidak	0.0335	0.1480	0.3040	0.4345	0.5895	0.7020

Table S3: Simulated FWER comparisons for single-step procedures with independent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
$m = 5$	MBonf	0.0020	0.0060	0.0075	0.0165
$\alpha = 0.05$	Tarone	0.0010	0.0030	0.0055	0.0105
	Bonf	0.0010	0.0020	0.0025	0.0030
	Sidak	0.0010	0.0020	0.0025	0.0030
$m = 10$	MBonf	0.0010	0.0045	0.0130	0.0160
$\alpha = 0.05$	Tarone	0.0000	0.0010	0.0050	0.0115
	Bonf	0.0000	0.0005	0.0025	0.0025
	Sidak	0.0000	0.0005	0.0025	0.0025
$m = 15$	MBonf	0.0010	0.0065	0.0045	0.0150
$\alpha = 0.05$	Tarone	0.0000	0.0010	0.0020	0.0070
	Bonf	0.0000	0.0005	0.0000	0.0000
	Sidak	0.0000	0.0005	0.0000	0.0000
$m = 5$	MBonf	0.0070	0.0125	0.0200	0.0365
$\alpha = 0.1$	Tarone	0.0020	0.0065	0.0110	0.0285
	Bonf	0.0020	0.0055	0.0065	0.0130
	Sidak	0.0020	0.0055	0.0065	0.0130
$m = 10$	MBonf	0.0040	0.0080	0.0275	0.0350
$\alpha = 0.1$	Tarone	0.0000	0.0030	0.0165	0.0195
	Bonf	0.0000	0.0015	0.0055	0.0060
	Sidak	0.0000	0.0015	0.0055	0.0060
$m = 15$	MBonf	0.0060	0.0155	0.0185	0.0315
$\alpha = 0.1$	Tarone	0.0005	0.0060	0.0045	0.0200
	Bonf	0.0000	0.0010	0.0020	0.0025
	Sidak	0.0000	0.0010	0.0020	0.0025

Table S4: Simulated minimal power comparisons for single-step procedures with independent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
$m = 5$	MBonf	0.9205	0.8805	0.7845	0.5565
	Tarone	0.8815	0.8240	0.7395	0.5235
$\alpha = 0.05$	Bonf	0.8735	0.8055	0.6610	0.4045
	Sidak	0.8735	0.8055	0.6610	0.4045
$m = 10$	MBonf	0.9850	0.9635	0.9035	0.7390
	Tarone	0.9470	0.9240	0.8630	0.6855
$\alpha = 0.05$	Bonf	0.9315	0.8635	0.7050	0.4775
	Sidak	0.9315	0.8635	0.7050	0.4775
$m = 15$	MBonf	0.9925	0.9810	0.9555	0.8210
	Tarone	0.9825	0.9500	0.9095	0.7845
$\alpha = 0.05$	Bonf	0.9820	0.9475	0.8560	0.6135
	Sidak	0.9820	0.9475	0.8560	0.6135
$m = 5$	MBonf	0.9680	0.9415	0.8615	0.6330
	Tarone	0.9410	0.9140	0.8240	0.5920
$\alpha = 0.1$	Bonf	0.9050	0.8375	0.7040	0.4520
	Sidak	0.9050	0.8375	0.7040	0.4520
$m = 10$	MBonf	0.9965	0.9875	0.9620	0.8315
	Tarone	0.9885	0.9660	0.9170	0.7835
$\alpha = 0.1$	Bonf	0.9870	0.9565	0.8690	0.6600
	Sidak	0.9870	0.9565	0.8690	0.6600
$m = 15$	MBonf	0.9995	0.9970	0.9830	0.9030
	Tarone	0.9960	0.9930	0.9605	0.8400
$\alpha = 0.1$	Bonf	0.9880	0.9615	0.8830	0.6515
	Sidak	0.9895	0.9635	0.8880	0.6590

Tables S5 and S6 provide numerical results of step-down procedures comparisons using Fisher Exact Test, which are also plotted as Figures S1 and S2. Tables S7 and S8 provide numerical results of step-up procedures comparisons using Fisher Exact Test, which are plotted as Figures S3 and S4.

Table S5: Simulated FWER comparisons for step-down procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MHolm	0.0030	0.0090	0.0065	0.0115	0.0150	0.0150
	TH	0.0015	0.0045	0.0030	0.0075	0.0090	0.0140
$\pi_0 = 0.2$	Holm	0.0010	0.0045	0.0030	0.0075	0.0090	0.0140
$m = 5$	MHolm	0.0055	0.0155	0.0135	0.0230	0.0225	0.0225
	TH	0.0030	0.0080	0.0080	0.0185	0.0140	0.0180
$\pi_0 = 0.4$	Holm	0.0020	0.0075	0.0080	0.0185	0.0140	0.0180
$m = 5$	MHolm	0.0100	0.0215	0.0215	0.0290	0.0305	0.0320
	TH	0.0065	0.0115	0.0115	0.0220	0.0185	0.0205
$\pi_0 = 0.6$	Holm	0.0030	0.0110	0.0115	0.0220	0.0185	0.0205
$m = 5$	MHolm	0.0155	0.0285	0.0285	0.0360	0.0375	0.0440
	TH	0.0115	0.0145	0.0160	0.0260	0.0240	0.0270
$\pi_0 = 0.8$	Holm	0.0050	0.0140	0.0160	0.0260	0.0240	0.0270
$m = 10$	MHolm	0.0020	0.0070	0.0125	0.0130	0.0160	0.0185
	TH	0.0005	0.0040	0.0070	0.0080	0.0115	0.0125
$\pi_0 = 0.2$	Holm	0.0005	0.0040	0.0070	0.0080	0.0115	0.0125
$m = 10$	MHolm	0.0050	0.0155	0.0200	0.0215	0.0280	0.0265
	TH	0.0025	0.0090	0.0125	0.0125	0.0200	0.0175
$\pi_0 = 0.4$	Holm	0.0025	0.0090	0.0125	0.0125	0.0200	0.0175
$m = 10$	MHolm	0.0095	0.0250	0.0285	0.0290	0.0360	0.0350
	TH	0.0060	0.0150	0.0185	0.0155	0.0220	0.0215
$\pi_0 = 0.6$	Holm	0.0045	0.0140	0.0185	0.0155	0.0220	0.0215
$m = 10$	MHolm	0.0175	0.0340	0.0360	0.0380	0.0420	0.0405
	TH	0.0090	0.0215	0.0235	0.0195	0.0255	0.0230
$\pi_0 = 0.8$	Holm	0.0055	0.0190	0.0225	0.0195	0.0255	0.0230
$m = 15$	MHolm	0.0045	0.0070	0.0070	0.0140	0.0125	0.0120
	TH	0.0025	0.0035	0.0030	0.0090	0.0060	0.0085
$\pi_0 = 0.2$	Holm	0.0005	0.0030	0.0030	0.0090	0.0060	0.0085
$m = 15$	MHolm	0.0095	0.0165	0.0145	0.0255	0.0255	0.0285
	TH	0.0060	0.0080	0.0075	0.0160	0.0175	0.0165
$\pi_0 = 0.4$	Holm	0.0010	0.0070	0.0075	0.0160	0.0175	0.0165
$m = 15$	MHolm	0.0165	0.0260	0.0215	0.0345	0.0350	0.0345
	TH	0.0090	0.0130	0.0105	0.0190	0.0215	0.0195
$\pi_0 = 0.6$	Holm	0.0020	0.0105	0.0100	0.0190	0.0215	0.0195
$m = 15$	MHolm	0.0215	0.0350	0.0315	0.0415	0.0465	0.0390
	TH	0.0120	0.0170	0.0165	0.0225	0.0290	0.0260
$\pi_0 = 0.8$	Holm	0.0020	0.0135	0.0165	0.0225	0.0290	0.0260

Table S6: Simulated minimal power comparisons for step-down procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MHolm	0.2555	0.5070	0.6855	0.8200	0.9145	0.9505
	TH	0.1945	0.3905	0.5780	0.7680	0.8660	0.9280
$\pi_0 = 0.2$	Holm	0.1130	0.3830	0.5770	0.7680	0.8660	0.9280
$m = 5$	MHolm	0.2120	0.4090	0.5790	0.7405	0.8375	0.9030
	TH	0.1605	0.3115	0.4725	0.6705	0.7695	0.8630
$\pi_0 = 0.4$	Holm	0.0880	0.3005	0.4710	0.6705	0.7695	0.8630
$m = 5$	MHolm	0.1555	0.3150	0.4330	0.5855	0.7035	0.7855
	TH	0.1185	0.2365	0.3375	0.5160	0.6265	0.7260
$\pi_0 = 0.6$	Holm	0.0605	0.2190	0.3360	0.5160	0.6265	0.7260
$m = 5$	MHolm	0.0950	0.1815	0.2585	0.3615	0.4690	0.5530
	TH	0.0745	0.1330	0.1920	0.2965	0.3960	0.4860
$\pi_0 = 0.8$	Holm	0.0330	0.1190	0.1920	0.2965	0.3960	0.4860
$m = 10$	MHolm	0.3160	0.6130	0.8095	0.9120	0.9765	0.9930
	TH	0.2075	0.4700	0.7220	0.8550	0.9415	0.9820
$\pi_0 = 0.2$	Holm	0.1575	0.4660	0.7220	0.8550	0.9415	0.9820
$m = 10$	MHolm	0.2705	0.5220	0.7185	0.8455	0.9445	0.9750
	TH	0.1770	0.3905	0.6065	0.7720	0.8905	0.9505
$\pi_0 = 0.4$	Holm	0.1235	0.3795	0.6065	0.7720	0.8905	0.9505
$m = 10$	MHolm	0.2010	0.4035	0.5615	0.7300	0.8450	0.9035
	TH	0.1330	0.2990	0.4525	0.6315	0.7590	0.8525
$\pi_0 = 0.6$	Holm	0.0800	0.2825	0.4525	0.6315	0.7590	0.8525
$m = 10$	MHolm	0.1115	0.2440	0.3500	0.4780	0.6145	0.6935
	TH	0.0760	0.1680	0.2645	0.3810	0.5175	0.6065
$\pi_0 = 0.8$	Holm	0.0390	0.1555	0.2645	0.3810	0.5175	0.6065
$m = 15$	MHolm	0.3375	0.6715	0.8820	0.9495	0.9915	0.9965
	TH	0.2520	0.4995	0.7530	0.8910	0.9765	0.9895
$\pi_0 = 0.2$	Holm	0.1390	0.4870	0.7515	0.8910	0.9765	0.9895
$m = 15$	MHolm	0.2885	0.5825	0.7915	0.9025	0.9635	0.9830
	TH	0.2110	0.4105	0.6475	0.8055	0.9335	0.9745
$\pi_0 = 0.4$	Holm	0.1030	0.3870	0.6460	0.8055	0.9335	0.9745
$m = 15$	MHolm	0.2135	0.4495	0.6575	0.7930	0.8840	0.9500
	TH	0.1495	0.3070	0.5085	0.6730	0.8315	0.9140
$\pi_0 = 0.6$	Holm	0.0700	0.2760	0.5065	0.6730	0.8315	0.9140
$m = 15$	MHolm	0.1205	0.2645	0.4280	0.5495	0.6730	0.7780
	TH	0.0835	0.1785	0.3055	0.4295	0.5890	0.7030
$\pi_0 = 0.8$	Holm	0.0335	0.1480	0.3045	0.4290	0.5890	0.7030

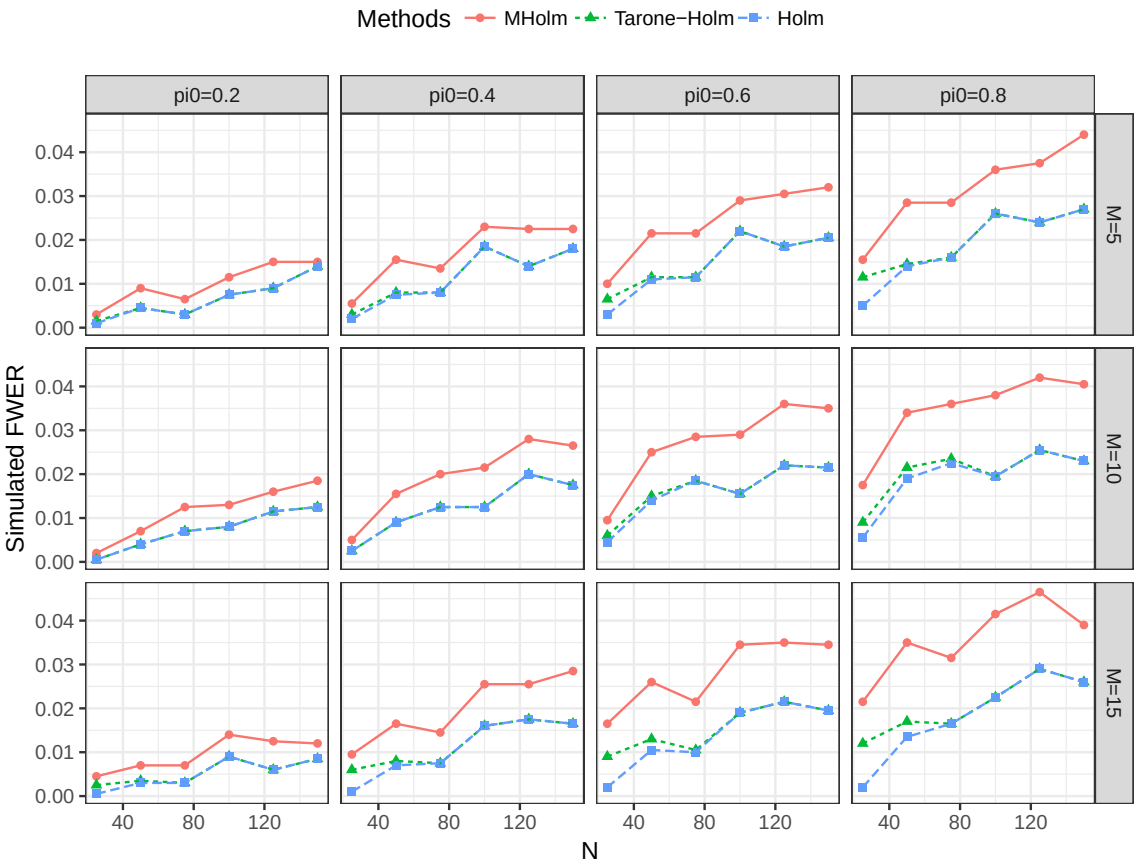

Figure S1: Simulated FWER comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

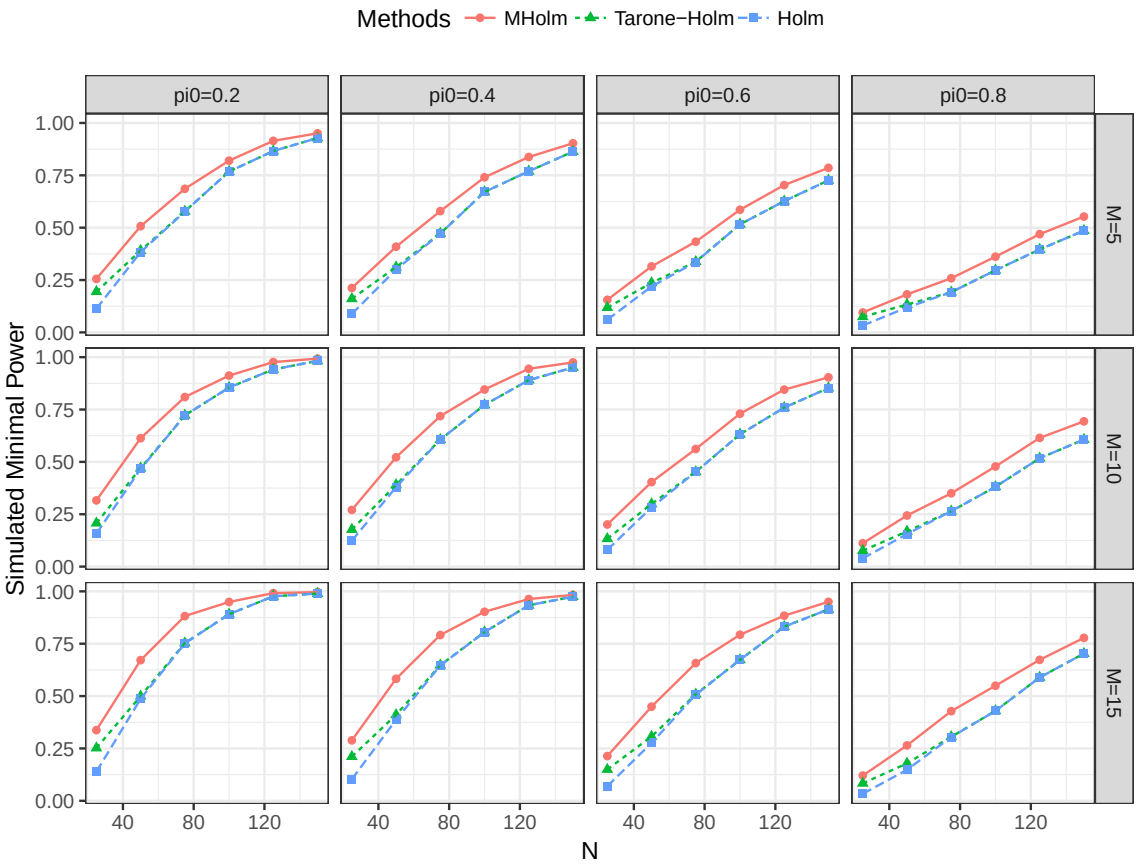

Figure S2: Simulated minimal power comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

Table S7: Simulated FWER comparisons for step-up procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MHoch	0.0030	0.0090	0.0070	0.0115	0.0150	0.0155
	Roth	0.0020	0.0045	0.0040	0.0085	0.0115	0.0155
$\pi_0 = 0.2$	Hoch	0.0015	0.0045	0.0040	0.0085	0.0115	0.0155
$m = 5$	MHoch	0.0060	0.0155	0.0140	0.0235	0.0230	0.0245
	Roth	0.0035	0.0080	0.0085	0.0185	0.0160	0.0200
$\pi_0 = 0.4$	Hoch	0.0025	0.0075	0.0085	0.0185	0.0160	0.0200
$m = 5$	MHoch	0.0105	0.0215	0.0215	0.0290	0.0305	0.0325
	Roth	0.0065	0.0115	0.0115	0.0220	0.0195	0.0215
$\pi_0 = 0.6$	Hoch	0.0030	0.0110	0.0115	0.0220	0.0195	0.0215
$m = 5$	MHoch	0.0160	0.0285	0.0285	0.0360	0.0380	0.0445
	Roth	0.0115	0.0145	0.0160	0.0265	0.0245	0.0280
$\pi_0 = 0.8$	Hoch	0.0050	0.0140	0.0160	0.0265	0.0245	0.0280
$m = 10$	MHoch	0.0025	0.0070	0.0125	0.0140	0.0170	0.0200
	Roth	0.0005	0.0040	0.0070	0.0080	0.0120	0.0135
$\pi_0 = 0.2$	Hoch	0.0005	0.0040	0.0070	0.0080	0.0120	0.0135
$m = 10$	MHoch	0.0055	0.0155	0.0200	0.0225	0.0290	0.0275
	Roth	0.0025	0.0090	0.0125	0.0125	0.0200	0.0185
$\pi_0 = 0.4$	Hoch	0.0025	0.0090	0.0125	0.0125	0.0200	0.0185
$m = 10$	MHoch	0.0095	0.0250	0.0285	0.0290	0.0360	0.0350
	Roth	0.0060	0.0150	0.0185	0.0155	0.0220	0.0215
$\pi_0 = 0.6$	Hoch	0.0045	0.0140	0.0185	0.0155	0.0220	0.0215
$m = 10$	MHoch	0.0180	0.0340	0.0360	0.0380	0.0420	0.0405
	Roth	0.0095	0.0210	0.0235	0.0195	0.0255	0.0235
$\pi_0 = 0.8$	Hoch	0.0055	0.0190	0.0225	0.0195	0.0255	0.0235
$m = 15$	MHoch	0.0045	0.0070	0.0070	0.0140	0.0125	0.0130
	Roth	0.0020	0.0035	0.0030	0.0090	0.0060	0.0095
$\pi_0 = 0.2$	Hoch	0.0005	0.0030	0.0030	0.0090	0.0060	0.0095
$m = 15$	MHoch	0.0100	0.0165	0.0145	0.0255	0.0255	0.0290
	Roth	0.0060	0.0080	0.0075	0.0160	0.0175	0.0165
$\pi_0 = 0.4$	Hoch	0.0010	0.0070	0.0075	0.0160	0.0175	0.0165
$m = 15$	MHoch	0.0175	0.0260	0.0215	0.0345	0.0350	0.0345
	Roth	0.0090	0.0130	0.0105	0.0190	0.0220	0.0195
$\pi_0 = 0.6$	Hoch	0.0020	0.0105	0.0100	0.0190	0.0220	0.0195
$m = 15$	MHoch	0.0215	0.0350	0.0315	0.0415	0.0465	0.0390
	Roth	0.0120	0.0170	0.0165	0.0225	0.0290	0.0265
$\pi_0 = 0.8$	Hoch	0.0020	0.0135	0.0165	0.0225	0.0290	0.0265

Table S8: Simulated minimal power comparisons for step-up procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$	MHoch	0.2600	0.5075	0.6885	0.8240	0.9170	0.9525
	Roth	0.1975	0.3915	0.5820	0.7685	0.8695	0.9300
$\pi_0 = 0.2$	Hoch	0.1170	0.3845	0.5810	0.7685	0.8695	0.9300
$m = 5$	MHoch	0.2155	0.4105	0.5810	0.7410	0.8400	0.9055
	Roth	0.1630	0.3115	0.4755	0.6705	0.7715	0.8660
$\pi_0 = 0.4$	Hoch	0.0885	0.3010	0.4740	0.6705	0.7715	0.8660
$m = 5$	MHoch	0.1580	0.3155	0.4340	0.5860	0.7045	0.7875
	Roth	0.1200	0.2365	0.3380	0.5165	0.6280	0.7275
$\pi_0 = 0.6$	Hoch	0.0605	0.2190	0.3365	0.5165	0.6280	0.7275
$m = 5$	MHoch	0.0955	0.1815	0.2585	0.3615	0.4695	0.5535
	Roth	0.0745	0.1330	0.1920	0.2970	0.3965	0.4870
$\pi_0 = 0.8$	Hoch	0.0330	0.1190	0.1920	0.2970	0.3965	0.4870
$m = 10$	MHoch	0.3215	0.6155	0.8110	0.9130	0.9765	0.9930
	Roth	0.2080	0.4685	0.7225	0.8555	0.9420	0.9820
$\pi_0 = 0.2$	Hoch	0.1580	0.4660	0.7225	0.8555	0.9420	0.9820
$m = 10$	MHoch	0.2735	0.5245	0.7200	0.8465	0.9450	0.9755
	Roth	0.1770	0.3840	0.6070	0.7720	0.8920	0.9510
$\pi_0 = 0.4$	Hoch	0.1240	0.3795	0.6065	0.7720	0.8920	0.9510
$m = 10$	MHoch	0.2030	0.4045	0.5615	0.7310	0.8450	0.9045
	Roth	0.1335	0.2910	0.4525	0.6315	0.7600	0.8530
$\pi_0 = 0.6$	Hoch	0.0800	0.2825	0.4525	0.6315	0.7600	0.8530
$m = 10$	MHoch	0.1135	0.2440	0.3500	0.4780	0.6150	0.6935
	Roth	0.0765	0.1625	0.2645	0.3810	0.5175	0.6075
$\pi_0 = 0.8$	Hoch	0.0390	0.1555	0.2645	0.3810	0.5175	0.6075
$m = 15$	MHoch	0.3405	0.6720	0.8830	0.9505	0.9915	0.9965
	Roth	0.2520	0.5010	0.7545	0.8910	0.9765	0.9900
$\pi_0 = 0.2$	Hoch	0.1390	0.4875	0.7535	0.8910	0.9765	0.9900
$m = 15$	MHoch	0.2895	0.5830	0.7925	0.9025	0.9635	0.9830
	Roth	0.2110	0.4115	0.6485	0.8060	0.9335	0.9745
$\pi_0 = 0.4$	Hoch	0.1030	0.3870	0.6470	0.8060	0.9335	0.9745
$m = 15$	MHoch	0.2150	0.4500	0.6595	0.7935	0.8845	0.9505
	Roth	0.1495	0.3080	0.5095	0.6730	0.8320	0.9150
$\pi_0 = 0.6$	Hoch	0.0700	0.2760	0.5075	0.6730	0.8320	0.9150
$m = 15$	MHoch	0.1210	0.2645	0.4285	0.5500	0.6730	0.7780
	Roth	0.0835	0.1785	0.3055	0.4295	0.5895	0.7035
$\pi_0 = 0.8$	Hoch	0.0335	0.1480	0.3045	0.4290	0.5895	0.7035

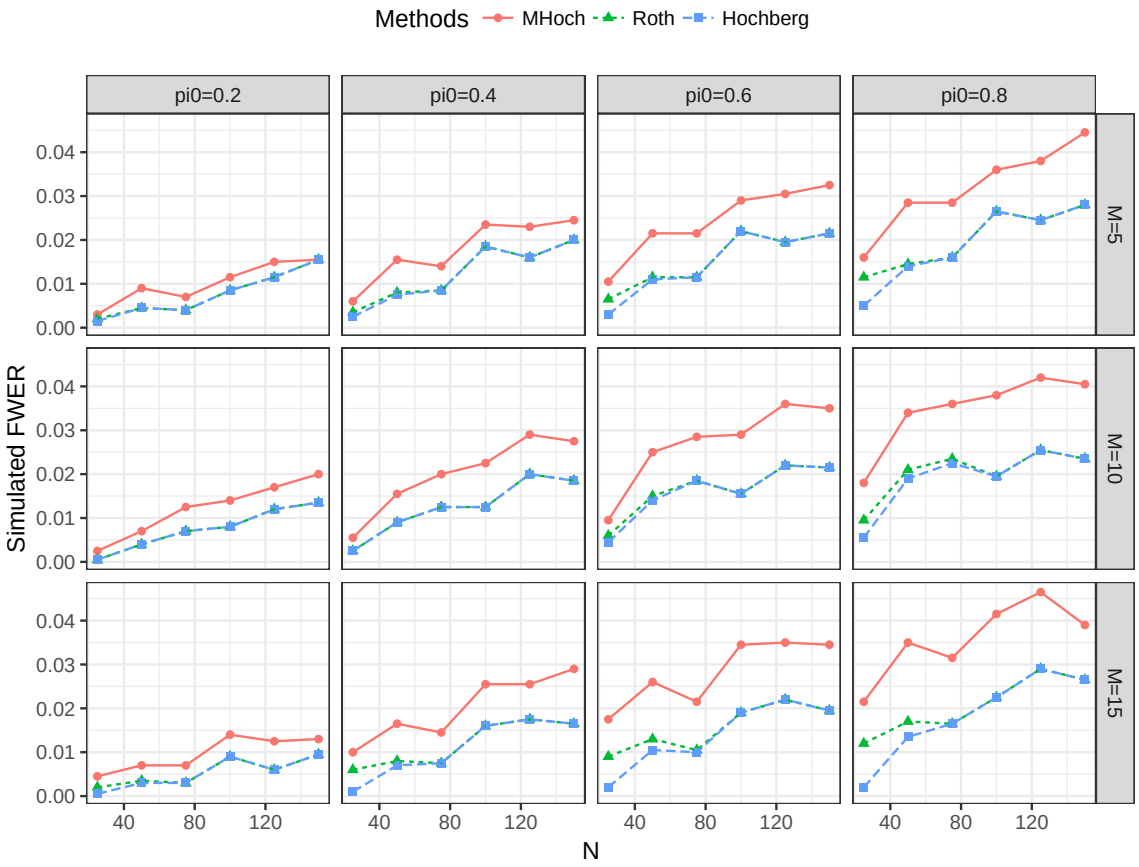

Figure S3: Simulated FWER comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

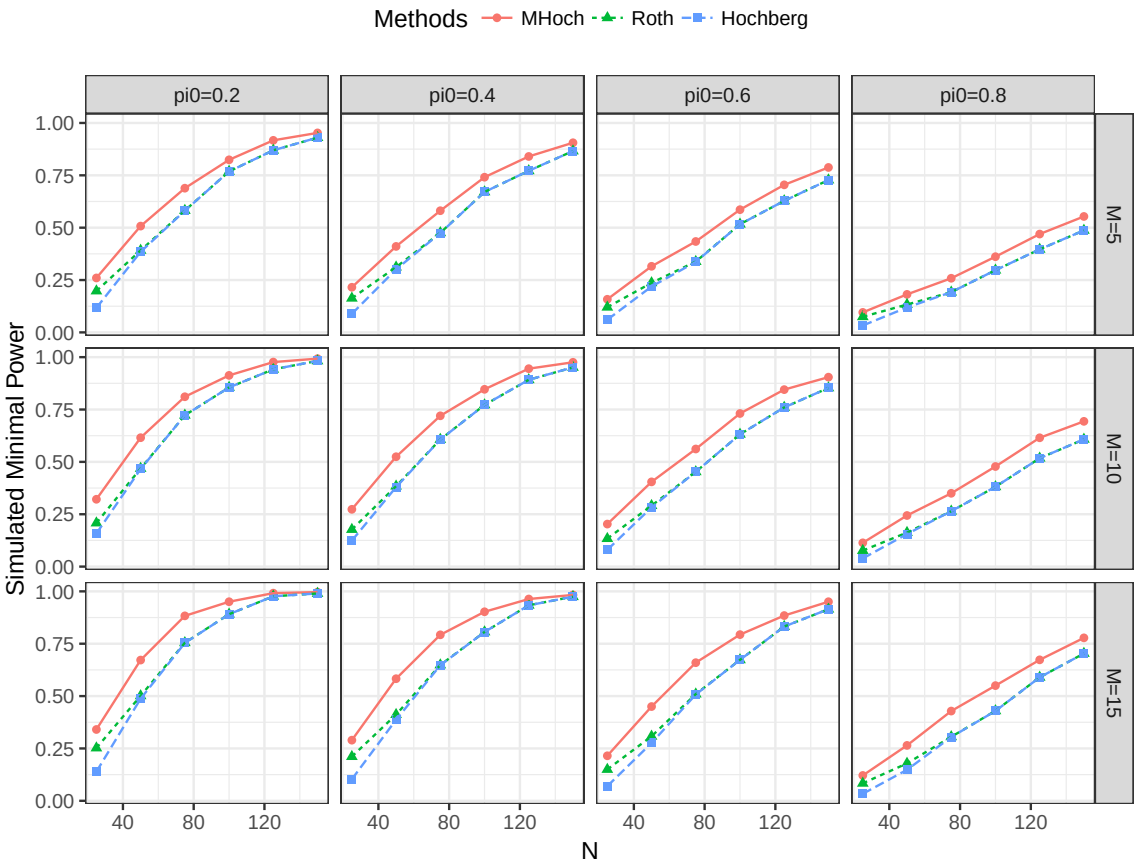

Figure S4: Simulated minimal power comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

S2 Results from Dependence Simulation Settings

In this section, we provide the details for simulating the block dependent binomial exact test (BET) statistics and the simulation results for the stepwise procedures comparisons. The following steps illustrate how to generate the dependent BET statistics and corresponding p-values.

Step 1. Generate dependent Poisson observed counts for each group

In order to generate m dependent BET statistics T_i, we use the following algorithm to generate m dependent Poisson random variables within each group, noting that the Poisson random variables between two groups are independent.

1. Let $\lambda_{i1} = 2$ for $i = 1, \ldots, m$, generate m independent Poisson random variable

$Y_{i1} \sim Poi((1-\rho)\lambda_{i1})$ and one $Y_{01} \sim Poi(2\rho)$.

2. Let $X_{i1} = Y_{i1} + Y_{01}$ for $i = 1, \ldots, m$, then $X_{i1} \sim Poi(2)$ and the correlation between X_{i1} and X_{j1} is $\dfrac{Cov(X_{i1}, X_{j1})}{\sqrt{Var(X_{i1})}\sqrt{Var(X_{j1})}} = \dfrac{Var(Y_{01})}{\sqrt{2}\sqrt{2}} = \dfrac{2\rho}{2} = \rho$ for $i, j = 1, \ldots, m$ and $i \neq j$.

3. Let $\lambda_{i2} = 2$ for $i = 1, \ldots, m_0$ and $\lambda_{i2} = 10$ for $i = m_0 + 1, \ldots, m$, generate m independent Poisson random variable $Y_{i2} \sim Poi((1-\rho)\lambda_{i2})$ for $i = 1, \ldots, m$, one $Y_{02} \sim Poi(2\rho)$, and one $Y'_{02} \sim Poi(10\rho)$.

4. Let $X_{i2} = Y_{i2} + Y_{02}$ for $i = 1, \ldots, m_0$ and $X_{i2} = Y_{i2} + Y'_{02}$ for $i = m_0 + 1, \ldots, m$, then $X_{i2} \sim Poi(2)$ for $i = 1, \ldots, m_0$ and $X_{i2} \sim Poi(10)$ for $i = m_0 + 1, \ldots, m$. For $i, j = 1, \ldots, m_0$ and $i \neq j$, the correlation between X_{i2} and X_{j2} is $\dfrac{Cov(X_{i2}, X_{j2})}{\sqrt{Var(X_{i2})}\sqrt{Var(X_{j2})}} = \dfrac{Var(Y_{02})}{\sqrt{2}\sqrt{2}} = \dfrac{2\rho}{2} = \rho$. Similarly, for $i, j = m_0 + 1, \ldots, m$ and $i \neq j$, the correlation between X_{i2} and X_{j2} is also equal to ρ; for $i = 1, \ldots, m_0$ and $j = m_0 + 1, \ldots, m$, the correlation between X_{i2} and X_{j2} is equal to zero.

Step 2. Obtain the conditional test statistics

Since the generated Poisson random variables between two groups are independent, we can directly conduct BET for each hypothesis. After generating Poisson observed counts x_{i1} and x_{i2}, let $c_i = x_{i1} + x_{i2}$ be the total observed count for two groups. Then the test statistics T_i is conditional test statistics X_{i1} given $X_{i1} + X_{i2} = c_i$ and the critical value is the observed count x_{i1} for Group 1.

Step 3. Conditional distribution of the test statistics

Based on the conditional inference in Lehmann and Romano [1], which is the BET in our paper, the conditional distribution of X_{i1} given $X_{i1} + X_{i2} = c_i$ is Binomial, $Bin(c_i, p_i)$, where $p_i = \dfrac{\lambda_{i1}}{\lambda_{i1} + \lambda_{i2}}$.

Step 4. Calculate available p-value P_i and attainable p-values

When H_i is true, i.e., $\lambda_{i1} = \lambda_{i2}$, $p_i = 0.5$. Thus, $X_{i1}|X_{i1} + X_{i2} = c_i \sim Bin(c_i, 0.5)$ under H_i. Therefore, the available conditional p-value for H_i can be calculated by

$$\begin{aligned} P_i &= \Pr_{H_i}\{X_{i1} \geq x_{i1} | X_{i1} + X_{i2} = c_i\} \\ &= \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^j (1-0.5)^{c_i - j} \\ &= \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^{c_i}. \end{aligned} \tag{1}$$

The corresponding attainable p-values can be calculated by

$$\Pr_{H_i}\{X_{i1} \geq x | X_{i1} + X_{i2} = c_i\} = \sum_{j=x}^{c_i} \binom{c_i}{j} 0.5^{c_i} \quad \text{for } x = 0, 1, \ldots, c_i. \tag{2}$$

The simulation results under the above simulation setting for stepwise procedures comparisons are shown in Tables S9 - S14 and Figures S5 - S8. It is easy to see that in such block dependence simulation setting, the p-values calculated based on the Poisson outcomes satisfies the PRDS Assumption 2.2, since $\rho \geq 0$ and the tests are one-sided.

R-package for MHTdiscrete: R-package MHTdiscrete [3] contains R code to implement our proposed methods and several existing FWER controlling procedures for discrete data, which are described in this paper. The package can be downloaded from `https://cran.r-project.org/web/packages/MHTdiscrete`.

Web Application for MHTdiscrete: A web application containing the proposed procedures and several comparable procedures can be accessed at `https://allen.shinyapps.io/MTPs`.

References

[1] Lehmann, E. L. and Romano, J. P. (2005). *Testing Statistical Hypotheses, 3rd Edition*. Springer.

[2] R Development Core Team (2018). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing.

[3] Zhu, Y. and Guo, W. (2017). *MHTdiscrete: Multiple Hypotheses Testing for Discrete Data*. R package version 1.0.0.

Table S9: Simulated FWER comparisons for single-step procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$	MBonf	0.0045	0.0050	0.0035	0.0045	0.0025	0.0055	0.0025	0.0080	0.0045	0.0040
	Tarone	0.0015	0.0010	0.0005	0.0020	0.0010	0.0030	0.0005	0.0040	0.0010	0.0005
$\pi_0 = 0.4$	Bonf	0.0015	0.0010	0.0005	0.0015	0.0010	0.0025	0.0005	0.0035	0.0000	0.0005
	Sidak	0.0015	0.0010	0.0005	0.0015	0.0010	0.0025	0.0005	0.0035	0.0000	0.0005
$m = 5$	MBonf	0.0100	0.0055	0.0080	0.0085	0.0080	0.0050	0.0095	0.0070	0.0045	0.0040
	Tarone	0.0060	0.0025	0.0060	0.0050	0.0050	0.0045	0.0045	0.0055	0.0040	0.0025
$\pi_0 = 0.6$	Bonf	0.0025	0.0015	0.0020	0.0035	0.0020	0.0010	0.0010	0.0000	0.0020	0.0010
	Sidak	0.0025	0.0015	0.0020	0.0035	0.0020	0.0010	0.0010	0.0000	0.0020	0.0010
$m = 5$	MBonf	0.0125	0.0145	0.0155	0.0160	0.0115	0.0135	0.0110	0.0060	0.0105	0.0045
	Tarone	0.0090	0.0105	0.0065	0.0095	0.0080	0.0085	0.0070	0.0045	0.0065	0.0030
$\pi_0 = 0.8$	Bonf	0.0035	0.0005	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
	Sidak	0.0035	0.0005	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
$m = 10$	MBonf	0.0035	0.0030	0.0025	0.0035	0.0030	0.0025	0.0025	0.0030	0.0025	0.0035
	Tarone	0.0005	0.0010	0.0010	0.0020	0.0010	0.0010	0.0010	0.0005	0.0010	0.0010
$\pi_0 = 0.4$	Bonf	0.0005	0.0005	0.0010	0.0005	0.0010	0.0010	0.0000	0.0005	0.0005	0.0005
	Sidak	0.0005	0.0005	0.0010	0.0005	0.0010	0.0010	0.0000	0.0005	0.0005	0.0005
$m = 10$	MBonf	0.0080	0.0065	0.0095	0.0095	0.0065	0.0035	0.0025	0.0055	0.0070	0.0030
	Tarone	0.0020	0.0030	0.0060	0.0050	0.0020	0.0020	0.0010	0.0030	0.0030	0.0025
$\pi_0 = 0.6$	Bonf	0.0000	0.0015	0.0020	0.0015	0.0000	0.0010	0.0005	0.0010	0.0010	0.0005
	Sidak	0.0000	0.0015	0.0020	0.0015	0.0000	0.0010	0.0005	0.0010	0.0010	0.0005
$m = 10$	MBonf	0.0185	0.0105	0.0115	0.0135	0.0135	0.0150	0.0100	0.0090	0.0050	0.0085
	Tarone	0.0120	0.0080	0.0075	0.0095	0.0075	0.0100	0.0065	0.0030	0.0040	0.0045
$\pi_0 = 0.8$	Bonf	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010
	Sidak	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010

Table S10: Simulated minimal power comparisons for single-step procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$	MBonf	0.9095	0.8880	0.8580	0.8455	0.8135	0.7745	0.7545	0.7295	0.6840	0.6305
	Tarone	0.8500	0.8350	0.8000	0.7945	0.7585	0.7070	0.6815	0.6630	0.6130	0.5605
$\pi_0 = 0.4$	Bonf	0.7530	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4685
	Sidak	0.7530	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4685
$m = 5$	MBonf	0.8150	0.8020	0.7755	0.7740	0.7655	0.7195	0.7255	0.6790	0.6750	0.6260
	Tarone	0.7635	0.7435	0.7210	0.7075	0.7185	0.6775	0.6815	0.6425	0.6255	0.5865
$\pi_0 = 0.6$	Bonf	0.6135	0.5985	0.5740	0.5615	0.5725	0.5410	0.5270	0.5070	0.4715	0.4365
	Sidak	0.6135	0.5985	0.5740	0.5615	0.5725	0.5410	0.5270	0.5070	0.4715	0.4365
$m = 5$	MBonf	0.5965	0.6055	0.5955	0.5925	0.6075	0.6095	0.5960	0.5960	0.6075	0.6120
	Tarone	0.5635	0.5730	0.5675	0.5600	0.5755	0.5880	0.5730	0.5730	0.5935	0.5985
$\pi_0 = 0.8$	Bonf	0.3825	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
	Sidak	0.3825	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
$m = 10$	MBonf	0.9760	0.9460	0.9175	0.8925	0.8570	0.8250	0.7885	0.7270	0.6895	0.6090
	Tarone	0.9470	0.8940	0.8535	0.8295	0.7875	0.7585	0.7120	0.6525	0.6040	0.5260
$\pi_0 = 0.4$	Bonf	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
	Sidak	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
$m = 10$	MBonf	0.9250	0.9125	0.8845	0.8425	0.8300	0.7920	0.7470	0.7160	0.6590	0.6180
	Tarone	0.8820	0.8630	0.8370	0.7705	0.7680	0.7285	0.6925	0.6645	0.6090	0.5745
$\pi_0 = 0.6$	Bonf	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
	Sidak	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
$m = 10$	MBonf	0.7675	0.7595	0.7390	0.7330	0.7320	0.6975	0.6865	0.6665	0.6340	0.6160
	Tarone	0.7145	0.7055	0.6910	0.6860	0.6885	0.6540	0.6445	0.6310	0.5925	0.5875
$\pi_0 = 0.8$	Bonf	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495
	Sidak	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

Table S11: Simulated FWER comparisons for step-down procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHolm	0.0085	0.0090	0.0070	0.0090	0.0105	0.0120	0.0095	0.0150	0.0115	0.0055
$m = 5$	TH	0.0055	0.0060	0.0040	0.0050	0.0065	0.0100	0.0040	0.0130	0.0095	0.0045
$\pi_0 = 0.4$	Holm	0.0025	0.0040	0.0035	0.0035	0.0035	0.0055	0.0015	0.0075	0.0040	0.0035
	MHolm	0.0170	0.0095	0.0125	0.0140	0.0125	0.0095	0.0160	0.0150	0.0090	0.0065
$m = 5$	TH	0.0110	0.0065	0.0095	0.0095	0.0090	0.0070	0.0115	0.0125	0.0075	0.0055
$\pi_0 = 0.6$	Holm	0.0030	0.0020	0.0035	0.0055	0.0040	0.0030	0.0045	0.0025	0.0040	0.0030
	MHolm	0.0160	0.0205	0.0200	0.0190	0.0170	0.0175	0.0135	0.0075	0.0120	0.0070
$m = 5$	TH	0.0110	0.0145	0.0120	0.0150	0.0120	0.0120	0.0115	0.0065	0.0100	0.0060
$\pi_0 = 0.8$	Holm	0.0035	0.0015	0.0020	0.0015	0.0025	0.0025	0.0025	0.0010	0.0025	0.0000
	MHolm	0.0130	0.0150	0.0115	0.0095	0.0090	0.0100	0.0115	0.0130	0.0095	0.0090
$m = 10$	TH	0.0060	0.0030	0.0075	0.0040	0.0040	0.0075	0.0080	0.0105	0.0070	0.0075
$\pi_0 = 0.4$	Holm	0.0005	0.0005	0.0015	0.0005	0.0015	0.0015	0.0010	0.0030	0.0010	0.0010
	MHolm	0.0160	0.0150	0.0185	0.0165	0.0175	0.0105	0.0125	0.0130	0.0140	0.0055
$m = 10$	TH	0.0055	0.0085	0.0115	0.0100	0.0125	0.0075	0.0065	0.0100	0.0125	0.0045
$\pi_0 = 0.6$	Holm	0.0000	0.0020	0.0020	0.0025	0.0000	0.0015	0.0005	0.0015	0.0025	0.0005
	MHolm	0.0230	0.0160	0.0195	0.0195	0.0200	0.0215	0.0145	0.0120	0.0090	0.0130
$m = 10$	TH	0.0160	0.0130	0.0145	0.0130	0.0145	0.0165	0.0130	0.0100	0.0075	0.0120
$\pi_0 = 0.8$	Holm	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0010

Table S12: Simulated minimal power comparisons for step-down procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHolm	0.9095	0.8880	0.8580	0.8455	0.8135	0.7745	0.7545	0.7300	0.6840	0.6305
$m = 5$	TH	0.8500	0.8350	0.8000	0.7945	0.7585	0.7070	0.6815	0.6630	0.6130	0.5605
$\pi_0 = 0.4$	Holm	0.7535	0.7465	0.7010	0.6920	0.6750	0.6205	0.5930	0.5675	0.5260	0.4690
	MHolm	0.8155	0.8020	0.7755	0.7740	0.7655	0.7195	0.7255	0.6795	0.6750	0.6260
$m = 5$	TH	0.7640	0.7440	0.7210	0.7080	0.7185	0.6775	0.6815	0.6425	0.6255	0.5865
$\pi_0 = 0.6$	Holm	0.6135	0.5985	0.5750	0.5615	0.5725	0.5415	0.5270	0.5070	0.4715	0.4365
	MHolm	0.5980	0.6060	0.5975	0.5930	0.6080	0.6095	0.5965	0.5960	0.6080	0.6125
$m = 5$	TH	0.5640	0.5740	0.5685	0.5600	0.5755	0.5880	0.5735	0.5730	0.5940	0.5990
$\pi_0 = 0.8$	Holm	0.3830	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3810
	MHolm	0.9760	0.9460	0.9175	0.8925	0.8570	0.8250	0.7885	0.7270	0.6895	0.6090
$m = 10$	TH	0.9470	0.8940	0.8535	0.8295	0.7875	0.7585	0.7120	0.6525	0.6040	0.5260
$\pi_0 = 0.4$	Holm	0.8805	0.8260	0.7625	0.7500	0.6845	0.6695	0.6075	0.5550	0.5045	0.4410
	MHolm	0.9265	0.9125	0.8845	0.8425	0.8300	0.7920	0.7470	0.7160	0.6590	0.6180
$m = 10$	TH	0.8820	0.8630	0.8380	0.7705	0.7680	0.7285	0.6925	0.6645	0.6090	0.5745
$\pi_0 = 0.6$	Holm	0.7420	0.7260	0.7030	0.6220	0.6285	0.5710	0.5425	0.4995	0.4440	0.4155
	MHolm	0.7680	0.7600	0.7390	0.7330	0.7325	0.6975	0.6870	0.6665	0.6340	0.6160
$m = 10$	TH	0.7165	0.7060	0.6925	0.6865	0.6895	0.6540	0.6450	0.6310	0.5925	0.5875
$\pi_0 = 0.8$	Holm	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

Table S13: Simulated FWER comparisons for step-up procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHoch	0.0085	0.0090	0.0070	0.0090	0.0105	0.0120	0.0110	0.0175	0.0130	0.0080
$m = 5$	Roth	0.0060	0.0070	0.0050	0.0065	0.0075	0.0095	0.0060	0.0140	0.0105	0.0070
$\pi_0 = 0.4$	Hoch	0.0025	0.0040	0.0035	0.0035	0.0035	0.0060	0.0020	0.0090	0.0055	0.0060
	MHoch	0.0170	0.0100	0.0125	0.0160	0.0130	0.0105	0.0165	0.0170	0.0120	0.0120
$m = 5$	Roth	0.0120	0.0075	0.0095	0.0110	0.0085	0.0070	0.0125	0.0140	0.0095	0.0105
$\pi_0 = 0.6$	Hoch	0.0030	0.0020	0.0035	0.0055	0.0040	0.0030	0.0050	0.0030	0.0050	0.0075
	MHoch	0.0165	0.0210	0.0200	0.0200	0.0170	0.0180	0.0135	0.0095	0.0140	0.0115
$m = 5$	Roth	0.0110	0.0145	0.0130	0.0150	0.0115	0.0130	0.0100	0.0075	0.0120	0.0085
$\pi_0 = 0.8$	Hoch	0.0035	0.0015	0.0020	0.0015	0.0025	0.0025	0.0030	0.0010	0.0030	0.0035
	MHoch	0.0145	0.0165	0.0115	0.0100	0.0105	0.0105	0.0135	0.0155	0.0110	0.0125
$m = 10$	Roth	0.0075	0.0045	0.0080	0.0035	0.0045	0.0075	0.0095	0.0120	0.0080	0.0115
$\pi_0 = 0.4$	Hoch	0.0005	0.0005	0.0015	0.0010	0.0015	0.0015	0.0015	0.0035	0.0025	0.0060
	MHoch	0.0165	0.0150	0.0185	0.0165	0.0185	0.0115	0.0150	0.0155	0.0190	0.0085
$m = 10$	Roth	0.0060	0.0070	0.0110	0.0100	0.0110	0.0070	0.0070	0.0105	0.0135	0.0070
$\pi_0 = 0.6$	Hoch	0.0000	0.0020	0.0020	0.0025	0.0005	0.0015	0.0010	0.0020	0.0045	0.0015
	MHoch	0.0235	0.0170	0.0205	0.0200	0.0210	0.0225	0.0165	0.0140	0.0105	0.0170
$m = 10$	Roth	0.0145	0.0120	0.0125	0.0130	0.0155	0.0155	0.0130	0.0075	0.0080	0.0110
$\pi_0 = 0.8$	Hoch	0.0005	0.0005	0.0005	0.0010	0.0010	0.0000	0.0020	0.0005	0.0010	0.0035

Table S14: Simulated minimal power comparisons for step-up procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHoch	0.9110	0.8905	0.8595	0.8490	0.8155	0.7780	0.7595	0.7370	0.6985	0.6525
$m = 5$	Roth	0.8565	0.8430	0.8075	0.8020	0.7640	0.7200	0.6945	0.6755	0.6300	0.5955
$\pi_0 = 0.4$	Hoch	0.7640	0.7555	0.7105	0.7025	0.6835	0.6310	0.6050	0.5860	0.5435	0.4945
	MHoch	0.8165	0.8065	0.7785	0.7775	0.7690	0.7255	0.7290	0.6880	0.6835	0.6425
$m = 5$	Roth	0.7670	0.7515	0.7245	0.7115	0.7250	0.6845	0.6840	0.6505	0.6325	0.5990
$\pi_0 = 0.6$	Hoch	0.6210	0.6045	0.5780	0.5665	0.5800	0.5460	0.5345	0.5140	0.4845	0.4530
	MHoch	0.5985	0.6065	0.5975	0.5940	0.6080	0.6095	0.5965	0.5965	0.6085	0.6130
$m = 5$	Roth	0.5490	0.5590	0.5520	0.5490	0.5610	0.5755	0.5560	0.5570	0.5775	0.5800
$\pi_0 = 0.8$	Hoch	0.3830	0.3845	0.3805	0.3760	0.3875	0.4000	0.3690	0.3735	0.3820	0.3815
	MHoch	0.9770	0.9465	0.9205	0.8945	0.8630	0.8300	0.7925	0.7380	0.7055	0.6305
$m = 10$	Roth	0.9495	0.8980	0.8595	0.8325	0.7910	0.7610	0.7190	0.6605	0.6190	0.5625
$\pi_0 = 0.4$	Hoch	0.8815	0.8260	0.7630	0.7520	0.6850	0.6705	0.6095	0.5565	0.5075	0.4485
	MHoch	0.9285	0.9155	0.8855	0.8475	0.8340	0.7985	0.7540	0.7235	0.6735	0.6405
$m = 10$	Roth	0.8870	0.8650	0.8425	0.7760	0.7735	0.7335	0.7000	0.6700	0.6145	0.5875
$\pi_0 = 0.6$	Hoch	0.7425	0.7260	0.7030	0.6225	0.6290	0.5710	0.5430	0.5000	0.4465	0.4205
	MHoch	0.7725	0.7625	0.7400	0.7380	0.7350	0.6995	0.6890	0.6710	0.6400	0.6285
$m = 10$	Roth	0.7200	0.7055	0.6945	0.6895	0.6905	0.6515	0.6460	0.6365	0.5970	0.5965
$\pi_0 = 0.8$	Hoch	0.4935	0.4880	0.4655	0.4710	0.4630	0.4235	0.4145	0.3975	0.3725	0.3495

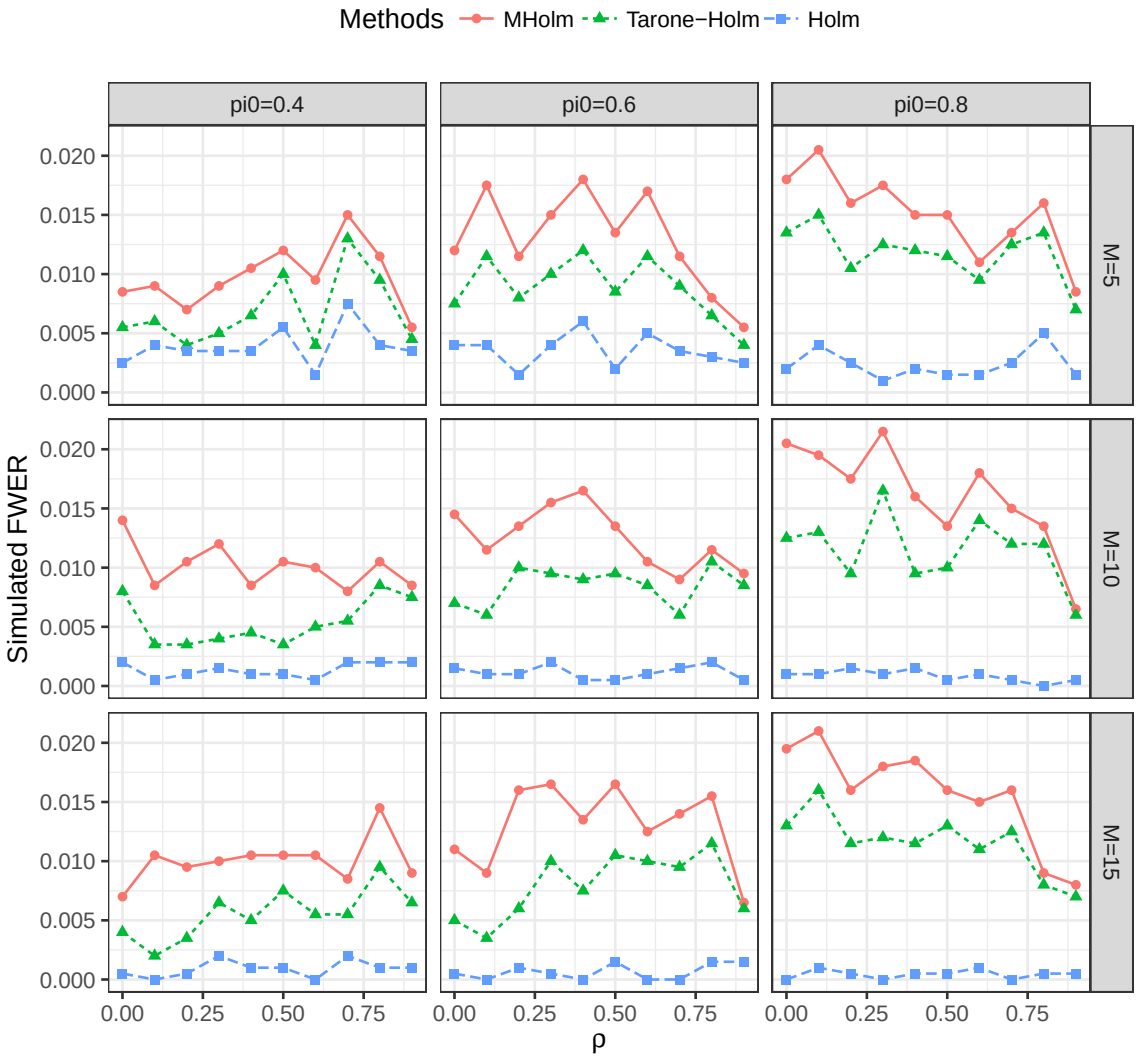

Figure S5: Simulated FWER comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

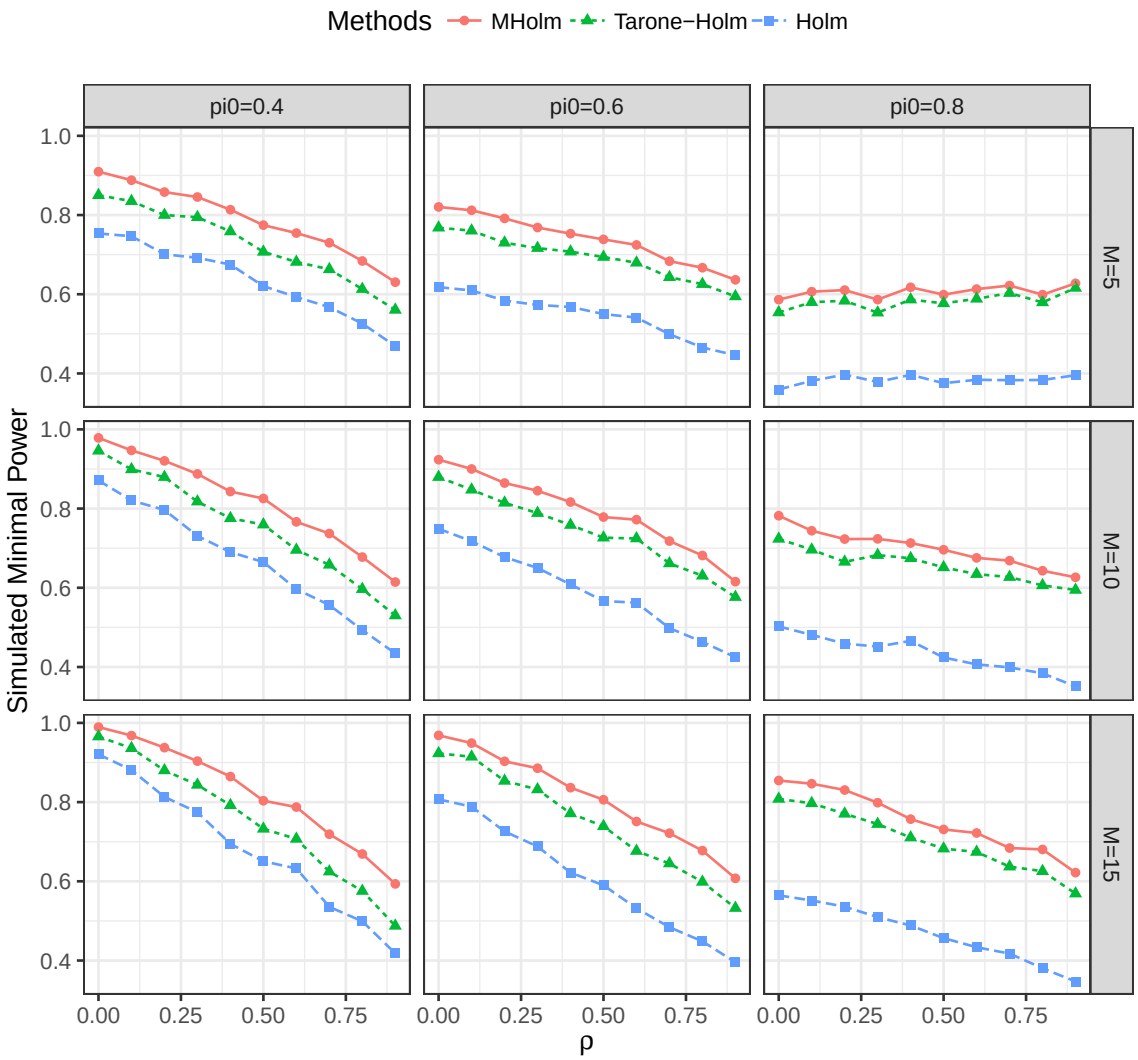

Figure S6: Simulated minimal power comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

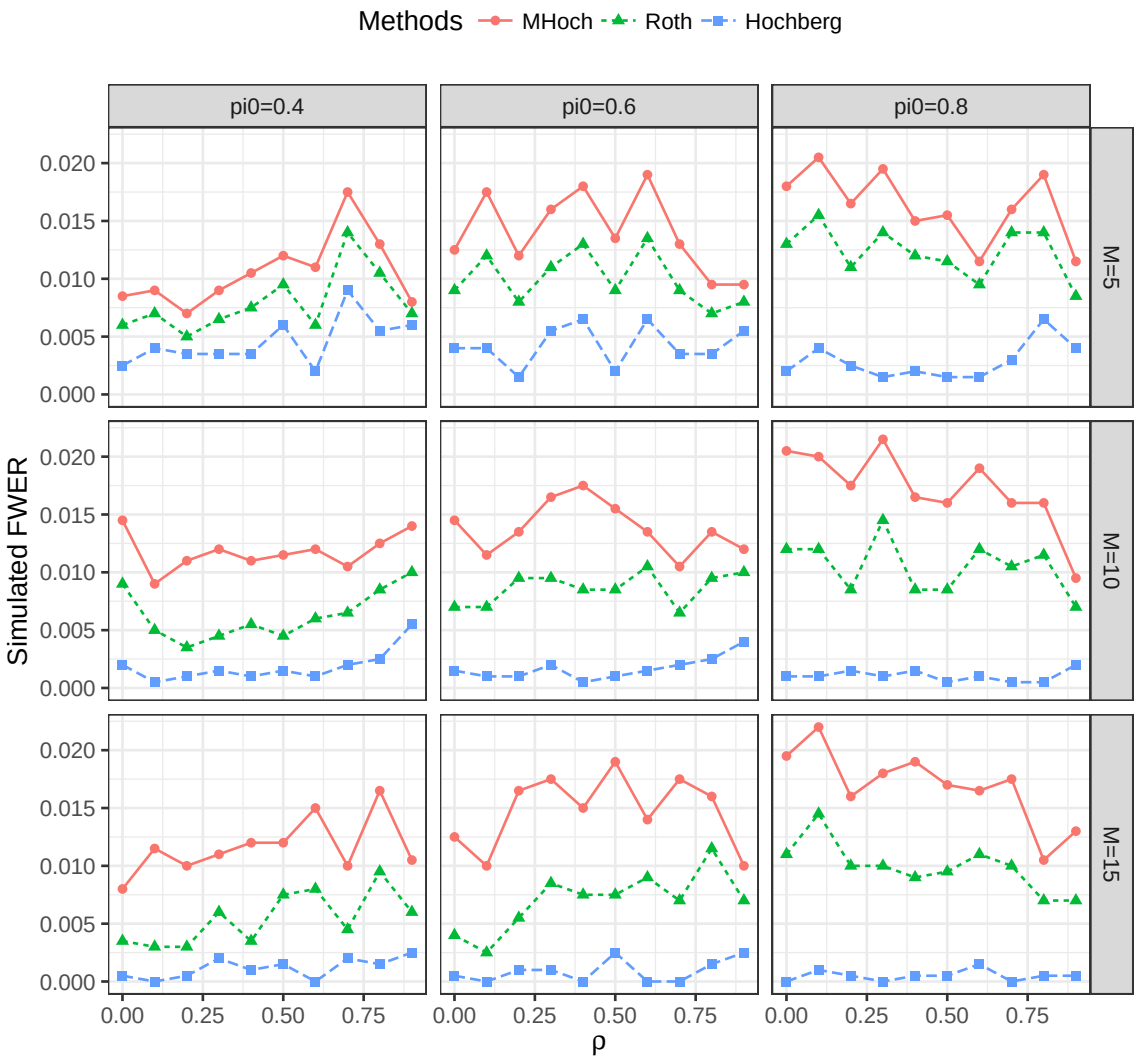

Figure S7: Simulated FWER comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

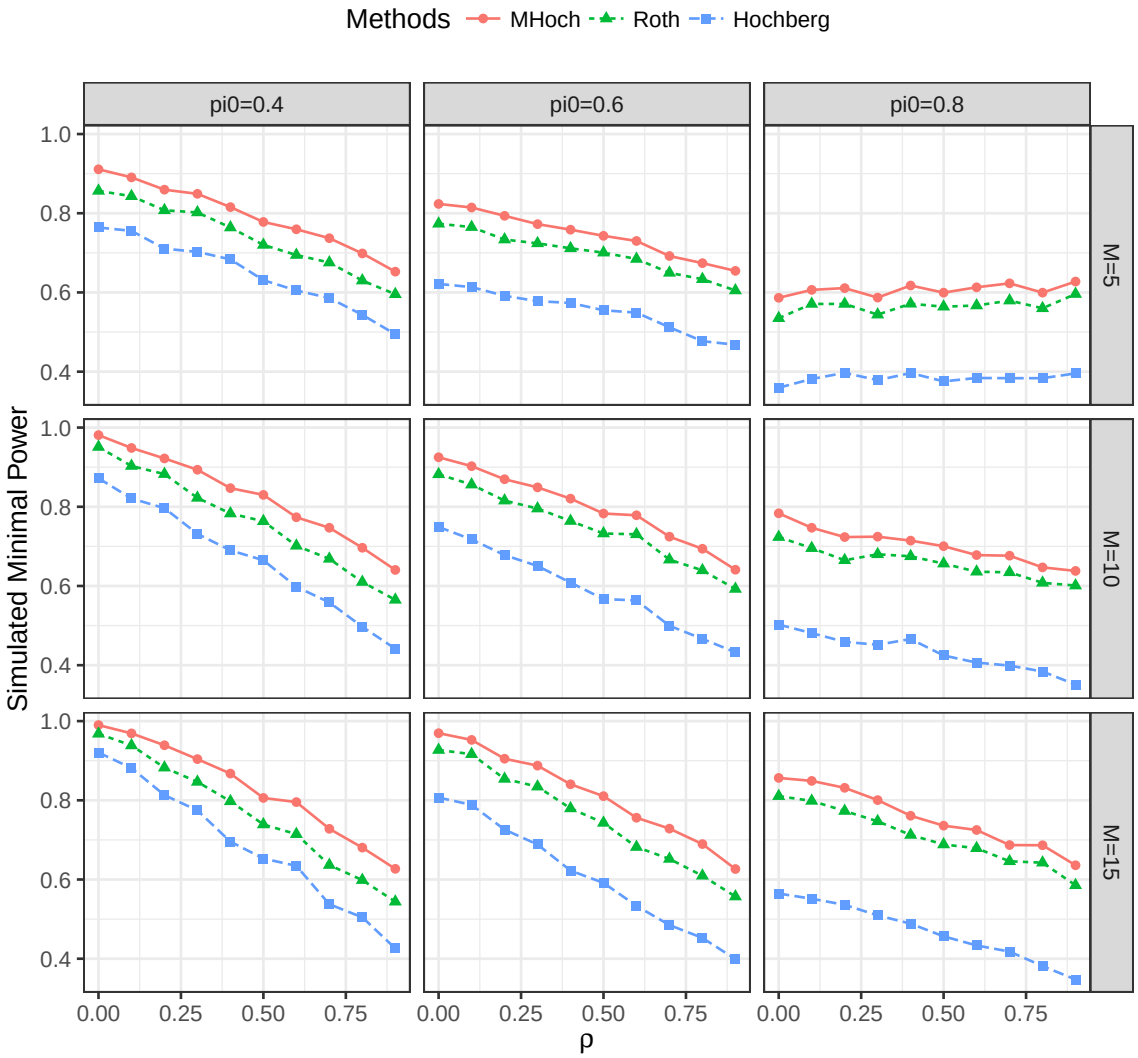

Figure S8: Simulated minimal power comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).