Math 222 Exam 1, September 28, 2016

Read each problem carefully. Show all your work. Turn off your phones. No calculators!

1. Consider the differential equation $y' = -y(y+1)(y-4)$.

   (a) (3 points) Find the equilibrium solution(s) ($y' = 0$).

   (b) (3 points) Sketch the direction field. (4 points) Sketch the solution $y(t)$ with initial condition $y(0) = -1$. Sketch another solution $y(t)$ with initial condition $y(1) = 3$.

2. (a) (4 points) Determine the order of the differential equation and state whether the equation is linear/nonlinear:

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \ln(xy).$$

   (b) (6 points) Verify that each given function (y_1 and y_2) is a solution of the differential equation: $y'' + 4y = 0, \quad t > 0; \quad y_1(t) = \cos(2t), \quad y_2(t) = -\sin(2t)$.

3. (a) (15 points) Find the solution of the IVP: $\frac{dy}{dx} = 2(1+x)(1-y^2), \quad y(0) = -2$. Express the solution in terms of an explicit function of x. Describe the behavior of the solution as x increases from zero.

   (b) (15 points) Find the solution of the IVP: $y' - y = -\frac{e^t}{2}, \quad y(0) = y_0$. Find y_0 such that the solution has a critical point at $t = 2$.

4. (15 points) A tank with a capacity of 9 liters initially contains 10 grams of salt and 1 liter of water. Water containing 1 g/L of salt enters the tank at a rate of 4 L/hour, and the well-mixed solution flows out of the tank at a rate of 2 L/hour. Find the amount of salt in the tank at any time prior to the instant when the tank is overflowing. What is the salt concentration at the point of overflowing?

5. (10 points) Consider the initial value problem: $y' = y^2 - t^2$, $y(0) = 1$. Use the Euler method to find an approximate value of the solution at $t = 0.3$ with a time step $h = 0.1$.

6. (a) (15 points) Find a linear homogeneous differential equation with constant coefficients that would have $r_1 = -1$ and $r_2 = 3$ for the roots of its characteristic equation. Write down the general solution for this differential equation, and let this general solution satisfy $y(0) = \alpha$ and $y'(0) = \beta$. Find conditions on α and β such that the solution will remain bounded as $t \to \infty$.

   (b) (10 points) Solve the IVP: $y'' - 4y = 0$, $y(0) = 1$, $y'(0) = 0$. Sketch the solution $y(t)$.