Math 222, Fall 2016.

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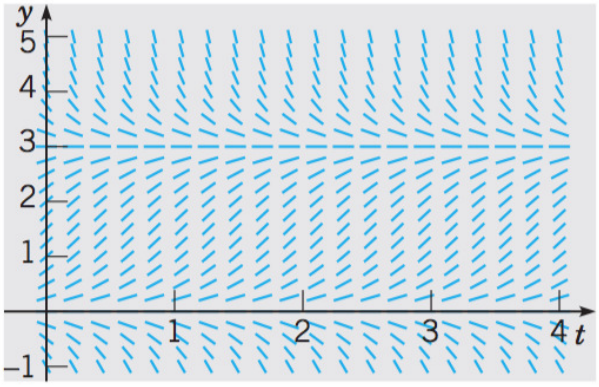
Extra Homework Problems for week 1

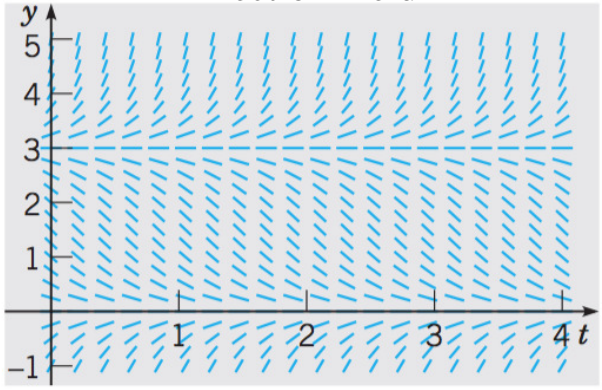
1. Consider the following list of differential equations. Identify the differential equation that corresponds to the given direction field.

$$(a) y' = y(3-y), \quad (b) y' = 2y(y-3).$$

Direction Field 1



Direction Field 2



2. Draw a direction field for $y' = y(3-y) - 2$. Based on your direction field, determine the behavior of y as $t \to \infty$ and its dependence on the initial value of y at $t = 0$. For example, what is the behavior of y as $t \to \infty$ when $y(t=0) = 1/2$? $y(t=0) = 3/2$? $y(t=0) = 3$?

3. Determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

$$(a) y^2 y' = t, \quad (b) yy'' = t, \quad (c) y'' - 2ty' + t^2 y = 2.$$

4. Use the integrating factor to solve the following initial value problem

$$y' = -y + be^{-t}, \quad y(0) = 0$$

with b a constant coefficient. Show that the solution will reach a maximum at $t = 1$ regardless of the value of b. If the maximum of the solution is 2, what should b be?

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Extra Homework Problems for week 2

1. Use the integrating factor to solve the following initial value problem for $t > 0$

$$ty' + (t+1)y = 2te^{-t}, \quad y(1) = a$$

   with a a constant coefficient. Show that the solution $y \to 0$ as $t \to \infty$ regardless of the value of a. If y touches the time axis at $t = 2$, what should a be? Instead if the solution has a critical point at $t = 1/2$, what should a be?

2. Problems 27 and 28 on page 49 (10th edition).

3. Two tanks are connected with solution pouring from Tank 1 to Tank 2. At $t = 0$, Tank 1 originally contains 100 gal of fresh water and Tank 2 originally contains 100 gal of salty water with 10 lb of salt. Then water containing 0.5 lb of salt per gallon is poured into Tank 1 at 1 gal/min, and the mixture is allowed to leave at 1 gal/min, pouring into Tank 2. In Tank 2 the mixture is leaving also at 1 gal/min. First find the total amount of salt in Tank 1. Then find the total amount of salt in Tank 2 at any time.

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Extra Homework Problems for week 3

1. Use Euler's method to find approximate values of the solution of the given initial value problem at $t = 0.2$.

$$y' = 2y - 1, \quad y(0) = 1,$$

with $h = 0.1$, 0.05, and 0.01. First calculate the exact solution using the integrating factor, and then compute the differences between the approximate values and the exact solution at $t = 0.2$. (Such difference is the error of the approximate solution from the Euler's method.) Show that the error decreases with h.

2. Use the characteristic equation to solve for the following linear differential equation with constant coefficients.

$$(a) ay' + by = 0, \;\; a \neq 0, \;\; y(0) = 1. \;\; (b) y'' + 8y' - 9y = 0, \;\; y(1) = 0, \;\; y'(1) = 1.$$

3. Problem 25 on page 144 (10th edition).

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Extra Homework Problems for week 4 and week 5

1. Use Abel's theorem to find the Wronskian for the differential equation $ty'' + 2y' + te^t y = 0$ for $t > 0$.
2. Problem 25 on page 164 (10th edition).
3. Problem 26 on page 165 (10th edition).
4. Problem 18 on page 173 (10th edition).
5. Problem 20 on page 173 (10th edition).

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Extra Homework Problems for week 6 and week 7

1. Use the method of undetermined coefficients to find the solution of the given initial value problem.
$$y'' + y = t(1 + \sin t), \quad y(0) = 0, \quad y'(0) = 0.$$

2. Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.
$$x^2y'' - 3xy' + 4y = x^2 \ln x, \quad x > 0; \quad y_1(x) = x^2, \quad y_2(x) = x^2 \ln x.$$
Finally find the solution for the initial values $y(1) = 0$ and $y'(1) = 0$.

3. The position of a certain spring-mass system satisfies the initial value problem
$$u'' + \frac{1}{4}u' + 2u = 0, \quad u(0) = 0, \quad u'(0) = 2.$$
Find the time T such that the oscillation amplitude at T first decreases below less than $1/2$ of the initial oscillation amplitude.

4. Problem 17 on page 218 (10th edition).

5. Problem 18 on page 218 (10th edition).

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Extra Homework Problems for week 8

1. Problem 22 on page 264 (10th edition).
2. Problem 35 on page 280 (10th edition).
3. Problem 36 on page 280 (10th edition).
4. Problem 37 on page 281 (10th edition).

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Extra Homework Problems for week 9 and week 10

1. Problem 11 on page 286 (10th edition).
2. Use Laplace transform to solve the initial value problem

$$y' + ay = e^{\lambda t}, \quad y(0) = c,$$

   with $a \neq 0$. Discuss the case when $\lambda + a \neq 0$ and the case when $\lambda + a = 0$.
3. Problem 25 on page 333 and 334 (10th edition).
4. Problem 32 on page 334 (10th edition).
5. Problem 16 on page 341 (10th edition).
6. Problem 14 on page 348 (10th edition).
7. Problem 15 on page 348 and page 349 (10th edition).

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Extra Homework Problems for week 11

1. Problem 13 on page 355 (10th edition).
2. Problem 20 on page 355 (10th edition).
3. Problem 22 on page 367 (10th edition).

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Extra Homework Problems for week 12

1. Problem 19 on page 405 and Problem 20 on page 406 (10th edition).
2. Problem 31 on page 407 (10th edition).

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Extra Homework Problems for week 13

1. Problem 21 on page 418 (10th edition).
2. Problem 22 on page 596 (10th edition).
3. Problem 29 on page 606 (10th edition).
4. Find the eigenvalues λ and eigenfunctions of the given boundary value problem (for a positive constant L)
$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(L) = 0.$$

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Extra Homework Problems for week 14

1. Problem 13 on page 612 (10th edition).

2. A function $f(x)$ is defined for $-\pi \leq x < \pi$ as

$$f(x) \quad = \quad \begin{cases} -\sin(x), & -\pi \leq x < 0, \\ \sin(x), & 0 \leq x < \pi, \end{cases} \tag{1}$$

   with $f(x + 2\pi) = f(x)$.

   (a) Sketch three periods of this function.

   (b) Find the Fourier series of this function.