Solution to Extra Homework for Week 1

Problem 1: (a)

$$y' = y(3-y).$$

at equilibrium $y'=0$, $y(3-y)=0$, $y=0, 3$

$y<0$, $y(3-y) < 0$

$0<y<3$, $y(3-y) > 0$ $\Rightarrow$ $\boxed{\text{direction field 1}}$

$y>3$, $y(3-y) < 0$

(b) $y' = 2y(y-3)$

at equilibrium $y'=0$, $2y(y-3)=0$, $y=0, 3$

$y<0$, $y(y-3) > 0$

$0<y<3$, $y(y-3) < 0$ $\Rightarrow$ $\boxed{\text{direction field 2}}$

$y>3$, $y(y-3) > 0$

Problem 2: $y' = y(3-y) - 2$

at equilibrium $y'=0$, $y(3-y)-2=0$, $-y^2+3y-2=0$

$y^2-3y+2=0$, $(y-2)(y-1)=0$, $y=1, 2$

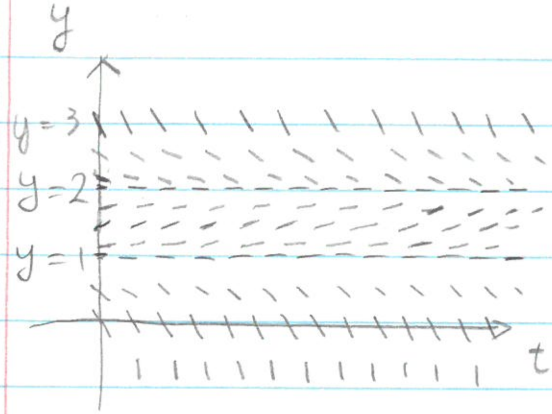
$y' = -(y-1)(y-2)$

$y<1$, $y' = -(y-1)(y-2) < 0$

$1<y<2$, $y' = -(y-1)(y-2) > 0$

$2<y$, $y' = -(y-1)(y-2) < 0$

p.02



$y=3, \quad y' = 3\cdot(3-3)-2 = -2 < 0$

$y = 3/2, \quad y' = \frac{3}{2}(3-\frac{3}{2}) - 2 = \frac{9}{4} - 2 = \frac{1}{4} > 0$

$y = 1/2, \quad y' = \frac{1}{2}(3-1/2) - 2 = \frac{5}{4} - 2 = -\frac{3}{4} < 0$

$y=0, \quad y' = 0(3) - 2 = -2 < 0$

If $y(0) > 2$, $\quad y \to 2$ as $t \to \infty$

$1 < y(0) < 2$, $\quad y \to 2$ as $t \to \infty$

$y(0) < 1$ $\quad y \to -\infty$ as $t \to \infty$

If $y(0) = 1$ $\quad$ y stay at 1 as $t \to \infty$

$y(0) = 2$, $\quad$ y stay at 2 as $t \to \infty$

Therefore if $y(0) = \frac{1}{2} < 1$, $y \to -\infty$ as $t \to \infty$.

$1 < y(0) = \frac{3}{2} < 2$, $y \to 2$ as $t \to \infty$.

Problem 3: (a) $y^2 y' = t$, $\quad$ 1st order non-linear diff. eq.

(b) $y'' - 2ty' + t^2 y = 2$, $\quad$ 2nd order, linear diff. eq.

Problem 4: $y' = -y + be^{-t}$, $\quad y(0) = 0$

$y' + y = be^{-t}$

$\mu y' + \mu y = b \cdot \mu \cdot e^{-t}$

$(\mu y)' = b\mu e^{-t}$ if $\mu' = \mu$, $\quad \mu = e^{t}$

$(e^t y)' = b$, $\quad e^t y = bt + c$, $\quad y = bte^{-t} + ce^{-t}$

P.03

$y(0) = 0, \quad c = 0, \qquad y = bte^{-t}$

$y' = -y + be^{-t}$

$= -bte^{-t} + be^{-t} = be^{-t}(1-t)$

$y' = 0$ at $t = 1$ regardless of values of b.

$y'(1) = 0, \quad y(1) = 2 = b \cdot 1 \cdot e^{-1}, \quad \boxed{b = 2e}$