Math 222, Fall 2016.

Present your work in an organized fashion. Make sure that your work is algebraically correct and logically sound. Show all your work. No calculator, notes, or books.

Quiz 09/12/2016 M222-001

1. Draw a direction field for the differential equation

$$y' = y(y-2)^2.$$

2. Solve the initial value problem

$$ty' + 2y = 4t^2, \quad y(1) = 4.$$

1. $y' = y(y-2)^2$, at equilibrium $y' = 0 = y(y-2)^2$, $y = 0, 2, 2$

$y < 0$, $y' = y(y-2)^2 < 0$
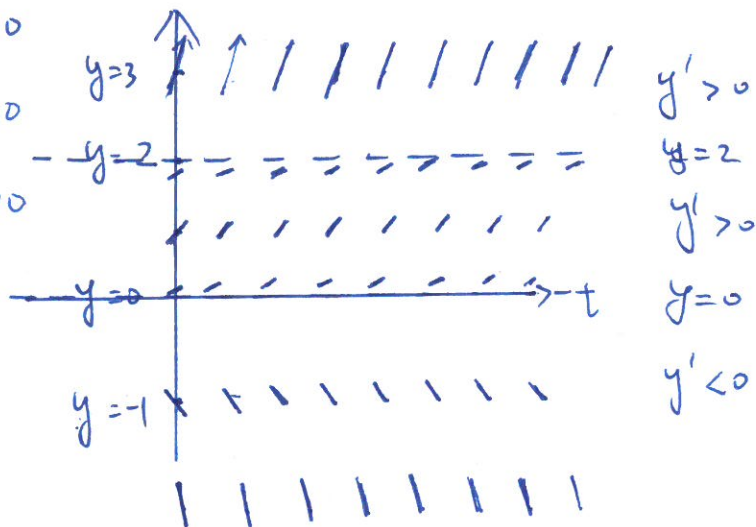
$0 < y < 2$, $y' = y(y-2)^2 > 0$

$2 < y$, $y' = y(y-2)^2 > 0$

$y = 1$, $y' = 1 \cdot (-1)^2 = 1 > 0$

$y = 3$, $y' = 3 \cdot 1^2 = 3 > 0$

2. $ty' + 2y = 4t^2$, $y(1) = 4$

$y' + \frac{2}{t}y = 4t$, $P = \frac{2}{t}$, $g = 4t$, $\mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$y = \frac{\int 4t \cdot t^2 dt + c}{t^2} = \frac{t^4 + c}{t^2} = t^2 + c \cdot t^{-2}$, $y(1) = 4 = 1 + c$, $c = 3$

$$\boxed{y(t) = t^2 + \frac{3}{t^2}}$$