

Math 222, Fall 2016.

Present your work in an organized fashion. Make sure that your work is algebraically correct and logically sound. Show all your work. No calculator, notes, or books.

Quiz 10/12/2016 M222-001

1. Use the method of reduction of order to find a second independent solution of the equation

$$t^2 y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}.$$

2. Find the solution of the IVP:

$$y'' + 4y = 3 \sin 2t, \quad y(0) = 2, \quad y'(0) = -1.$$

Solution: Problem 1: $y_2 = v(t)y_1$, $y_2' = v'y_1 + vy_1'$, $y_2'' = v''y_1 + 2v'y_1' + vy_1''$

$$t^2(v''y_1 + 2v'y_1' + vy_1'') + 3t(v'y_1 + vy_1') + vy_1 = 0$$

$$(t^2y_1)'' + (2t^2y_1' + 3ty_1)v' = 0, \quad t v'' + v' = 0, \quad \frac{v''}{v'} = -\frac{1}{t},$$

$$\ln v' = -\ln t = \ln \frac{1}{t}, \quad v' = \frac{1}{t}, \quad v = \ln t, \quad \boxed{y_2 = t^{-1} \ln t}$$

Problem 2: $y'' + 4y = 3 \sin 2t$, $y(0) = 2$, $y'(0) = -1$

$$y_c \Rightarrow y_c'' + 4y_c = 0, \quad r^2 + 4 = 0, \quad r = \pm 2i, \quad y_c = C_1 \cos 2t + C_2 \sin 2t$$

$$Y = t(A \cos 2t + B \sin 2t)$$

$$Y' = A \cos 2t + B \sin 2t + 2t(-A \sin 2t + B \cos 2t)$$

$$Y'' = -2A \sin 2t + 2B \cos 2t + 2(-A \sin 2t + B \cos 2t) + 4t(-A \cos 2t - B \sin 2t)$$

$$Y'' + 4Y = 4t(-A \cos 2t - B \sin 2t) - 4A \sin 2t + 4B \cos 2t + 4t(A \cos 2t + B \sin 2t) = 3 \sin 2t,$$

$$-4A = 3, \quad 4B = 0 \Rightarrow A = -\frac{3}{4}, \quad B = 0$$

$$y = C_1 \cos 2t + C_2 \sin 2t - \frac{3}{4} t \cos 2t$$

$$y(0) = 2 = C_1$$

$$y' = -2C_1 \sin 2t + 2C_2 \cos 2t - \frac{3}{4} \cos 2t + \frac{3}{2} t \sin 2t$$

$$y'(0) = -1 = -2C_1 \cdot 0 + 2C_2 - \frac{3}{4}$$

$$2C_2 - \frac{3}{4} = -1, \quad 2C_2 = -\frac{1}{4},$$

$$C_2 = -\frac{1}{8}$$

$$\boxed{y = 2 \cos 2t - \frac{1}{8} \sin 2t - \frac{3}{4} t \cos 2t}$$