Solutions to HW #1 M222 Spring 2016

Problem 1: $y' = 2y - 3$

(a) 1st order, linear differential equation

(b) $y = \frac{3}{2} + (c - \frac{3}{2})e^{2t}$, $y' = (c - \frac{3}{2}) \cdot 2 \cdot e^{2t}$

$y' = (c - \frac{3}{2}) \cdot 2 \cdot e^{2t}$

$2y - 3 = 2\left[\frac{3}{2} + (c - \frac{3}{2})e^{2t}\right] - 3 = 2 \cdot (c - \frac{3}{2})e^{2t} = y'$

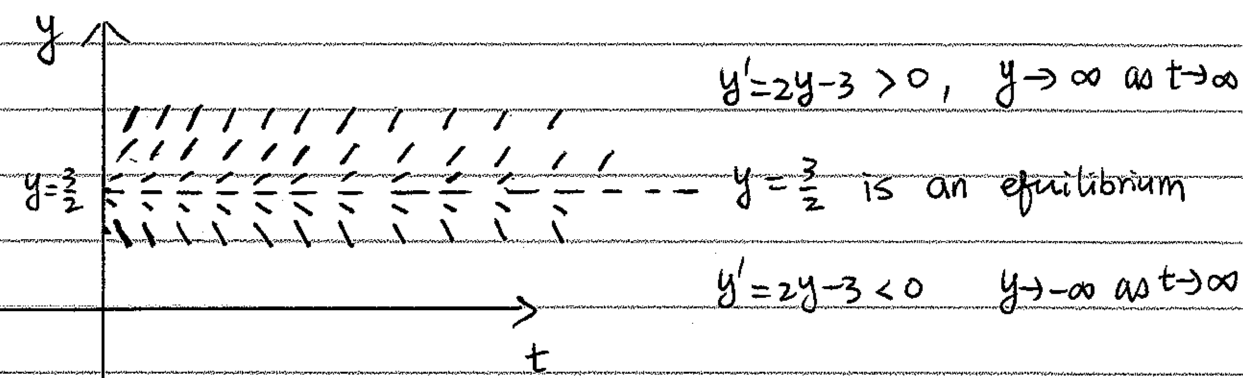
therefore, $y = \frac{3}{2} + (c - \frac{3}{2})e^{2t}$ is a solution

(c) $y = \frac{3}{2}$, $y' = 0$,

$y = \frac{3}{2}$, $2y - 3 = 3 - 3 = 0$ $\Rightarrow$ therefore, $y = \frac{3}{2}$ is a soluti

(d) when $c - \frac{3}{2} > 0$, $y \to +\infty$ as $t \to \infty$

when $c - \frac{3}{2} < 0$, $y \to -\infty$ as $t \to \infty$



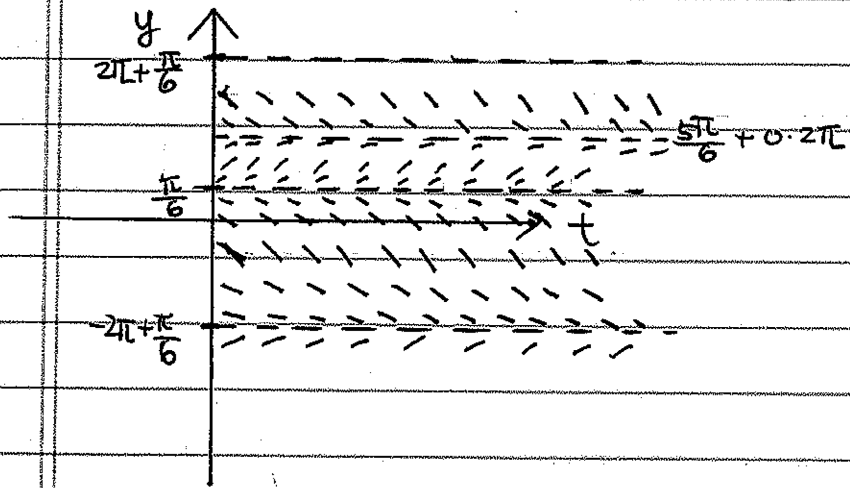
Problem 2: $y' = \sin(y) - a$

(a) 1st order, nonlinear differential equation

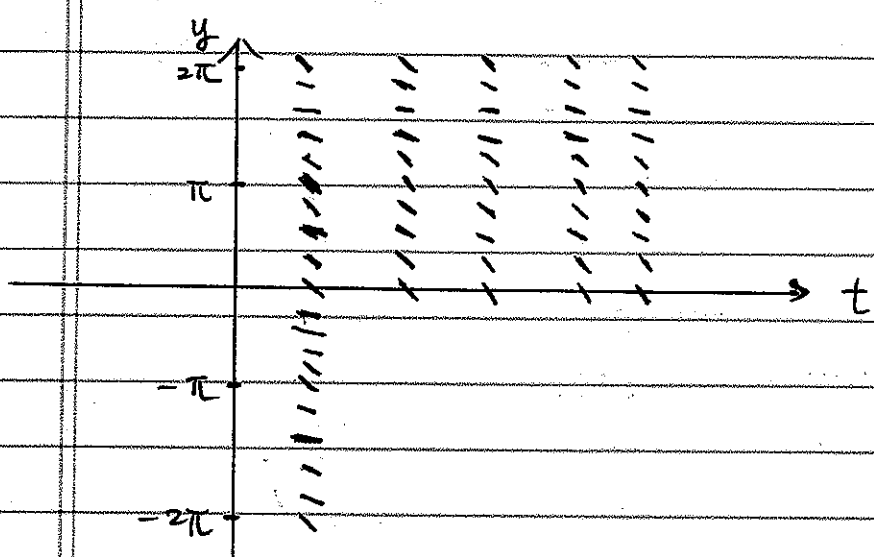
(b) $a = 0.5$

$y' = \sin y - 0.5$

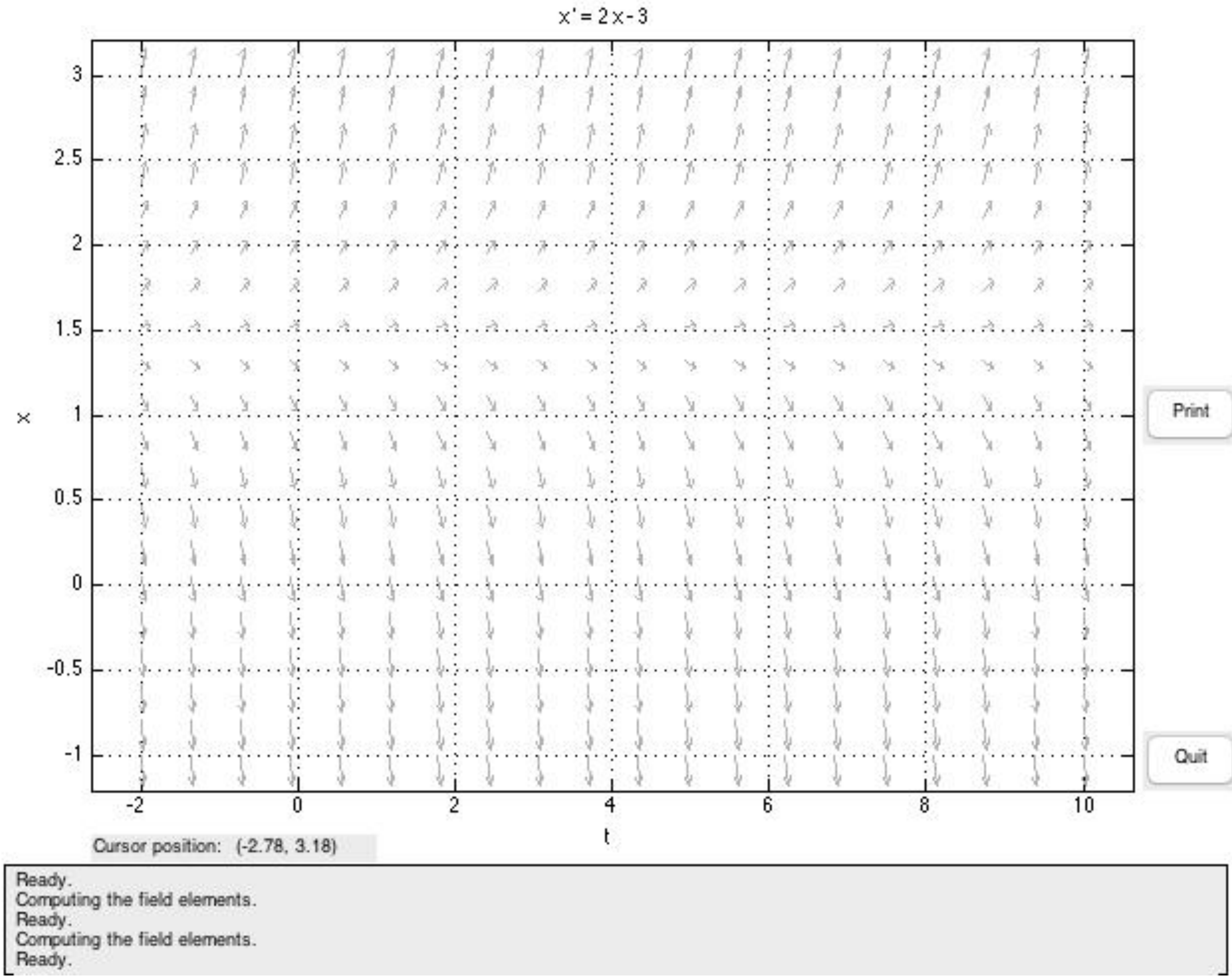
$\sin y = 0.5$, $\quad y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi$

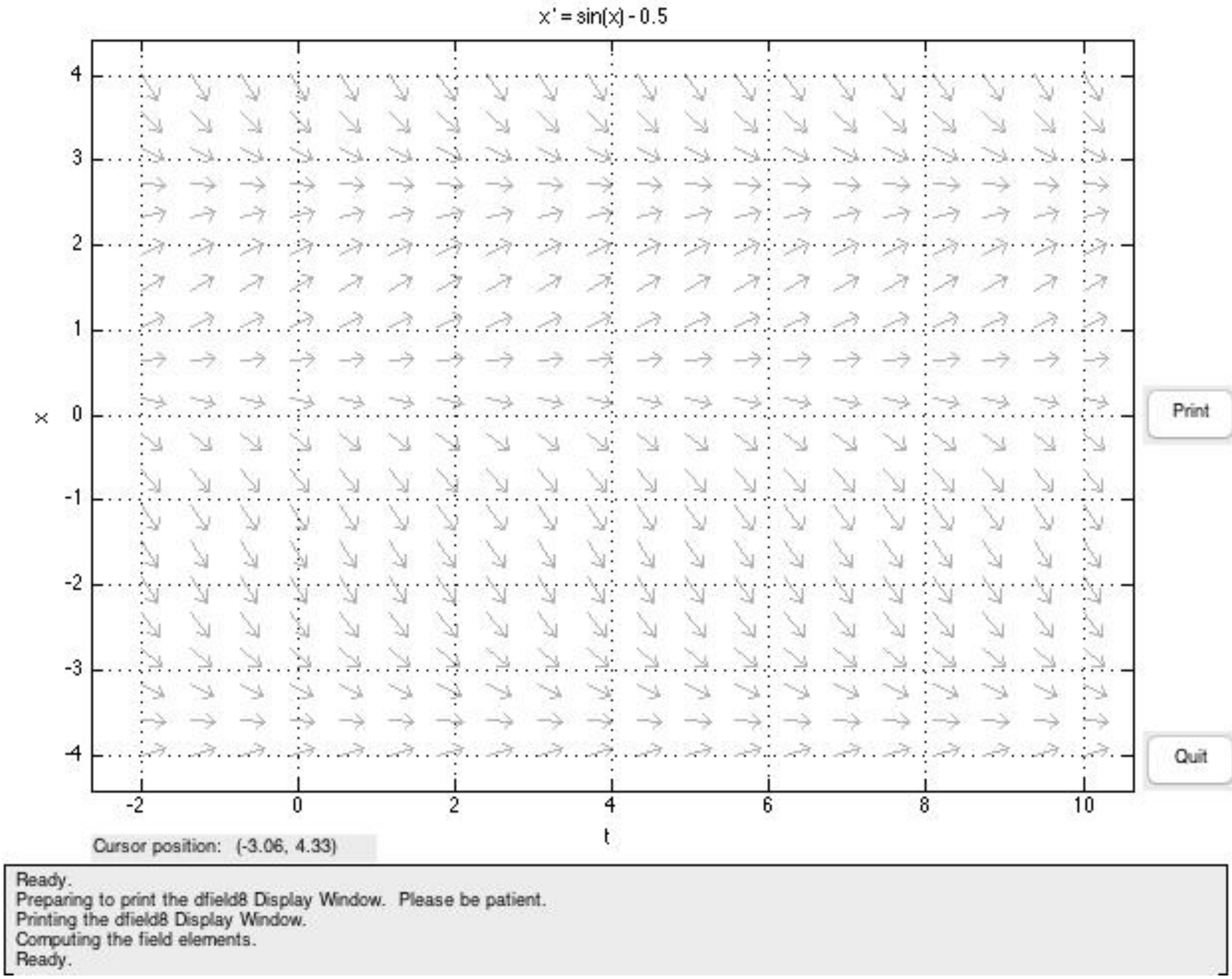


(c) $a = 1.5$ $\qquad y' = \sin y - 1.5 < 0$ for any value of y



(d) when a is greater than 1, there exists no equilibrium solution such that $y' = 0$


x' = 2 x - 3
3
2.5
2
1.5
1
0.5
0
-0.5
-1
x
-2
0
2
4
6
8
10
t
Print
Quit
Cursor position: (-2.78, 3.18)
Ready.
Computing the field elements.
Ready.
Computing the field elements.
Ready.



x' = sin(x) - 0.5
x
t
4
3
2
1
0
-1
-2
-3
-4
-2
0
2
4
6
8
10
Print
Quit
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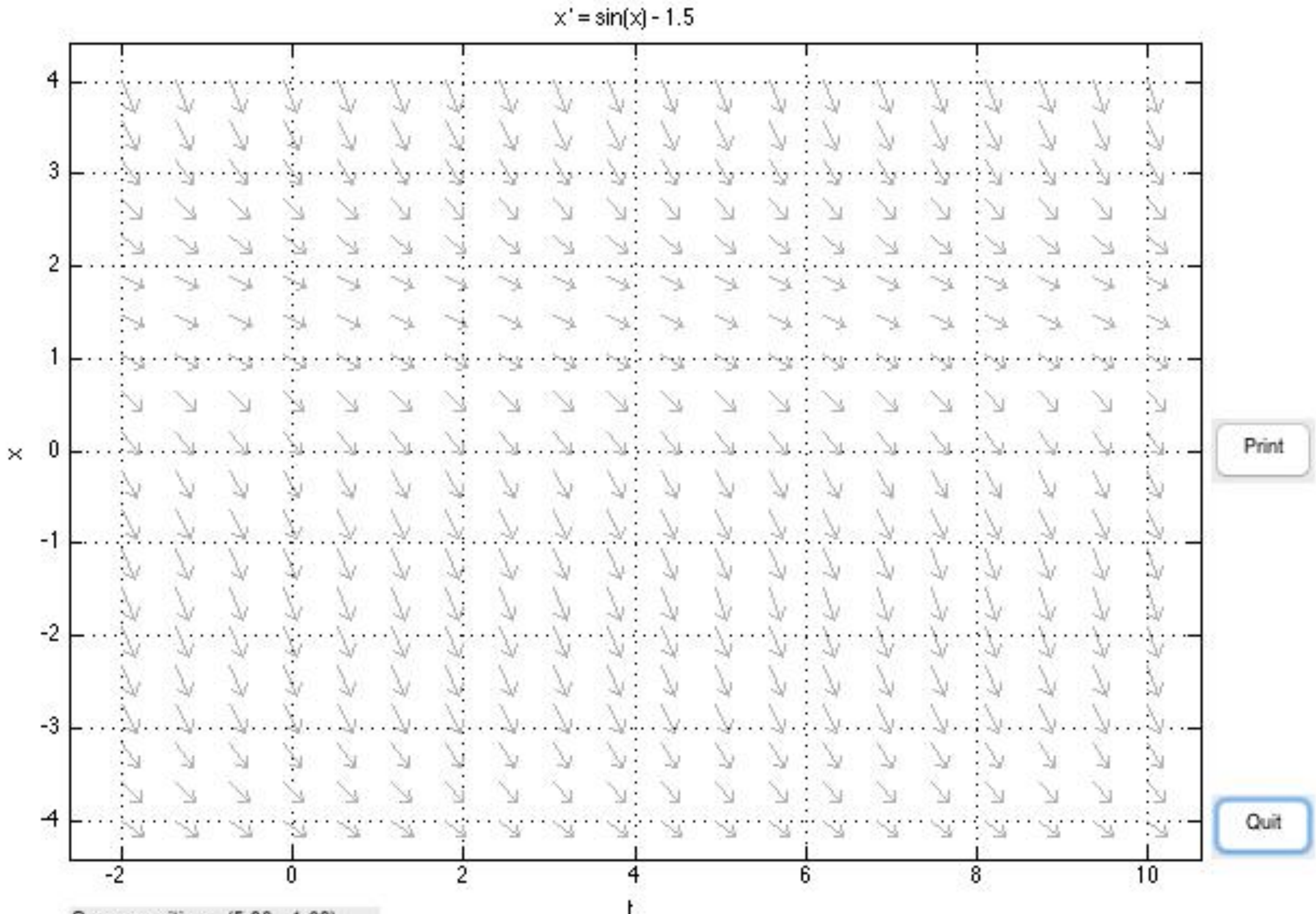
Preparing to print the dfield8 Display Window. Please be patient.
Printing the dfield8 Display Window.
Computing the field elements.
Ready.



Cursor position: (5.36, -1.92)

Computing the field elements.
Ready.
Preparing to print the dfield8 Display Window. Please be patient.
Printing the dfield8 Display Window.