Math 222, Spring 2016
Solutions Problems for Week 3

1. Solve problem 24 from Section 2.3 in your textbook.

Solution: (a) We are given that

$$\frac{dv}{dt} = -\mu v^2. \tag{1}$$

Substitute

$$\frac{dv}{dt} = v\frac{dv}{dx},$$

cancel one factor of v and obtain

$$\frac{dv}{dx} = -\mu v. \tag{2}$$

(b) 2000 feet are required to slow the speed from 150 mi/hr to 15 mi/hr where 2000 ft = 0.3787 mi. Thus $v(0) = 15$ and $v(0.3787) = 15$. Integrating equation (2), we find $v(x) = cexp(-\mu x)$. Use $v(0) = 150$ to find that $c = 150$. Then use $v(0.3787) = 15$ to find that $\mu = 6.08$ mi^{-1}. Note that x is the indepenedent variable.

(c) Now we solve equation (1) with $\mu = 6.08$, $v(0) = 150$. We want to find τ such that $v(\tau) = 15$. Note here that t is the independent variable. Integrating we find

$$v = \frac{1}{\mu t + c}. \tag{3}$$

Using the initital condition, we find $c = 1/150$. Then using $v(\tau) = 15$, we find that $\tau = 9/150\mu = 0.0098$ hrs. Multiply by 3600 seconds, we find that $\tau = 35.5 sec$.

2. Consider the following two equations

$$y' = 4 - 2t - 3y, \;\; y(0) = 1 \tag{4}$$

$$y' = 4 - 2t + 3y, \;\; y(0) = 1 \tag{5}$$

If we were to use Euler's method to numerically solve each of these equations, would you expect the method to be more accurate for equation (4) or (5), or equally accurate for both? Carefully explain why.

Solution: The only way the two eequations differ is in the sign of the coefficient of the linear term. The general solutions are

$$y(t) = \frac{14}{9} - \frac{2}{3}t + ce^{-3t} \tag{6}$$

$$y(t) = -\frac{14}{9} + \frac{2}{3}t + ce^{3t} \tag{7}$$

The solutions in equation (6) form a converging family since the term $ce^{-3t} \to 0$ as $t \to \infty$, while those in equation (7) form a diverginging family since the term $ce^{3t} \to \pm\infty$ as $t \to \infty$, depending on the sign of c. If we were to use Euler's method with the same step size h and solve for the same amount of time, then we would expect the method to give a better approximation for the converging family (6) rather than the diverging family (7). By this we mean that if we were to measure error at any time t by taking the absolute value of the difference from the numerical solution from the actual solution, this difference would be smaller for the converging family. We can still get a pretty good approximation for the solution (7) if we choose a smaller and smaller step size h. Please note that the problem as stated asked for whether Euler's method would be more or less accurate or the same when solving either problem. The term "accurate" has a specific mathematical meaning when it comes to numerical methods for solving differential equations involving how the the error is related to the step size h. You can learn more about this in section 8.1 of your text book or in Math 340. There you will learn that Euler's method is equally accurate for both equations.