Solutions to HW Week 8

Problem 1: $2k = 400$, $k = 200$

$m = 50$ kg,

$$50u'' + 200u = 0, \quad u(0) = 0, \quad u'(0) = -10.$$

$$u'' + 4u = 0, \quad u = A\cos 2t + B\sin 2t$$
$$u' = -2A\sin 2t + 2B\cos 2t$$

$u(0) = 0 = A$

$u'(0) = -10 = 2B$, $\boxed{u = -5\sin 2t}$

Problem 2: $1k = 2$, $k = 2$

$$m = \frac{3.2}{32} = \frac{1}{10}$$

$$0.1u'' + 0.4u' + 2u = 0$$

(a) $u'' + 4u' + 20u = 0$, $u(0) = -1$, $u'(0) = 0$

(b) $r^2 + 4r + 20 = 0$, $(r+2)^2 + 16 = 0$, $r = -2 \pm 4i$

$u = e^{-2t}(A\cos 4t + B\sin 4t)$

$u' = -2e^{-2t}(A\cos 4t + B\sin 4t) + e^{-2t}(-4A\sin 4t + 4B\cos 4t)$

$u(0) = A = -1$

$u'(0) = 0 = -2(-1) + 4B$, $B = \frac{1}{2}$

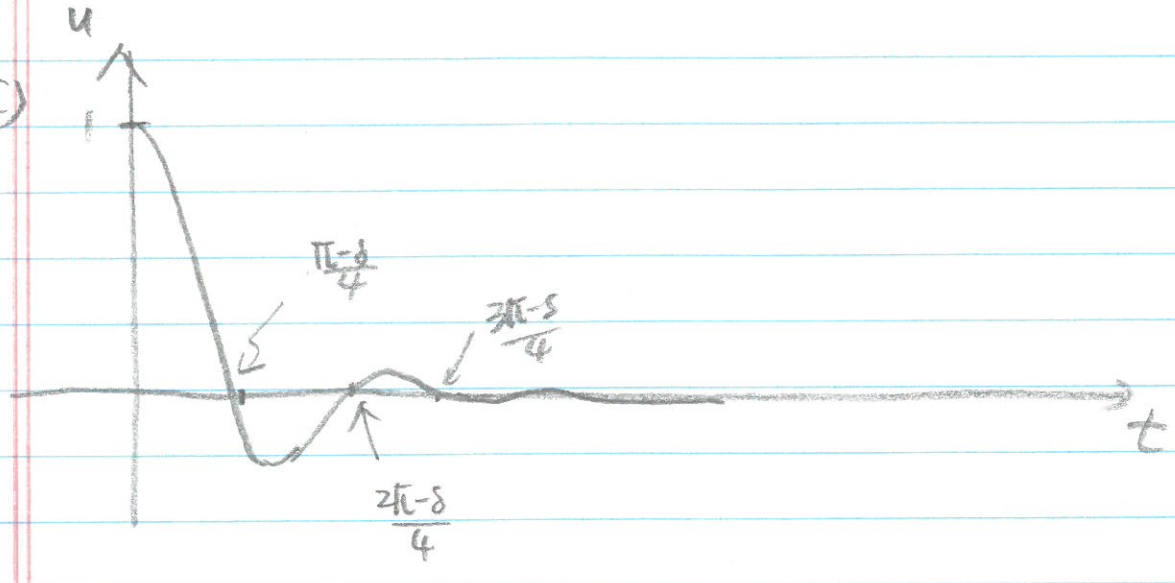
$$u(t) = e^{-2t}\left(-\cos 4t + \frac{1}{2}\sin 4t\right)$$

$$= \sqrt{\frac{5}{4}}\, e^{-2t}\left(-\frac{1}{\sqrt{5/4}}\cos 4t + \frac{1/2}{\sqrt{5/4}}\sin 4t\right) = \sqrt{\frac{5}{4}}\, e^{-2t}\sin(4t + \delta)$$

$\delta = \tan^{-1}(2)$

P.2

(c)



$4t+\delta=\pi$

$t=(\pi-\delta)/4$

Problem 3:

(a) $L=1\,H,\ R=2\Omega,\ C=\frac{1}{4}f,\ E=50\cos t)\ V$

$q''+2q'+4q=50\cos t$

$r^2+2r+4=0,\quad (r+1)^2+3=0,\quad r=-1\pm\sqrt{3}i$

$q_c=e^{-t}(A\cos\sqrt{3}t+B\sin\sqrt{3}t)$

$Q=C\cos t+D\sin t$

$Q'=-C\sin t+D\cos t$

$Q''=-C\cos t-D\sin t$

$-C\cos t-D\sin t+2(-C\sin t+D\cos t)+4(C\cos t+D\sin t)=50\cos t$

$-C+2D+4C=50,\quad \begin{cases} 3C+2D=50 & \times 2 \\ -2C+3D=0 & \times 3 \end{cases}$

$D=\frac{100}{13}$

$C=\frac{150}{13}$

$q(t)=e^{-t}(A\cos\sqrt{3}t+B\sin\sqrt{3}t)+\frac{150}{13}\cos t+\frac{100}{13}\sin t$

as $t\to\infty$, $q(t)\to\frac{150}{13}(3\cos t+2\sin t)$

P.3

(b) $q' = i$,

$L i' + R i + \frac{1}{C} q = E$

$L i'' + R i' + \frac{1}{C} i = E'$, $E' = -50\sin t$

$i = e^{-t}(\alpha \cos\sqrt{3}t + \beta \sin\sqrt{3}t) + \underbrace{E\cos t + F\sin t}_{I}$

$I' = -E\sin t + F\cos t$

$I'' = -E\cos t - F\sin t$

$-E\cos t - F\sin t + 2(-E\sin t + F\cos t) + 4(E\cos t + F\sin t) = -50\sin t$

$-E + 2F + 4E = 0$ $\quad 2F + 3E = 0$ $\quad \times 2$

$-F - 2E + 4F = -50$ $\quad 3F - 2E = -50$ $\quad \times 3$

$(4+9)F = -50 \times 3$, $F = -\frac{150}{13}$

$-\frac{300}{13} + 3E = 0$, $E = \frac{100}{13}$

$i = e^{-t}(\alpha \cos\sqrt{3}t + \beta \sin\sqrt{3}t) + \frac{100}{13}\cos t - \frac{150}{13}\sin t$

$t \to \infty$, $i(t) \to \frac{50}{13}(-3\sin t + 2\cos t)$