Math 222, Spring 2016.

Present your work in an organized fashion. Make sure that your work is algebraically correct and logically sound. Show all your work. Discussion (if necessary) with others is encouraged, while copying other's solution is a violation of NJIT student honor code. Do not forget that you should also be able to do (but not hand in) the homework problems listed on the syllabus.

Homework Problems for Chapter 7

1. Consider the equation
$$ay'' + by' + cy = 0,$$
where a, b and c are constants with $a \neq 0$. In Chapter 3 it was shown that the general solution depended on the roots of the characteristic equation
$$ar^2 + br + c = 0,$$
where r is the exponent in the ansatz $y \sim e^{rt}$.

   (a) Transform the second order ODE into a system of first order equations by letting $x_1 = y$, $x_2 = y'$. Find the system of equations $\mathbf{x}' = \mathbf{A}\mathbf{x}$ satisfied by $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

   (b) Find the equation that determines the eigenvalues of the coefficient matrix $\mathbf{A}$ in part (a). Note that this equation is just the characteristic equation of the second order ODE.

2. Convert each linear equation into a system of first-order equations.

   (a) $y'' - 4y' + 6y = 0$.

   (b) $y''' + 5y'' + 7y' - 9y = 5t^3 \cos(2t)$.

3. Suppose a square 2 by 2 matrix $\mathbf{A}$ has eigenvalues 5 and -4, with corresponding eigenvectors $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, respectively. What is the general solution of the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$?

4. Find the solution to
$$\mathbf{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$
Describe the behavior of the solution as $t \to \infty$.