Illustration of using dfield8.m to draw direction field

Example 1: $2x' + x = 3t^2$

$x' + \frac{1}{2}x = \frac{3}{2}t^2, \quad \mu' = \frac{1}{2}\mu, \quad \mu = e^{\frac{t}{2}}$

$\left(e^{\frac{t}{2}}x\right)' = \frac{3}{2}t^2 \cdot e^{\frac{t}{2}}, \quad e^{\frac{t}{2}}x = \int \frac{3}{2}t^2 e^{\frac{t}{2}}dt$

$u = t^2, \quad v' = e^{\frac{t}{2}}$

$u' = 2t, \quad v = 2e^{t/2}$

$e^{\frac{t}{2}}x(t) = \frac{3}{2}\left[t^2 \cdot 2 \cdot e^{t/2} - \int 4te^{t/2}dt\right]$

$= 3t^2e^{t/2} - 6\int te^{t/2}dt$

$u = t, \quad v' = e^{t/2}$

$u' = 1, \quad v = 2e^{t/2}$

$= 3t^2e^{t/2} - 6\left(t \cdot 2 \cdot e^{t/2} - \int 2e^{t/2}dt\right)$

$= 3t^2e^{t/2} - 12te^{t/2} + 12\int e^{t/2}dt$

$e^{t/2}x(t) = 3t^2e^{t/2} - 12te^{t/2} + 24e^{t/2} + c$

$\boxed{x(t) = 3t^2 - 12t + 24 + c \cdot e^{-t/2}}$

Example 2: $\frac{dy}{dx} = \frac{x^2}{1-y^2} \Rightarrow \frac{dx}{dt} = \frac{t^2}{1-x^2}$

page 43 $\Rightarrow \boxed{-t^3 + 3x - x^3 = c}$

Example 3: $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$

$\Leftrightarrow \frac{dx}{dt} = \frac{3t^2+4t+2}{2(x-1)}, \quad x(0) = -1$

page 45 $\Rightarrow \boxed{x(t) = 1 - \sqrt{x^3+2x^2+2x+4}}$